## Direct Variation \& Partial Variation

## Direct Variation

- Passes through $(0,0)$ / origin
- Does NOT have an initial value / fixed cost


## Partial Variation

- Does NOT pass through $(0,0)$ / origin
- Has an initial value / fixed cost


Direct / Partial


Direct / Partial


Direct / Partial


Direct / Partial
Direct / Partial
Direct / Partial

9 | Hours | Cost |
| :---: | :---: |
| 0 | 0 |
| 1 | 10 |
| 2 | 20 |
| 3 | 30 |

Direct / Partial

$\downarrow$|  | $\begin{array}{l}\text { Direct / Partial }\end{array} \underbrace{-}+100$ |
| :---: | :---: |


$h \quad$|  |  |
| :---: | :---: |
|  | Hours |
| 0 | Cost |
| 1 | 15 |
| 2 | 30 |
| 3 | 45 |

Direct / Partial


Direct / Partial

| Hours | Cost |
| :---: | :---: |
| 0 | 40 |
| 1 | 90 |
| 2 | 140 |
| 3 | 190 |

Direct / Partial

$$
C=70-10 n
$$

Direct / Partial

2
Scarlett and Thomas both have jobs working at different stores in the mall. If they both work a 5 hour shift, they each earn $\$ 100$.
a) Predict how much you think they will each earn if they work a 10 hour shift.
b) Scarlett earns $\$ 20$ per hour at her job. Complete the following table to show her earnings.

| Hours <br> Worked | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Money <br> Earned |  |  |  |  |  |  |  |  |  |  |  |

c) Thomas earns $\$ 40$ each shift, plus $\$ 12$ per hour. Complete the following table to show his earnings.

| Hours <br> Worked | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Money <br> Earned |  |  |  |  |  |  |  |  |  |  |  |

d) Was your prediction for both Scarlett and Thomas correct? Explain.
e) Determine which of the following is a direct variation and which is a partial variation. Explain.

Kelly earns $\$ 50$ when she works 5 hours, and $\$ 80$ when she works 8 hours.

A moving company charges $\$ 270$ for a 3 hour move and $\$ 390$ for a 5 hour move.

3 The following graph shows the cost to hire a mechanic, where C is the cost and n is the number of hours worked.
a) Is this an example of direct or partial variation? Justify your answer.
b) What is the fixed cost? What does it represent?
c) What is the rate of change? What does it represent?
d) Write an equation for the relation.
e) Predict the cost of repair is the mechanic worked for 10 hours.
f) How long did the repairs take if it costs $\$ 365.00$ to repair your car?


## Tricky Questions

4 Determine the initial value of each of the following and state whether it is an example of direct or partial variation.


Determine the missing values in the following tables. State whether it is an example of direct or partial variation.

| \# of People | Cost |
| :---: | :---: |
| 0 | 200 |
| 10 |  |
| 20 |  |
| 30 |  |
| 40 | 1000 |


| Hours | Temperature |
| :---: | :---: |
| 0 |  |
| 2 | 15 |
| 7 | 18 |
| 15 | 25 |

## Types of Variation

## Direct Variation

No flat fee or initial cost.

## Partial Variation

Has a flat fee or initial cost.

For each of the following, determine whether they are examples of direct or partial variation.


Direct Partial

| A painter charges $\$ 50$ |
| :---: |
| per hour. |

Direct Partial

| Hours | Cost |
| :---: | :---: |
| 0 | 0 |
| 1 | 10 |
| 2 | 20 |
| 3 | 30 |

Direct Partial


Direct/Partial


Direct Partial)

A moving company charges a fee o $\$ 100$ plus $\$ 75$ per hour.

Direct/Partial

| Hours | Cost |
| :---: | :---: |
| 0 | 0 |
| 1 | 15 |
| 2 | 30 |
| 3 | 45 |

Direct/ Partial


Direc) / Partial


Direct Partial

A banquet hall charges a $\$ 700$ booking feefand $\$ 30$ per person.

Direct Partial

| Hours | Cost |
| :---: | :---: |
| 0 | 40 |
| 1 | 90 |
| 2 | 140 |
| 3 | 190 |

Direct $\langle$ Partial


Direct Partial

Variation - Why Does It Matter?
2 Scarlett and Thomas both have jobs working in different stores at the mall. If they both work for a 5 hour shift, they each earn $\$ 100$.
a) Predict how much you think they will each earn if they work a 10 hour shift.

$$
\begin{aligned}
& 100 \times 2 \\
& =\$ 200
\end{aligned}
$$

$\therefore$ I predict that they will earn $\$ 200^{\circ}$
b) Scarlett earns $\$ 20$ per hour. Complete the following table showing her earnings.

| Hours <br> Worked | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Money <br> Earned | 0 | 20 | 40 | 60 | 80 | 100 | 120 | 140 | 160 | 180 | 200 |

c) Thomas earns $\$ 40$ each shift, plus $\$ 12$ per hour. Complete the following table showing his earnings.

| Hours <br> Worked | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Money <br> Earned | 40 | 52 | 64 | 76 | 88 | 100 | 112 | 124 | 136 | 148 | 160 |

d) Was your prediction correct for both Scarlett and Thomas? Explain.

No. My prediction was correct for Scarlett, but not for Thomas. Thomas is partial variation.
e) Determine which of the following are direct variation, and which are partial.
i) Kelly earns $\$ 50$ when she works 5 hours, and $\$ 80$ when she works 8 hours.

$$
\begin{aligned}
& \frac{50}{5}=10 \mathrm{r} \quad \frac{80}{8}=10^{2} \\
& \therefore \text { Direct Variation }
\end{aligned}
$$

ii) A moving company charges $\$ 270$ for a 3 hour move and $\$ 390$ for a 5 hour move.

$$
\frac{270}{3}=90 \quad \frac{390}{5}=78
$$

$\therefore$ Partial Variation

The following graph shows the cost of hiring a mechanic, where C is the cost and n is the number of hours that the mechanic spends working on your car.
a) Is this an example of direct or partial variation? Justify your answer.
Partial Variation. The graph does not start at $(0,0)$
b) What is the initial value of the graph? What does it represent?
$\$ 75^{\circ}$
The service fee to bring the car in.
c) What is the rate of change of the graph? What does it represent?

$$
+\begin{gathered}
\left.\left.\begin{array}{c}
h \mid c \\
\hline 0 \\
2
\end{array} \right\rvert\, \begin{array}{c}
155
\end{array}\right)+80
\end{gathered}
$$

$$
\begin{aligned}
R O C & =\frac{\Delta C}{\Delta h} \\
& =\frac{80}{2} \\
& =\$ 40 / \mathrm{hr} .
\end{aligned}
$$

This is the cost $\frac{5}{5}$ per hour for labour.
d) Write the equation of the cost.

$$
c=75+40 h
$$

e) Predict the cost if it takes 10 hours to repair your car.

$$
\begin{aligned}
& C=75+40(10) \\
& C=75+400 \quad \therefore \text { It will } \cos .475 . \\
& C=475 \quad \$ 4 .
\end{aligned}
$$

f) How long did the repairs take if it cost $\$ 365.00$ to repair your car?


$$
\begin{aligned}
& 365=75+40 h \\
& 290=40 h \\
& 7.25=h
\end{aligned}
$$

$\therefore$ It took 7.25 hours

Tricky Questions
Determine the initial value of each of the following and state whether it is an example of direct or partial variation.

| Temperature <br> $\left({ }^{\circ} \mathbf{C}\right)$ | Height <br> $(\mathbf{c m})$ |
| :---: | :---: |
| -4 | 3 |
| -2 | 8 |
| 0 | 13 |
| 2 | 18 |
| 4 | 23 |

$I V=13 \mathrm{~cm}$
Partial Variation

$N=\$$
Athene Variation Direct


$$
N=155
$$

Partial Variation.

5 Determine the missing values in the following tables:

$+40$| \# of People | Cost |
| :---: | :---: |
| 0 | 200 |
| 10 | 400 |
| 20 | 600 |
| 30 | 800 |
| 40 | 1000 |

$$
\begin{aligned}
R O C & =\frac{800}{40} \\
& =\$ 0 \$ / \text { person }
\end{aligned}
$$

$$
20 \times 10=200
$$

Partial Variation.


$$
\begin{aligned}
\operatorname{ROC} & =\frac{10}{5} \\
& =2^{\circ} \mathrm{C} / \mathrm{hr} \quad C=11+2 n
\end{aligned}
$$

Partial Variation.

