

Optimizing Perimeter and Area

Example 1 A park worker has 32 m of fencing to build a rectangular pen for rabbits. What is the **maximum** area that she can provide for the rabbits?

Complete the table

| Length (m) | Width (m) | Area = $l * w$ | Perimeter = $2l + 2w$ |
|------------|-----------|----------------|-----------------------|
| 1 | 15 | $15m^2$ | 32m |
| 2 | 14 | $28m^2$ | |
| 3 | 13 | $69m^2$ | |
| 4 | | | |
| 5 | | | |
| 6 | | | |
| 7 | | | |
| 8 | | | |
| 9 | | | |
| 10 | | | |
| 11 | | | |
| 12 | | | |
| 13 | | | |
| 14 | | | |
| 15 | | | |

What type of “rectangle” maximizes the Area?

Example 2: A park worker is to build a rectangular pen for rabbits with an area of 36 m^2 . What is the **minimum** length of fencing he needs for the project?

Complete the table

| Length (m) | Width (m) | Area = $l * w$ | Perimeter = $2l + 2w$ |
|------------|-----------|----------------|-----------------------|
| 1 | 36 | 36m^2 | |
| 2 | 18 | 36m^2 | |
| 3 | | | |
| 4 | | | |
| 6 | | | |
| | | | |
| | | | |

What type of “rectangle” minimizes the perimeter?

Do you think this is always true? Consider the next Example:

Example 3: A hobby farmer is creating a fenced exercise yard for her horses. She has 900 m of flexible fencing and wishes to maximize the area. She is going to fence a rectangular or a circular area. Determine which figure encloses the greater area.

Rectangular

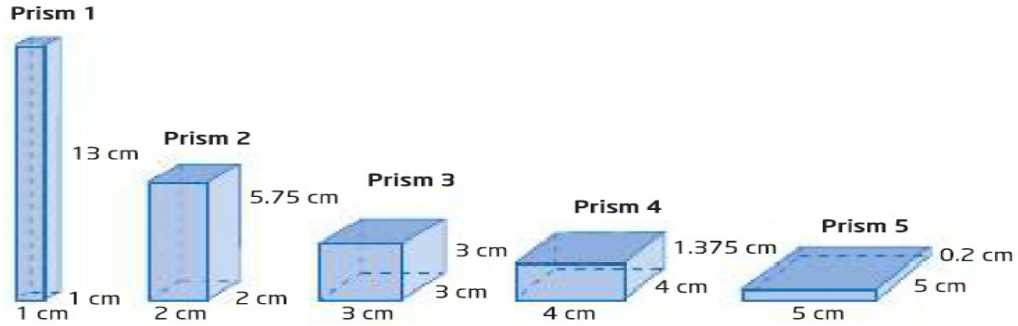
1. What type of rectangle maximizes the area?
2. What’s the formula for the perimeter of this object?
3. Find the length of the sides using the formula (substitute 900 into P).
4. Use the Area formula to find the area.

Circular

1. What’s the formula for perimeter of a circle?
2. Find Radius (Use formula and substitute 900 into C)
3. Use area formula to find the area.

Maximizing Volume and Minimizing Surface Area of 3D Object

1. Using the diagrams provided fill in the table below.



| Prism | Length | Width | Height | Surface Area | Volume |
|-------|--------|-------|--------|--------------|--------|
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |

2. Which square based prism gives you the largest volume? Why is this important to know?

3. Fill in the table provided using the information given.

| Prism | Length | Width | Height | Surface Area | Volume |
|-------|--------|-------|--------|--------------|--------|
| 1 | 1 | 1 | 64 | | |
| 2 | 1 | 2 | 32 | | |
| 3 | 1 | 4 | 16 | | |
| 4 | 1 | 8 | 8 | | |
| 5 | 2 | 2 | 16 | | |
| 6 | 2 | 4 | 8 | | |
| 7 | 4 | 4 | 4 | | |

4. Which prism gives you the smallest surface area? Why is this important to know?

5. What are the formulas for surface area and volume of an optimized prism?

Prisms versus Cylinder



1. Your task is to design a can that uses no more than 375 cm^2 of aluminum.

1. Write down surface area formula
2. Fill in the surface area as that is the same for all of our cans.
3. Rearrange the formula to solve for h.
4. Now you have a general formula for h.

Let's complete the first one in the table together.

1. Find h, using the formula from above.
2. Now use Volume formula to find the volume.

| Cylinder | Radius | Height | Volume | Surface Area |
|----------|--------|--------|--------|--------------------|
| 1 | 1 | | | 375 cm^2 |
| 2 | 2 | | | 375 cm^2 |
| 3 | 3 | | | 375 cm^2 |
| 4 | 4 | | | 375 cm^2 |
| 5 | 5 | | | 375 cm^2 |

What will be the relation between radius and height that will maximize the volume of a cylinder?

2. Your task is to design a can that has the least surface area but with a volume of 500 cm^3

1. Use the volume formula and substitute the known volume.
2. Solve for h.
3. Now you have a general formula for h.

| Cylinder | Radius | Height | Surface Area | Volume |
|----------|--------|--------|--------------|--------------------|
| 1 | 1 | | | 500 cm^3 |
| 2 | 2 | | | 500 cm^3 |
| 3 | 3 | | | 500 cm^3 |
| 4 | 4 | | | 500 cm^3 |
| 5 | 5 | | | 500 cm^3 |

What is the relation between radius and height that will minimize surface area?

What other shape will do an even better job of maximizing volume and minimizing the surface area?
Why is that shape not used by pop-can manufacturers?