Trigonometric Functions Unit 6

Tentative TEST date_______________

Big idea/Learning Goals

In this unit you will study trigonometric functions from grade 11, however everything will be done in radian measure. Just like length can be measured in meters or yards, angles can be measured using different units of measure. Radian measure, as you will learn, actually has no units at all. You will review primary and secondary trigonometric ratios, contrast reciprocal trig functions with inverse trig function, and graph these functions with and without transformations. You will once again use the unit circle to find solutions of the angles as well as use special triangles to find exact values for ratios – all of this in radian measure now. Finally you will again apply the knowledge of sinusoidal functions of grade 11 to solve real life word problems as well as review rates of change with trigonometric functions.

Corrections for the textbook answers:
Sec 6.1 #8h)covert angle to a positive by adding period #9b) 81.25 #16 86.8 rad/sec
Sec 6.4 #5b) no neg on 6 #9b) 50 beats/min
Sec 6.6 #10a) y=3.7cos[2pi/365(x-172)]+12 b) 9.16 #11 using months y=16.2cos(2pi/12(x-7))+1.4 where x=1 is January
Sec 6.7 #9a)shift for cosine 22 b) fast at 4, 16, 28,… slow at 10,22, …
Review #16 y= 30sin[5pi/3(t-0.3)]+150

Success Criteria

- ☐ I understand the new topics for this unit if I can do the practice questions in the textbook/handouts

Date	pg	Topics	# of quest. done? You may be asked to show them	**Questions I had difficulty with** ask teacher before test!
	2-3	Review Radicals and Unit Circle TWO Handouts		
	4-7	Radian Measure – 2 days Section 6.1 & THREE Handouts		
	8-9	Exact Values & Other Questions Section 6.2 (SKIP equivalent expressions for now – covered in the next unit) & THREE Handouts		
	10-12	Sketching Sinusoidals Section 6.4 & Handout		
	13-15	Parent Graphs of Trig Functions & Transformations of Trig Functions Section 6.3 & 6.5 & TWO Handouts		
	16-17	Word Problems with Sinusoidals Section 6.6 & TWO Handouts		
	18-19	Rates of Change Section 6.7		
		REVIEW		

Reflect – previous TEST mark ___________, Overall mark now___________.

Review Radicals and Unit Circle

1. Summarize the rules of simplifying radicals: How to reduce, how to add/subtract/multiply/divide and rationalize. Use the following examples in your explanations.

a. $3\sqrt{32}$

b. $2\sqrt{12}+\sqrt{48}-5\sqrt{175}$

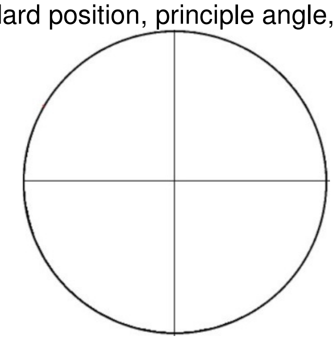
c. $3\sqrt{12}\times4\sqrt{6}$

d. $\dfrac{4\sqrt{32}}{12\sqrt{12}}$

e. $\dfrac{2}{\sqrt{6}}$

g. $\dfrac{4}{\sqrt{2}-5\sqrt{3}}$

2. Review the following by stating a definition or by showing on a diagram: (terminal arm, initial position, positive angles, negative angles, coterminal position, standard position, principle angle, related acute angle)



3. Explain the signs of primary trig ratios by CAST or by their definition.(It is important to know the definitions since CAST doesn't work for angles on the x&y axes.)

4. Predict whether each value will be positive or negative without using the calculator.

a. $\tan 195^o$ b. $\sin(-115^o)$ c. $\cos 670^o$

5. Find the following ratios without using the calculator.

a. $\cos(-90^o)$ b. $\cos 180^o$ c. $\tan 270^o$ d. $\sin 360^o$

6. For the ratio $\sin\theta = -\frac{2}{5}$, the angle θ is in standard position $0^o \le \theta \le 360^o$.

a. How many answers for θ are there?

b. Is θ acute, obtuse or reflex in Quadrant III or reflex in Quadrant IV?

c. Find all possible measures of θ in the given domain.

7. For the point $P(-2,6)$

a. Sketch the angle, θ, in standard position

b. Find $\cot\theta$

c. Find the angle θ.

Radian Measure

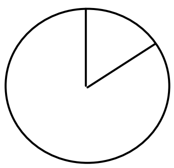
Just like when you measure length, where you can use centimeters or inches, you can measure the angles using different systems of measure. You can use:

- Degrees, or
- Revolutions, or
- Radians

Degrees and Revolutions

Degrees and revolutions are simple, look at the following example to see if you understand these two systems:

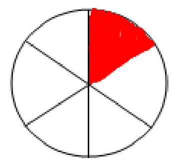
1. 60^o is equivalent to $\frac{1}{6}$ of a revolution



This is because 1revolution = 360^o

So to convert 60^o you need to multiply it by the ratio of $\frac{1rev}{360^o}$

$60^o \times \frac{1rev}{360^o} = \frac{1}{6} rev$ (degree units will cancel)



This should make sense since there are 6 sections of 60^o that make a full revolution of 360^o.

2. $\frac{5}{4}$ revolution is equivalent to 450^o

To convert $\frac{5}{4} rev$ you need to multiply it by the ratio of $\frac{360^o}{1rev}$

$\frac{5}{4} rev \times \frac{360^o}{1rev} = \frac{1800^o}{4} = 450^o$ (*rev* will cancel)

NOTE

How do you decide which ratio to multiply by?

Well, you should look at units.

If you want to

get rid of degrees

put the degrees in the ratio on top/bottom so that they can cancel

get rid of revolutions

put the *rev* on on top/bottom so that they can cancel

Notice that in both examples we multiplied by a ratio that is equivalent to ONE

$\frac{360^o}{1rev}$ and $\frac{1rev}{360^o}$ since $1rev = 360^o$

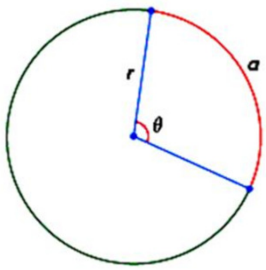
3. Convert 125^o to revolutions

4. Convert 2.4 revolutions to degrees

Radians

Unlike degrees and revolutions, radian measure doesn't have any units at all. It is a ratio of lengths.

Definition of radian ratio is the following formula. (It is not derived from anything; we define it to be this ratio.)



$$\text{angle (in radians)} = \frac{\text{arc length}}{\text{radius}}$$

$$\theta = \frac{a}{r}$$

Let's figure out how to convert radians to degrees or revolutions.

Consider a full revolution. Arc length will be the full circumference ($C = 2\pi r$) and the angle in degrees for a full revolution is 360°.

$$\theta = \frac{a}{r}$$

$$360^\circ = \frac{2\pi r}{r} \quad \text{(radius cancels)}$$

$$360^\circ = 2\pi \quad \text{or} \quad 180^\circ = \pi$$

Since 1revolution = 360°=2π, you can convert from one measure to the next by using the following ratios that are equivalent to ONE.

$$\frac{360^\circ}{1rev} \qquad \frac{1rev}{2\pi} \qquad \frac{2\pi}{360^\circ}$$

or reciprocals of these (in these ratios use π as 3.14)

NOTE

We just found that $\pi = 180^\circ$

How can π be 3.1415… AND 180°?

3.14 and 180 are not the same numbers and so are not equal, however look at units. 180 has a degree symbol and 3.14 doesn't.

So π can be replaced with

- 3.14 if you want to use radians (no units) or
- 180° when you want to use degrees.

Notice that radians will not have any units. Look at the following example to see this.

5. Arc length is 4.58cm and radius is 1.5cm.
   a) What is the angle in radians?

$$\theta = \frac{a}{r}$$

$$\theta = \frac{4.58cm}{1.5cm}$$

$$\theta \approx 3.05 \qquad \text{no units!! (centimeters cancel)}$$

   b) Convert to degrees

$$3.05 \times \frac{360^\circ}{2\pi} = \frac{1098^0}{2\pi} \approx 175^\circ$$

   c) Convert to revolutions.

$$3.05 \times \frac{1rev}{2\pi} = \frac{3.05rev}{2\pi} \approx 0.48rev$$

NOTE

Remember radians have no units at all and so if converting to radians try to cancel all units.

6. Convert 125° to radians

7. Convert 2.4 revolutions to radians

8. What is the arc length if the angle is 65° and the radius is 4cm

Linear and Angular Velocities

Let's use the following variables:

$t =$ time (sec, min, hr)

$a =$ arc length (mm, cm, m, km)

$\theta =$ angle (degrees, revolutions, radians)

$v =$ linear velocity/speed (cm/sec, km/hr, ...)

$\omega =$ angular velocity/speed (^{o}/sec, rev/sec, rad/sec, rpm, ...

- Angle in radians is $\theta = \frac{a}{r}$ by definition.
- Linear velocity is $v = \frac{a}{t}$ since speed is distance divided by time (here distance travelled would be the arc length)
- Angular velocity is $\omega = \frac{\theta}{t}$ by definition.

Deriving a formula for ω in terms of v and r :

rewrite the angle and the linear velocity formulas

$a = \theta r$ and $a = vt$

equating these

$\theta r = vt$

solving for θ

$$\theta = \frac{vt}{r}$$

substituting this into the angular velocity formula

$$\omega = \frac{\left(\frac{vt}{r}\right)}{t} = \frac{vt}{r} \div t = \frac{vt}{r} \times \frac{1}{t} = \frac{v}{r} \qquad \text{therefore } \omega = \frac{v}{r}$$

Here are all the formulas that you will need and all the ratio conversions that you may need.

$$\theta = \frac{a}{r} \qquad \omega = \frac{\theta}{t} \qquad v = \frac{a}{t} \qquad \omega = \frac{v}{r}$$

$$\frac{2\pi}{360^o} \quad \frac{360^o}{1rev} \quad \frac{1rev}{2\pi} \quad \frac{100cm}{1m} \quad \frac{1000m}{1km} \quad \frac{60\sec}{1\min} \quad \frac{60\min}{1hr}$$

9. Suppose a 150cm diameter wheel is rotating at 25 rpm. At what rate is the wheel moving along the road in m/hr?

10. A bicycle has 70cm wheel diameter, how many rotations per second does the cyclist have to achieve to push the bicycle along a flat surface at 25km/hr?

11. Find the angular velocity in radians/sec of a point on a water wheel if the wheel makes 100 revolutions in 1 minute.

12. A large clock has its seconds hand travelling at 6cm/sec. Find the length of the second hand.

13. A plane travels in a circular path very quickly at 650km/hr, on a circle with radius 8m, find the number of rotations that the plane makes per second.

Exact Values & Other Questions

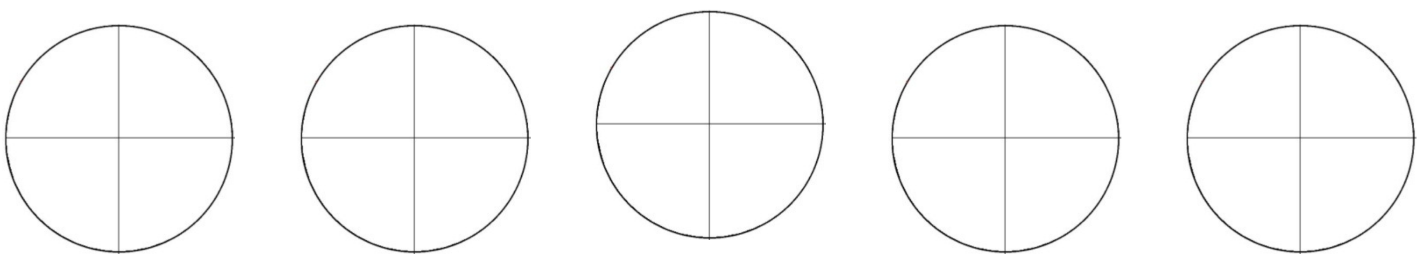
1. What answer is better to record of the two below, and why?

$\cos\frac{\pi}{6} = 0.866025403...$ or $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

2. Almost everytime trig functions are used there is rounding error. However, it is possible to find exact values for some special angles. Recall the two special triangles you have learned in grade 11, use radians this time.

3. You should get comfortable at drawing angles given in radian measure. Explain using the following examples how looking at the denominator helps you decide: how many pieces should you 'cut' pi into.

a. $\frac{7\pi}{4}$ b. $\frac{5\pi}{6}$ c. 4.05 d. $-\frac{4\pi}{3}$ e. $\frac{5\pi}{2}$



4. For each of the questions above, use the related acute angle to draw the special triangle, if possible, then for one of the questions find all primary trig ratios and for one of the questions find the secondary trig ratios.

5. For each of the following draw the terminal arms in correct quadrants then find all answers for the possible angles within first positive revolution. (Find exact angles if possible)

a. $\cos\theta = -\frac{\sqrt{2}}{2}$

b. $\sin\theta = \frac{-\sqrt{3}}{2}$

c. $\sin\theta = -\frac{1}{2}$

d. $\cos\theta = \frac{\sqrt{3}}{2}$

e. $\tan\theta = \frac{\sqrt{3}}{5}$

f. $\tan\theta = -2$

6. P(3, -4) forms a principal angle θ, find exact values of secondary trig ratios of θ

7. $\cot\theta = -5$ find exact values of primary trig ratios of θ

Sketching Sinusoidals

To sketch transformed sine or cosine:
You can also apply the previous technique to sinusoidals, but it maybe quicker to do the following

- Sketch three horizontal lines and label the values on them (Don't bother drawing the x and y axes)

$$\text{MAX} = c + |a| \text{------------}$$

$$\text{axis} = c = \frac{\text{MAX} + \text{MIN}}{2} \text{------------}$$

$$\text{MIN} = c - |a| \text{------------}$$

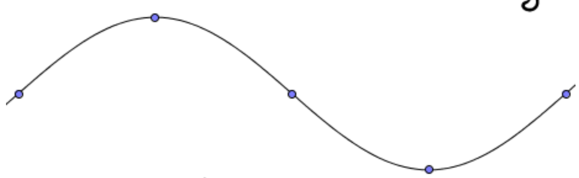
recall that

$$\text{amplitude} = |a| = \frac{\text{MAX} - \text{MIN}}{2}$$

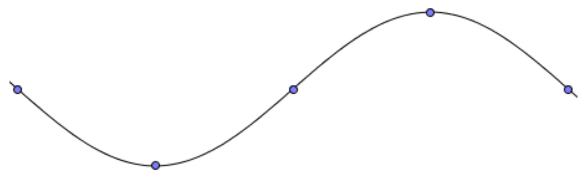
$$\text{period} = p = \frac{360^o}{|k|} = \frac{2\pi}{|k|}$$

- Decide on the shape of the wave and sketch it between the lines you just drew

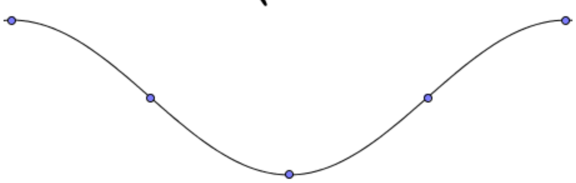
Sine with a and K SAME sign



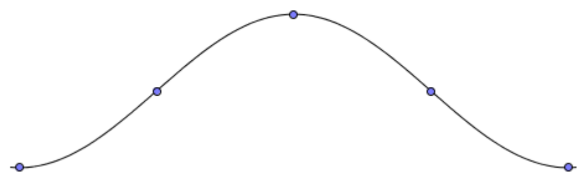
Sine with a and K opposite signs



Cosine with positive a



Cosine with negative a



- Label the 5 points of the cycle as follows

$$\text{1st point} = d$$

$$\text{last point} = d + \textit{period}$$

$$\text{middle point(s)} = \frac{\text{left point} + \text{right point}}{2}$$

1. Sketch the following

a. $y = 6\sin(\pi x + 13\pi) + 22$

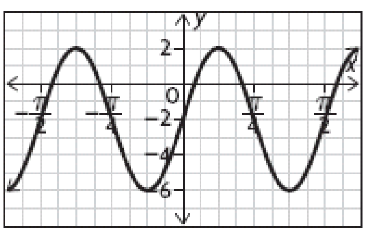
b. $y = -18\cos\left(\frac{\pi x}{4}\right) - 14$

c. $y = 0.4\sin(\pi - 2x) - 2.5$

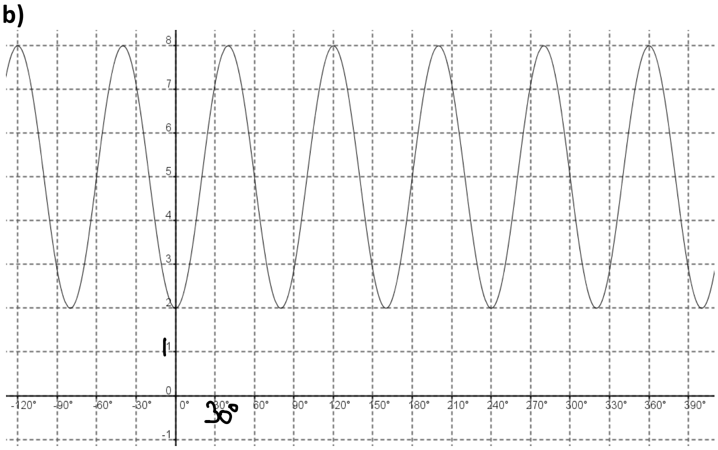
d. $y = -\frac{1}{2}\cos\left(\frac{1}{2}x - \frac{\pi}{6}\right)$

2. For each of the following find a few equations that model the graph.

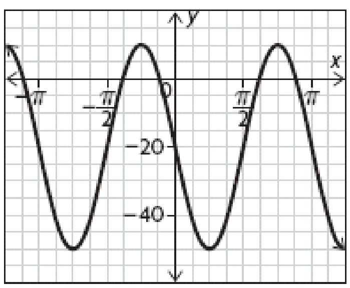
a)



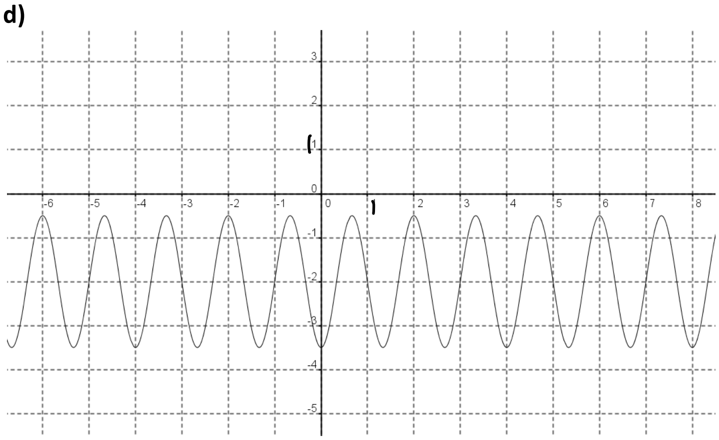
b)


8
7
6
5
4
3
2
1
0
-1
-120°
-90°
-60°
-30°
0°
30°
60°
90°
120°
150°
180°
210°
240°
270°
300°
330°
360°
390°
30°

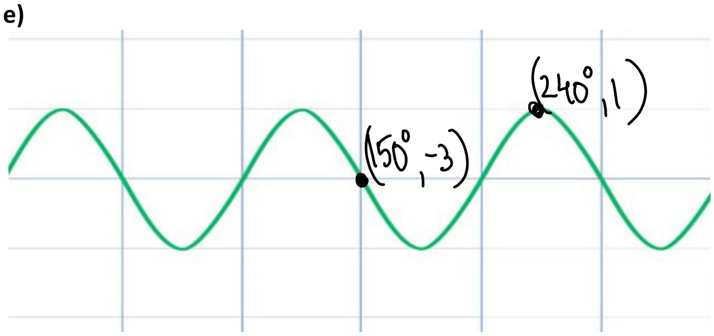
c)


y
x
−π
−π/2
0
π/2
π
−20
−40

d)


3
2
1
0
-1
-2
-3
-4
-5
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8

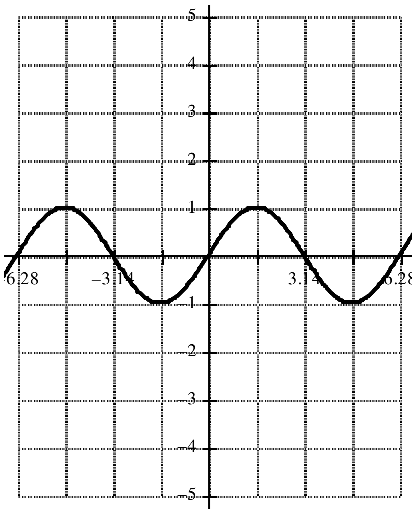
e)


(150°, -3)
(240°, 1)

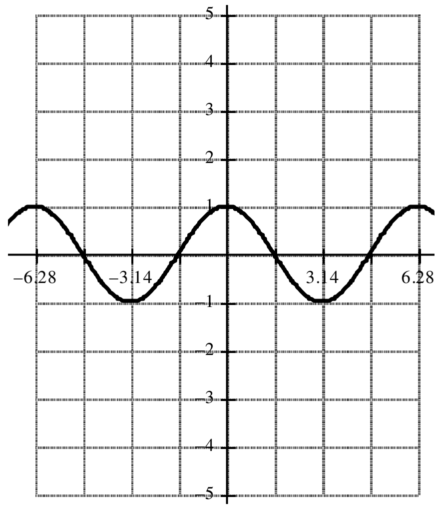
Parent Graphs of Trig Functions

1. Recall from the rationals unit what the relationship is between characteristics of $f(x)$ graph and its reciprocal, $\frac{1}{f(x)}$, graph. Use that knowledge to sketch the secondary trig functions overtop of the drawn primary trig functions.

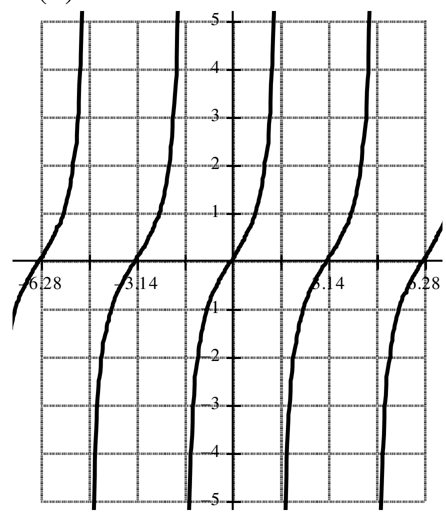
$\frac{1}{f(x)} = \frac{1}{\sin x} =$



$\frac{1}{g(x)} = \frac{1}{\cos x} =$

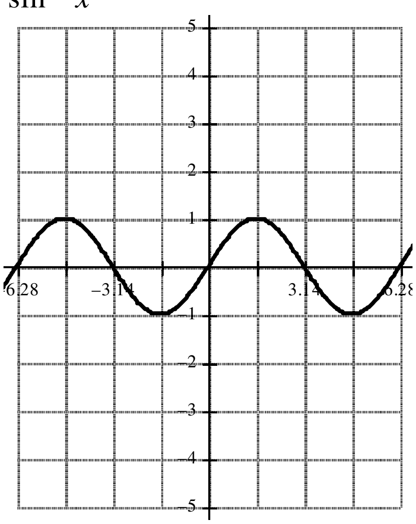


$\frac{1}{h(x)} = \frac{1}{\tan x} =$

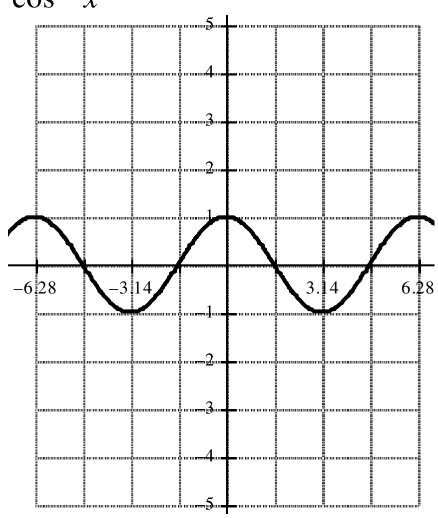


2. Recall from the functions unit what the relationship is between characteristics of $f(x)$ graph and its inverse, $f^{-1}(x)$, graph. Use that knowledge to sketch the inverse trig functions overtop of the drawn primary trig functions

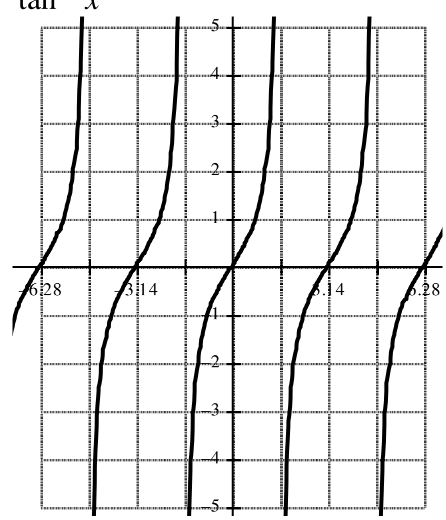
$\sin^{-1} x$



$\cos^{-1} x$



$\tan^{-1} x$



3. As you can see, reciprocals and inverses are not the same. $f^{-1}(x) \neq [f(x)]^{-1}$ The notation for the inverse can be confused with a negative exponent, which DOES make a reciprocal, hence there are other names you should know for the inverse functions.

$\sin^{-1} x =$ $\cos^{-1} x =$ $\tan^{-1} x =$

Transformations of Trig Functions (all types except sinusoidal)

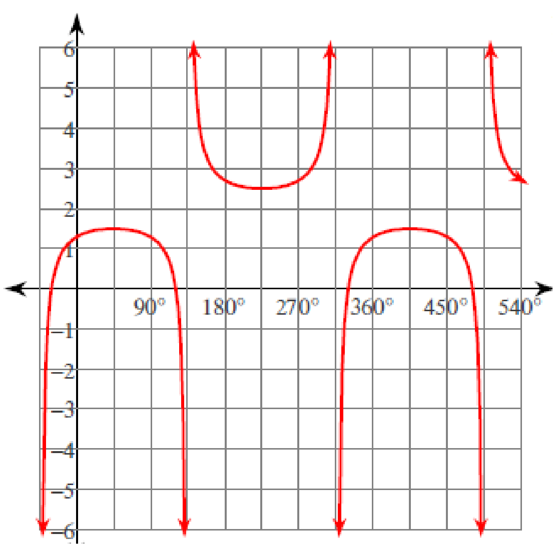
4. How can you remember the parent shapes and key values?

5. Summarize how to sketch any transformed trig function. $y = af[k(x-d)]+c$

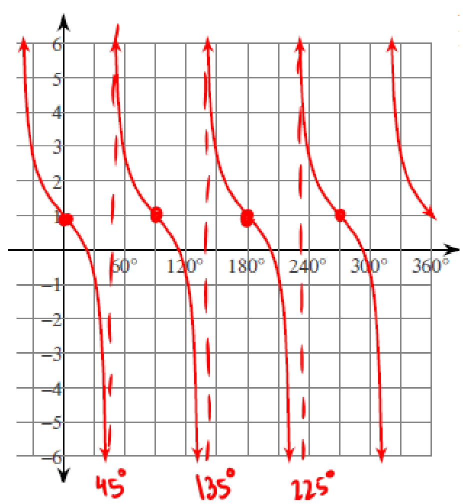
Note that some handouts online have constants d and c switched. Don't worry about the name of the constant so much as the placement of it in the equation.

Find two different equations of the following

6.



7.



8. Sketch the following functions

a. $2\tan\frac{1}{2}x$

b. $\csc(\pi x+3\pi)+1$

c. $\cot 2x-1$

d. $-2\sec(x-\pi)$

 12AdvF Date:____________ Name: ________________________

Word Problems with Sinusoidals

1. The piston in the engine of a small aircraft moves horizontally relative to the crankshaft, from a minimum distance of 25cm to a maximum distance of 75cm. During normal cruise power settings, the piston completes 2100 rpm (revolutions per minute). Assume starts at minimum
   a. Sketch the horizontal distance position, h, in centimeters, of the piston as a function of time, t, in seconds.
   b. What is the equation that models this?

2. Commercial bottling machines often use a circular drum as part of a mechanism to install tops on bottles. One such machine has a diameter of 120 cm, and makes a complete turn every 5 sec. A sensor at the left side of the drum monitors its movement. Take the sensor position as zero.
   a. Sketch the graph of the horizontal position of a point on the drum, h, in centimeters, as a function of time, t, in seconds. Assume starts at far right side
   b. What is the equation that models this?
   c. At what times in the first 10 second interval is a point on the drum 50 cm away horizontally from the sensor?
   d. What is the speed of the rotating sensor in cm/sec?

3. A buoy bobs up and down in the lake. The distance between the highest and lowest points is 1.5 m. It takes 6 seconds for the buoy to move from its highest point to its lowest point and back to its highest point. Suppose the depth of the water is 7m.
   a. Sketch the vertical displacement, v, in meters, of the buoy as a function of time, t, in seconds. Assume that the buoy is at its equilibrium point at t = 0 sec and that the buoy is on its way down at that time.
   b. What is the equation that models this?

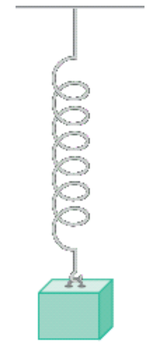
4. A salesperson selling a car alarm reports that the sound has a minimum frequency of 250 Hz, the maximum being 1150 Hz, and the frequency at t=0 being 700 Hz. The salesperson reports that the car alarm reaches its maximum frequency after 1 second and that the frequency increases before it decreases.
   a. Sketch the graph of the frequency, f, in Hertz, as a function of time, t, in seconds.
   b. What is the equation that models this?
   c. What is the frequency of the alarm at 1.2 seconds?
   d. At what times in the first 7 seconds, is the frequency at 1000 Hz?

Rates of Change

1. Explain why it is not possible to find the exact instantaneous rate of change of trig functions by using the difference quotient.

2. For $y = -25\cos\left(\frac{x}{2}\right) + 60$ find the average rate on $\frac{\pi}{2} \le x \le \pi$. Use special triangles to keep answer exact.

3. A mass on a spring is pulled toward the floor and released, causing it move up and down. Its height, h, in centimeters, above the floor after t seconds is given by the function
$h(t) = 10\sin(2\pi t + 1.5\pi) + 15, 0 \le t \le 2$
   a. Sketch the graph of height versus time.
   b. What does the rate of change of this function represent?
   c. Predict for which two points the instantaneous rate be zero.
   d. Predict for which two intervals is the average rate positive on one and negative on the other but same value otherwise.
   e. What is the speed of the spring in cm/s?



4. A car moving at 60km/hr has wheels of radius 40cm. Model the height of a point on this wheel as a function of time, assuming the point starts at minimum height.
   a. Graphical model
   b. Algebraic model