

Rational Functions Unit 5

Tentative TEST date _____



Big idea/Learning Goals

This unit begins with the study of how to graph rational functions that are reciprocals of linear and quadratic functions. Once you understand the idea of how zeros and vertical asymptotes are related, you will learn how to sketch rational functions with a non-constant numerator – this will involve the study of the main characteristics of the rational function: the y-intercepts, zeros, holes, vertical asymptotes, horizontal asymptotes and oblique asymptotes. You will learn to solve rational equations and inequalities as well as word problems that are modeled by rational functions. At the end you will yet again revisit the rates of change but with rational functions.

Corrections for the textbook answers:

- Sec 5.1 #9c) $y\text{-int}=-12$, d) range ≤ 0.5
- Sec 5.2 #2i) HA at $y=2$
- Sec 5.3 2e) $x \neq 2$ #5c) $y=(x+5)/(4x-1)$ and $y\text{-int} (0, -5)$ #8 HA not VA at $y=1/2$
- Sec 5.4 #13b) 1 min 32 sec or 1.04 min
- Sec 5.5 #4a) $-5 < x < -4.5$ #4f) $-1 < x < 7/8, x > 4$
- #5d) $t < -2, 0 < t < 3$ #5e) $t < 0$ #5f) $(-\infty, -2), (0, 3)$
- #6a) $(-6, -11/6], (4, \infty)$ #6b) $(-\infty, -3)$ #6c) $(-4, -2]$ or $(-1, 2]$
- #7 $x < -6, -1 < x < 1/2, 2 < x$ #9 there is a solution $0 < t < 0.31$
- #11. $1 < x < 5$
- Sec 5.6 #6a) $-15/49$ #9b) -1.2 #10a) 11.39 , b) 1.29



Success Criteria

- I understand the new topics for this unit if I can do the practice questions in the textbook/handouts

Date	pg	Topics	# of quest. done? <small>You may be asked to show them</small>	Questions I had difficulty with <small>ask teacher before test!</small>
	2-4	Reciprocal Graphs Section 5.1 & Handout		
	5-8	INVESTIGATION of Other Rational Graphs Section 5.2 & Handout		
	9-11	Graphing Rational Functions Section 5.3 & THREE Handouts		
	12-14	Solve Rational Inequalities Section 5.5 & TWO Handouts		
	15-16	Solve Rational Equations Section 5.4 & THREE Handouts		
	17-19	Problem Solve – 2 days TWO Handouts		
	20-21	Rates of Change of Rationals Section 5.6		
		REVIEW		



Reflect – previous TEST mark _____, Overall mark now _____.

Reciprocal Graphs



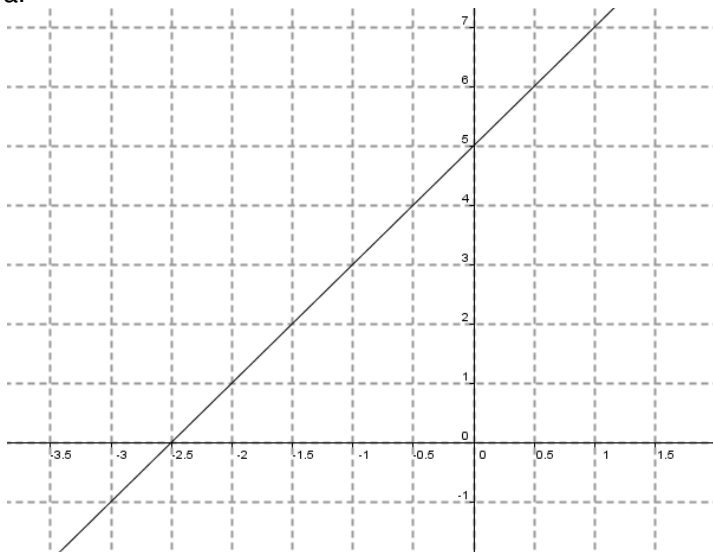
1. Describe what is the relationship between characteristics of $f(x)$ graph and its reciprocal, $\frac{1}{f(x)}$, graph.

(Include the following characteristics: zeros, y-intercepts, vertical asymptotes, end behaviour, horizontal asymptotes, increase and decrease intervals, points where the two graphs will meet.)

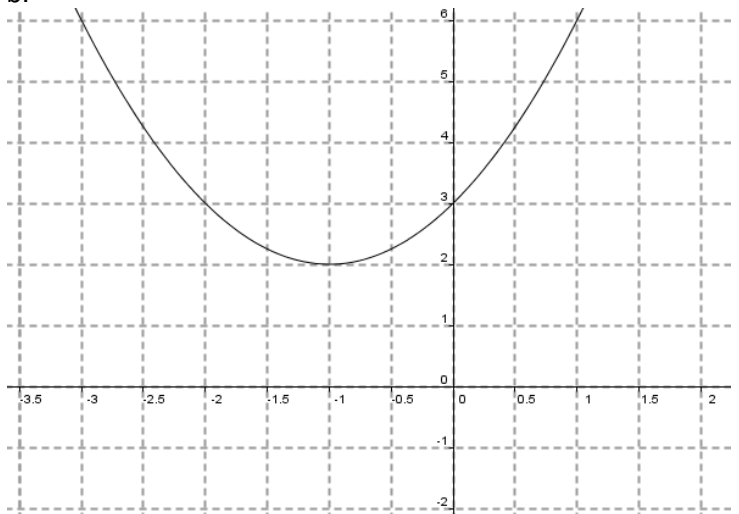
2. Sketch the reciprocal graphs for the following



a.



b.



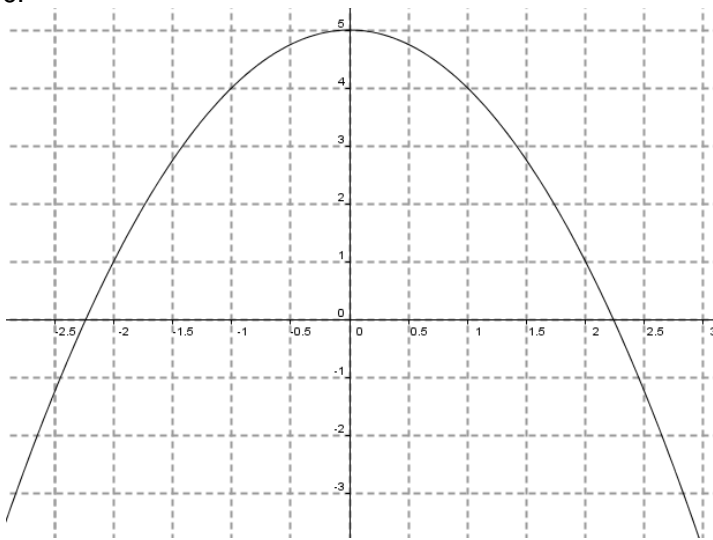


c. $y = 2x^2 + x + 1$

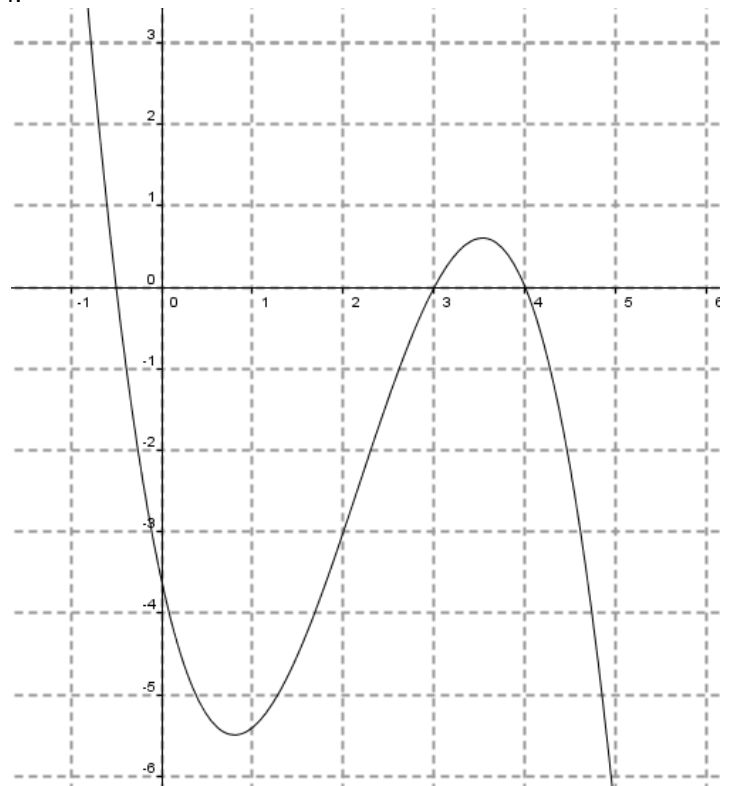
d. $y = 9x^2 + 8x - x^3$



e.



f.



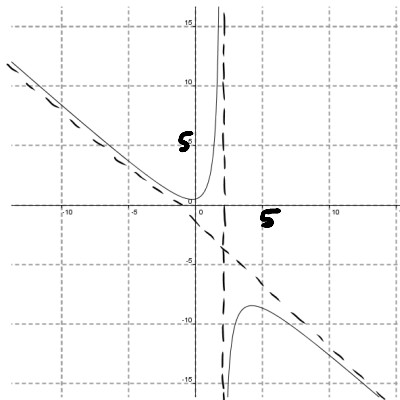


g. $y = 2x - 5$

h. $y = 2x^2 - 6x + 5$



3. You will learn how to sketch rational functions that have x 's in the numerator. You will see that some functions will have an **oblique asymptote**. See the example graph \rightarrow



INVESTIGATION of Other Rational Graphs



1. Fill out the chart by looking at graphs to discover how to find the key characteristics from the equation without the use of graphs. Always try to factor both the numerator and the denominator to see if there are any cancellations. If things don't simplify and the numerator has a higher degree than the denominator do long division.

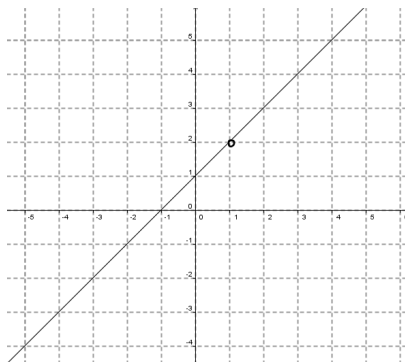
Graph

Factor & simplify or do long division

State all the key characteristics:

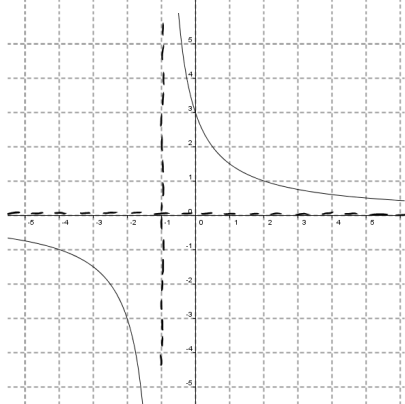
y-intercept, zeros, holes, VA, HA, OA

a



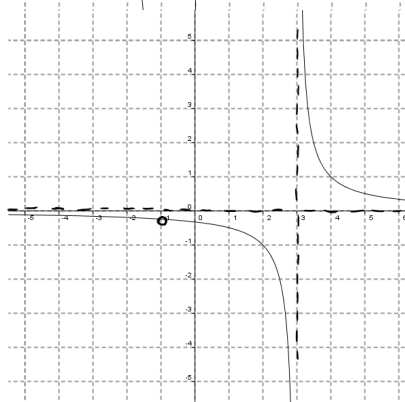
$$f(x) = \frac{x^2 - 1}{x - 1}$$

b



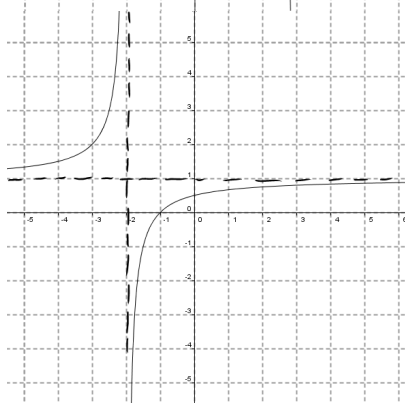
$$f(x) = \frac{3}{x + 1}$$

c



$$f(x) = \frac{x + 1}{x^2 - 2x - 3}$$

d



$$f(x) = \frac{x + 1}{x + 2}$$



Graph

Factor & simplify or do long division

State all the key characteristics:
y-intercept, zeros, holes, VA, HA, OA

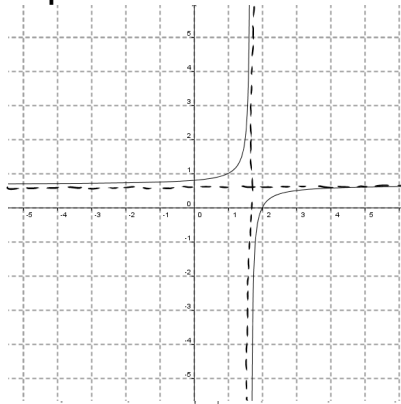
$$f(x) = \frac{2x-4}{3x-5}$$

$$f(x) = \frac{1+x^2}{2-x}$$

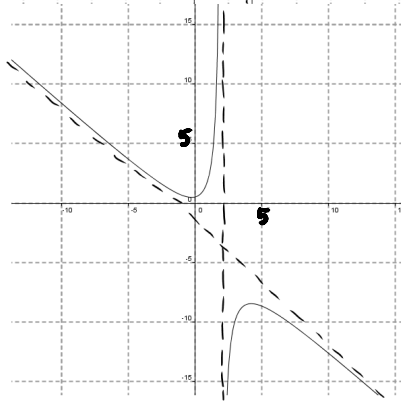
$$f(x) = \frac{x^2 + 2x}{x + 1}$$

$$f(x) = \frac{x^3 - 8}{x + 1}$$

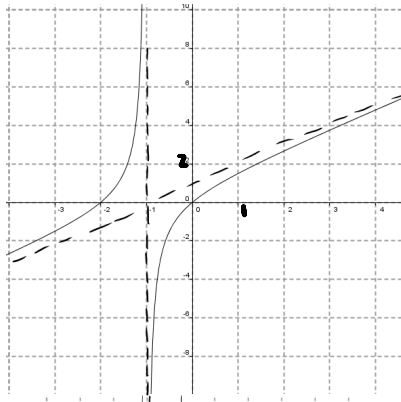
e



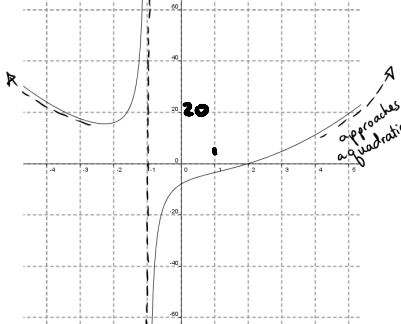
f



g



h



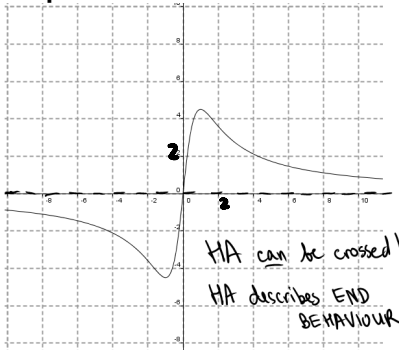


Graph

Factor & simplify or do long division

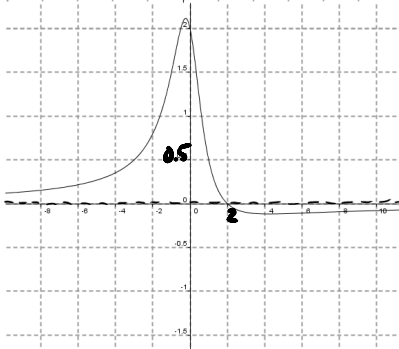
State all the key characteristics:
y-intercept, zeros, holes, VA, HA, OA

i



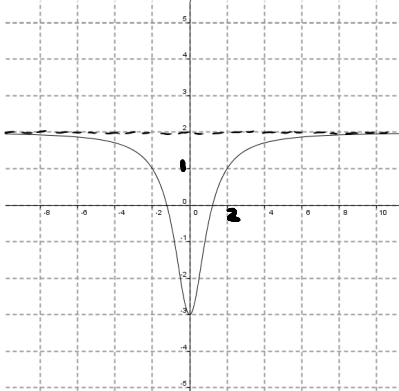
$$f(x) = \frac{9x}{1 + x^2}$$

j



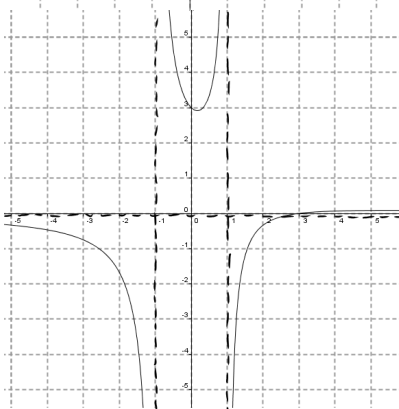
$$f(x) = \frac{2-x}{1+x^2}$$

k



$$f(x) = \frac{2x^2 - 3}{x^2 + 1}$$

l



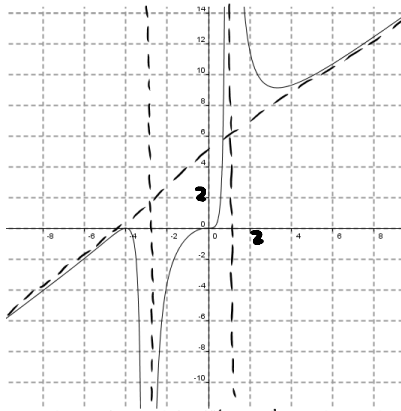
$$f(x) = \frac{x-3}{x^2 - 1}$$



2. EXTENSION

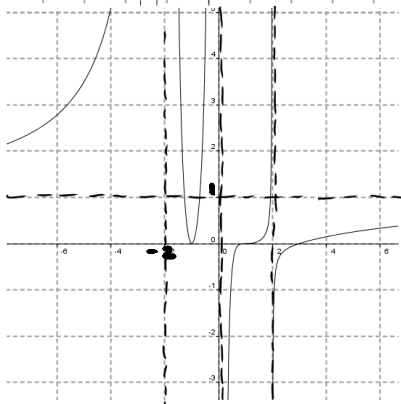
Note the orders of each factor and how it affects the behaviour near zeros and VAs

a



$$f(x) = \frac{x^3(x+4)^2}{(x-1)^2(x+3)^2}$$

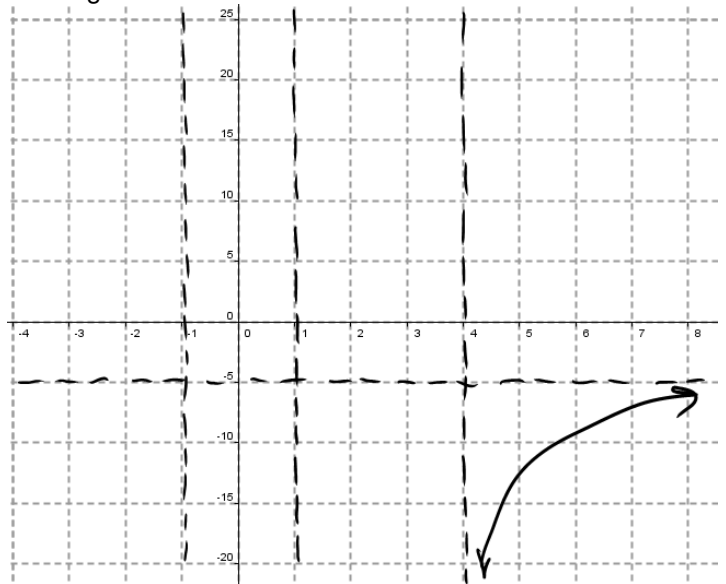
b



$$f(x) = \frac{(x+1)^2(x-3)(x-1)^3}{x^3(x-2)(x+2)^2}$$

3. Determine the behaviour near zeros and near VAs for the following and then finish off the started sketch.

$$f(x) = \frac{-5x^3(x-3)^2(x+3)}{(x-1)^3(x+1)^2(x-4)}$$



Graphing Rational Functions



1. Summarize the steps of sketching rational functions

RECIPROCAL graphs $\frac{1}{f(x)}$

OTHER rationals

TRANSFORMED parent $y = \frac{a}{k(x-d)} + c$

2. Sketch the following



a. $f(x) = \frac{-2}{3x+6} + 4$

b. $f(x) = \frac{2+5x}{3-4x}$



c. $f(x) = \frac{1}{-2x^2 + 12x - 19}$



d. $f(x) = \frac{x^2 - x + 8}{x}$



e. $f(x) = \frac{4}{1-x} - 3$

f. $f(x) = \frac{3x-1}{6x-2}$



g. $f(x) = \frac{1}{x^2 + 4x + 8}$

h. $f(x) = \frac{2x - 8}{3x}$



i. $f(x) = \frac{x^3 - x^2 - 2x}{x^2 + 4x - 12}$

j. $f(x) = \frac{3x^2 - 75}{4 - x}$

Solve Rational Inequalities



1. Recall the rules of working with inequalities. Discuss why in addition to those rules you cannot cross multiply questions like the ones below.

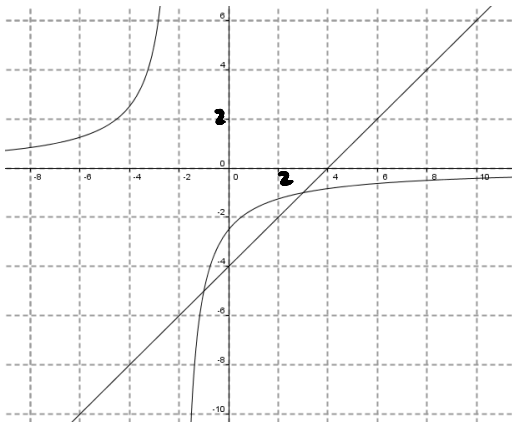


2. There are 3 ways to solve inequalities:

- A. Graph original 2 equations, and see where one is above/below the other
- B. Graph the related equation with ZERO on one side, and see where the graph is above/below x-axis
- C. Use +/- table on the equation with ZERO on one side, and pick positive/negative intervals

A: Graphing the original

$$x - 4 \leq \frac{-5}{x + 2}$$



a. How do you find the solution to this question?

Show the derivation of the related function that will be used in the next method:

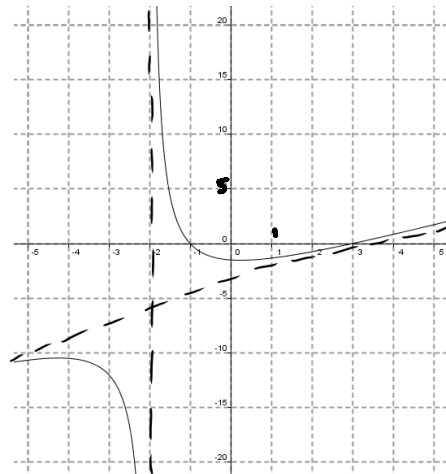
$$x - 4 \leq \frac{-5}{x + 2} \xrightarrow{\text{show steps}} \frac{(x - 3)(x + 1)}{(x + 2)} \leq 0$$

b. Show steps

B: Graphing the related function (can move everything to either side)

choose to move terms to the left:

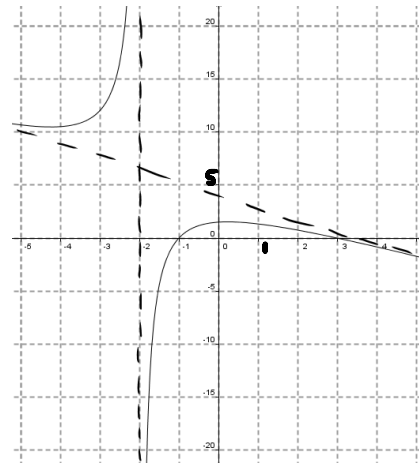
$$x - 4 + \frac{5}{x + 2} \leq 0 \longrightarrow \frac{(x - 3)(x + 1)}{(x + 2)} \leq 0$$



OR

choose to move terms to the right:

$$0 \leq -x + 4 - \frac{5}{x + 2} \longrightarrow 0 \leq \frac{-(x - 3)(x + 1)}{(x + 2)}$$



**C: Solving the related function with +/- table**

choose to move terms to the right:

$$0 \leq -x + 4 - \frac{5}{x+2} \longrightarrow 0 \leq \frac{-(x-3)(x+1)}{(x+2)}$$



3. Summarize the steps of Method C: Using the +/- table for solving an inequality



4. Solve

$$\frac{2}{x+3} \leq \frac{1}{x^2+1}$$

5. Solve the following



a. $\frac{2x+1}{2x-3} \geq \frac{x+1}{x-5}$



b. $-\frac{2}{x} < x+1$

Solve Rational Equations

1. Unlike inequalities, equations can be cross multiplied. The only thing you must watch out for is whether your final solution is part of restriction or not. Solve the following questions and check for extraneous solutions.



a. $\frac{2x}{5} = \frac{x^2 - 5x}{5x}$



b. $\frac{2+x}{3x} = \frac{x+1}{x+2}$



2. Dan and Sue set off at the same time on a 42 km go-cart race. Dan, drives 0.4 km/min faster than Sue, but has to stop en route and fix his go-cart for one-half hour. This stop costs Dan to arrive 15 min after Sue. How fast was each person driving?



3. To travel 60 miles, it takes Sue, riding a moped, 2 hours less than it takes Doreen to travel 50 miles on a bicycle. Sue travels 10 miles per hour faster than Doreen. Find the times and rates of speed of both women.
4. A tank has a capacity of 10 gallons. When it is full, it contains 15% alcohol. How many gallons must be replaced by an 80% alcohol solution to give 10 gallons of 70% solution?



5. Walt can mow a lawn in 1 hour, while his son, Malik, can mow the same lawn in 50 minutes. One day Malik started mowing the lawn by himself and worked for 30 minutes. Then Walt joined him and they finished the lawn. How long did it take them to finish mowing the lawn after Walt started to help?
6. A rock is dropped into a well, and 3 seconds later the sound of its splash is heard. How deep is the water in the well? Assume the sound travels at 1100 feet per second and that an object falls a distance of $16t^2$ feet in t seconds. (t represents the time for the rock to reach the water)
7. A homemade loaf of bread turns out to be a perfect cube. Five slices of the bread, each 0.6 inch thick, are cut from one end of the loaf. The remainder of the loaf now has a volume of 700 cubic inches. What were the dimensions of the original loaf?

Rates of Change of Rationals



1. Recall the formulas of rate of change

2. After you eat something that contains sugar, the pH of acid level in your mouth changes. This can be modeled by the function below where L is the pH level and m is the number of minutes that have elapsed since eating. Find the average rate of change in the first 5 minutes

$$L(m) = \frac{-20.4m}{m^2 + 36} + 6.5$$



3. Find the turning points of $f(x) = \frac{x^2 + 1}{2 - x}$