nNOTESfixed2012

newRatesUnitNOTES

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see below for last years answers

 12AdvF Date:___________ Name: ______________________

Rates of Change – Unit 2

Big idea

Not a lot of things in real life stay constant all the time. Things change depending on many variables. In this unit you will learn about how to calculate the rate at which things change over an interval as well as at one specific point. The rate of change over an interval is called **average rate of change**, while the rate at one specific point is called **instantaneous rate of change**. Finding rate of change is like finding the slope which you learned in grade 9 when you studied lines. Since we will study mainly nonlinear functions, how do you think you can do slope of a curve? Slope between two points is not a problem however the problem will arise as to how to find slope at one specific point. This is an introduction to calculus which is a study of change. In calculus you will learn fast ways of finding rates of change of specific functions, called the derivatives, in this class you will learn the foundations upon which calculus principles are based.

4 methods

Corrections for the textbook answers:

Success Criteria

- ☐ I am ready for this unit if I am confident in the following review topics
  - Slope formula
  - Function notation
  - Domain & range
  - Simplifying expressions
  - Solving equations
  - Expanding & Pascal's triangle
  - Subtracting rational expressions
  - Rationalizing numerator
  - Problem solving with lines, quadratics, exponentials and trig.
- ☐ I understand the new topics for this unit if I can do the practice questions in the textbook/handouts

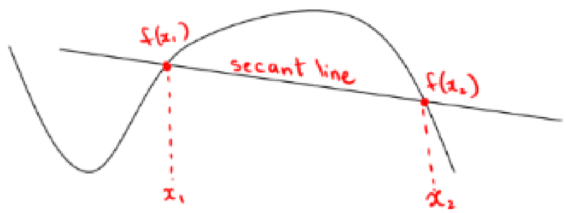
Date	pg	Topics	# of quest. done? You may be asked to show them
	2-3	Average Rate of Change Section 2.1 & Handout	
	4-6	Instantaneous Rate of Change Section 2.3 & Handout	
	7-10	Difference Quotient – 2 days Section 2.2 & Handout	
	11-13	Graphical Models Section 2.4 & Handout	
	14-16	Absolute and Local Extreme Points Section 2.5 & Handout	
		EXTRA (if there is time) 1.Regression using technology (Instructions for Excel, CurveExpert, Graphing Calculator TI83, Practice & Group Handouts) 2. Rates Extra Assignment	

Reflect – previous TEST mark ___________, Overall mark now___________.

Looking back, what can you improve upon?

Average Rate of Change → 1 method.

You should remember how to find slope of a line from grade 9. To find slope/rate of change of a curve, you still will be using the same formula. The slope between two points of a curve is the average of all the slopes in between the points. The line that you use to calculate the average slope on the interval is called a **secant line**. You will learn how to find the average rate of change from table of values, from the graph and from the given equation. → touches the curve at two points.



1. Recall the slope formula from grade 9. Write it using function notation, specifying what interval it is used on.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{change in } y}{\text{change in } x}$$

$f(x_2)$

m_{sec}

$$\boxed{\text{ave.r.o.c} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} \text{ for interval } x \in [x_1, x_2]}$$

2. A group of children set up a lemonade stand in their neighbourhood. The number of cups of lemonade that they sell is given by the following table of values

hrs

Time (t)	Cups (C)
0.5	4
1.0	9 C(1)
1.5	13
2.0	20 C(2)
2.5	31
3.0	36
3.5	30
4.0	30

Find how the cups sold change over time by calculating the following rates of change

a. $t \in [1, 2]$

b. $t \in [2, 4]$

c. What could account for the difference in answers?

ⓐ $\text{a.r.o.c} = \frac{C(2) - C(1)}{2 - 1} = \frac{20 - 9}{1} = 11$ cups / hour

∴ from hour 1 to hour 2 there was an increase in sales by 11 cups/hr.

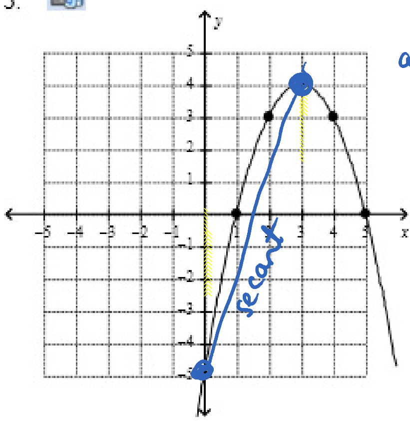
ⓑ $\text{a.r.o.c} = m_{sec} = \frac{C(4) - C(2)}{4 - 2} = \frac{30 - 20}{2} = \frac{10}{2} = 5$ cups/hour

ⓒ Different times of day can cause less/more people on the street.

- Change in weather

Find the average rate of change of the following on the given intervals

3.



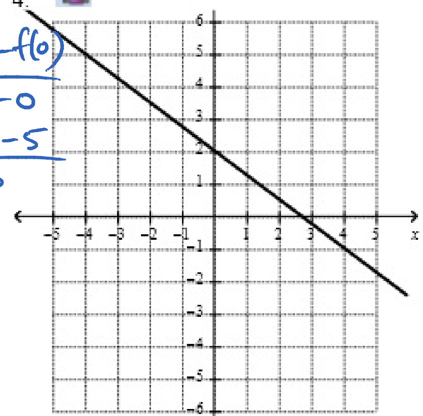
a) $a.r.o.c = \frac{f(3)-f(0)}{3-0}$

$= \frac{4--5}{3}$

$= \frac{9}{3}$

$m_{sec} = 3$

4.



a. $x \in [0,3]$

b. $x \in [2,4]$

c. Give an interval where the average rate of change (ave.r.o.c) is negative

b) $a.r.o.c = \frac{f(4)-f(2)}{4-2}$

$= \frac{3-3}{2}$

$= 0$

c) same as interval of decrease

$x \in [3,5]$

a. $x \in [0,2]$

b. $x \in [2,4]$

c. explain these results

(a) $a.r.o.c = \frac{0.5-2}{2} = -0.75$

(b) $a.r.o.c = \frac{-1-0.5}{2} = -0.75$

(c) same slope since linear

5. The deer population of a country is modeled by $P(t) = 20t^2 + 200t + 10500$, where $P(t)$ is the size of the population and t is the number of years since 1995.

$20(5)^2 + 200(5) + 10500$

a. Do you expect the rate of change to remain the same for this function or not? Explain.

b. Calculate average rate of change in each time period

i. 1995 – 2000 $\rightarrow t \in [0,5]$

ii. 2005 – 2015

iii. 1995 – 2015

a) function is a parabola

$\therefore$ r.o.c will always change

b) i) $\frac{P(5)-P(0)}{5-0} = \frac{(12000-10500)}{5} = \frac{1500}{5} = 300 \quad \frac{\Delta P}{\Delta t}$

$\therefore$ a.r.o.c. is 300 deer/year

ii) $\frac{P(20)-P(10)}{20-10} \quad \frac{22500-14500}{10} = 800$ deer/year

iii) $\frac{P(20)-P(0)}{20-0} = \frac{22500-10500}{20} = 600$ deer/year

Instantaneous Rate of Change

In the last question of the last lesson you found the average rates of change of deer population over given intervals of time, which was in years. What if you needed to know how the deer population is changing exactly at one specific point in time? How is this question relevant? Well, what if it was not deer population that you were studying but how fast an infectious disease is spreading. It may be very important to know the rate of things changing in the present time. So how can one find the rate of change or slope at one point? The answer is not a simple one.

4 methods

In this course you will learn how to approximate the answer using the following methods:

1 • Graphing the function and looking at the slope of the **tangent line** at the point you are interested in.

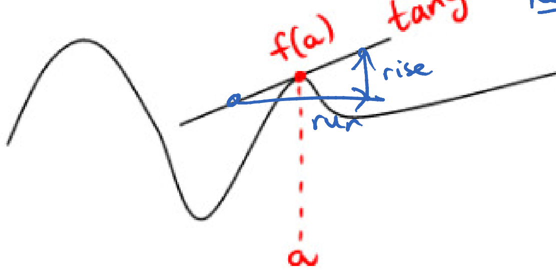
2 • Taking the average of the *ave.r.o.c* calculations on the **preceeding and following intervals** of the point you are interested in.

3 • **Squeezing intervals** – doing at least two calculations on intervals that shrink in size near the point you are interested in and looking at the decimals to see how accurate your prediction is.

And you will learn how to find an exact answer by the following algebraic method

4 • The **difference quotient** – but it only works for polynomial, square root and rational functions. (This particular method is what calculus is based upon. What you will learn in calculus will allow you to find exact rates of change of ALL functions, not just polynomial, square root and rational functions.)

The line that you use to calculate the instantaneous rate of change at ONE point is called a **tangent** line.

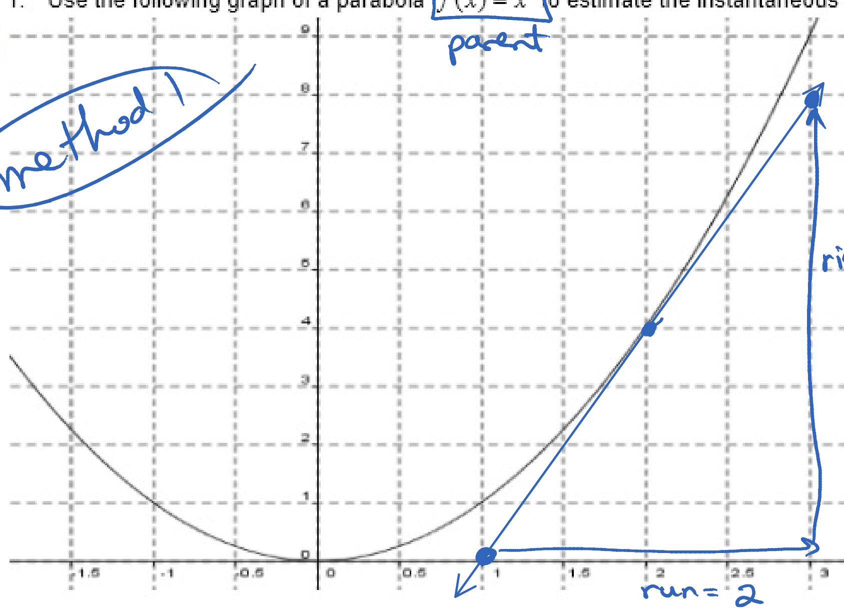


near the point
the tangent line just "skims" the curve and has the same slope as the curve

1. Use the following graph of a parabola $\boxed{f(x)=x^2}$ to estimate the instantaneous rate of change at x=2.

parent

method 1



$\therefore m = \frac{rise}{run}$

$= \frac{8}{2}$

$m = 4$

$m = 3.5$

2. Find the approximate instantaneous rate of change (inst.r.o.c) of $f(x) = x^2$ at x=2 by doing the following methods

a. Preceeding and following intervals method   method 2

i.r.o.c = ? at $x = 2$

preceeding interval $x \in [1.5, 2]$ $a.r.o.c = \frac{f(2) - f(1.5)}{2 - 1.5} = \frac{4 - 2.25}{0.5} = \frac{1.75}{0.5} = 3.5$

same size

following interval $x \in [2, 2.5]$ $a.r.o.c = \frac{f(2.5) - f(2)}{2.5 - 2} = \frac{6.25 - 4}{0.5} = \frac{2.25}{0.5} = 4.5$

$\therefore$ i.r.o.c $\sim \doteq \frac{3.5 + 4.5}{2} = \frac{8}{2} = 4$

b. Squeezing the intervals method   method 3

i.r.o.c = ? at $x = 2$

tiny interval close to 2
$x \in [2, 2.01]$

$$a.r.o.c = \frac{f(2.01) - f(2)}{2.01 - 2} = \frac{4.0401 - 4}{0.01} = \frac{0.0401}{0.01} = 4.01$$

To know how accurate the answer is do a second calculation with even smaller interval + compare decimal places

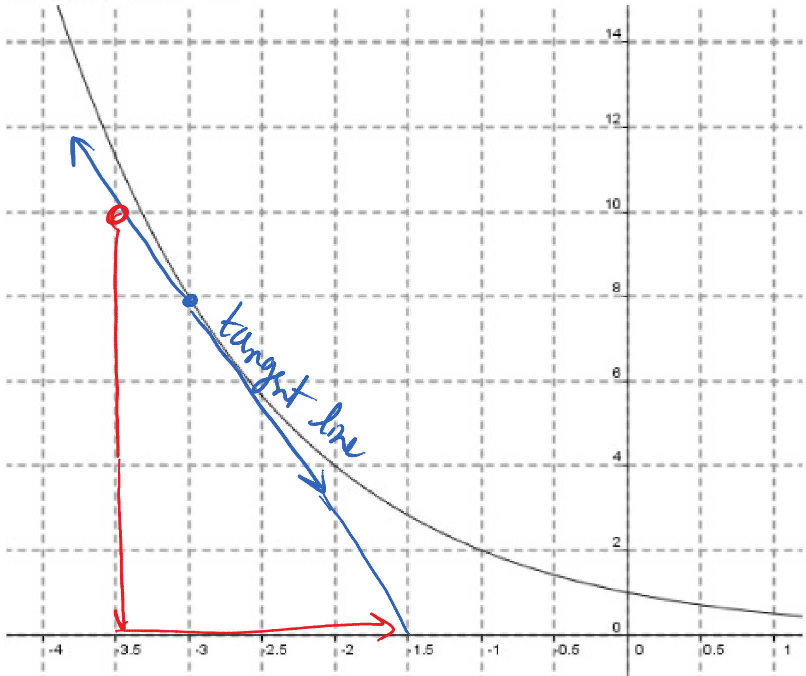
$x \in [2, 2.001]$

a.r.o.c. = 4.001

$\therefore$ i.r.o.c $\doteq 4.0$

c. Compare your results to the graphing method, how close was your answer there to the ones you get with these methods?

3. Find the approximate *inst.r.o.c* of the following by performing **ALL THREE** methods.
$g(x) = 0.5^x$ at x= – 3.



$m_{tan} = i.r.o.c$

$m_{tan} = \frac{rise}{run}$

$m_{tan} = \frac{-10}{2}$

$m_{tan} = -5$

(2) average of preceeding + following

$x \in [-4,-3]$ a.r.o.c = -8

$\frac{8-16}{1}$

$x \in [-3,-2]$ a.r.o.c = -4

$\frac{4-8}{1}$

$\therefore$ i.r.o.c = $\frac{-8+-4}{2}$

iroc $\doteq -6$ at $x = -3$

(3) squeezing intervals

$x \in [-3, -2.99]$

aroc $\doteq -5.5260036\ldots$

$x \in [-3, -2.999]$

aroc $\doteq -5.543256$

$\therefore$ iroc $\doteq -5.5$ at $x = -3$

Difference Quotient

The concept of sqeezing the intervals of *ave.r.o.c.* calculations to approximate *inst.r.o.c.* is what will be used for this new topic. Take the formula you already know for average rate of change

on the interval $x \in [x_1, x_2]$, $ave.r.o.c = m_{sec} = \frac{f(x_2)-f(x_1)}{x_2-x_1}$ and replace the interval to be of a general size, h.

Let $x_1 = a$ and $x_2 = a+h$. The formula then becomes, what we call the **difference quotient**:

on the interval $x \in [a, a+h]$, $m_{sec} = \frac{f(a+h)-f(a)}{\cancel{a}+h-\cancel{a}} = \frac{f(a+h)-f(a)}{h}$

This is still an average rate of change calculation, however if you shrink h all the way to zero the secant line becomes a tangent line and it will then represent the instantaneous rate of change. (SHOW this with technology!)

Problem is, you cannot divide by zero, so if you make h zero at the beginning you will run into mathematical errors. BUT if you algebraically manage to cancel the h out and THEN sub in h as zero, you will have exact instantaneous rate of change. This is how we record the answers:

$$ave.r.o.c. = m_{sec} = \frac{f(a+h)-f(a)}{h} \xrightarrow[\text{as } h \to 0]{\text{approaches}} m_{tan} = inst.r.o.c.$$

As mentioned before, this algebraic manipulation only works for polynomial, square root and rational functions. It will not work for exponential or trig functions, for these you will need the rules of calculus.

1. Review how to use Pascal's triangle to expand polynomials, how to subtract rational expressions, and how to rationalize the square root functions. All of these are key components of getting the h to cancel out in the functions you'll be given.

a. expand $\frac{(x+h)^4 - x^4}{h}$

$(x+h)^4$

Pascal's Δ

1 1
1 2 1
1 3 3 1
$\boxed{1 \quad 4 \quad 6 \quad 4 \quad 1}$
1 5 10 10 5 1

$= 1x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + 1h^4$

$(a+b)^5$

$= 1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1b^5$

b. simplify $\left(\frac{1}{x+h} - \frac{1}{x}\right)\frac{1}{h}$

L.C.D.

$= \left(\frac{x - 1(x+h)}{x(x+h)}\right)\frac{1}{h}$

$= \left(\frac{x - x - h}{x(x+h)}\right)\frac{1}{h}$

never expand denom.

$= \left(\frac{-h}{x(x+h)}\right)\frac{1}{h}$

$= \frac{-1}{x(x+h)}$

c. rationalize numerator $\frac{\sqrt{x+h}-\sqrt{x}}{h} \times \frac{(\sqrt{x+h}+\sqrt{x})}{(\sqrt{x+h}+\sqrt{x})}$

$(\sqrt{x+h})^2 \qquad (\sqrt{x})^2$

$= \frac{(x+h) + \sqrt{x}\sqrt{x+h} - \sqrt{x}\sqrt{x+h} - x}{h(\sqrt{x+h}+\sqrt{x})}$

$= \frac{\cancel{x}+h-\cancel{x}}{h(\sqrt{x+h}+\sqrt{x})}$

$= \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h}+\sqrt{x})}$

$= \frac{1}{\sqrt{x+h}+\sqrt{x}}$

Exact method #4.

2. Find the instantaneous rate of change of $y=5\sqrt{x}$ at x=4. Sketch an explanation of the result.

i.r.o.c.=? at $x=4=a$ $f(x)$

$$\text{a.r.o.c} = \frac{f(4+h)-f(4)}{h} = \frac{5\sqrt{4+h} - \overset{10}{5\sqrt{4}}}{h}$$

common factor before rationalizing

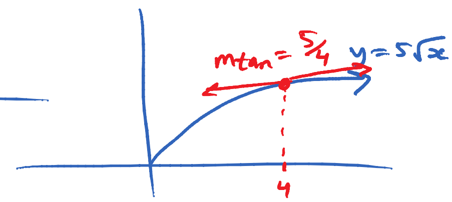
$$\text{a.r.o.c} = \frac{5\left[\sqrt{4+h}-2\right]}{h} \times \frac{\left(\sqrt{4+h}+2\right)}{\left(\sqrt{4+h}+2\right)}$$

$$\text{a.r.o.c.} = \frac{5\left[\cancel{4}+h+\cancel{2\sqrt{4+h}}-\cancel{2\sqrt{4+h}}-\cancel{4}\right]}{h\left(\sqrt{4+h}+2\right)}$$

$$\text{a.r.o.c} = \frac{5\cancel{h}}{\cancel{h}\left(\sqrt{4+h}+2\right)}$$

$$\text{a.r.o.c.} = \frac{5}{\left(\sqrt{4+h}+2\right)} \xrightarrow[h\to 0]{} \text{i.r.o.c.} = \frac{5}{\sqrt{4+0}+2} = \frac{5}{\sqrt{4}+2} = \frac{5}{2+2}$$

i.r.o.c at $x=4$ = $\frac{5}{4}$



3. Find the slope of the tangent line of $f(x)=-2x^4$ at x=-2. Sketch an explanation of the result.

i.r.o.c=? at $x=-2=a$ Pascal's Δ.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

$$\text{a.r.o.c} = \frac{f(-2+h)-f(-2)}{h}$$

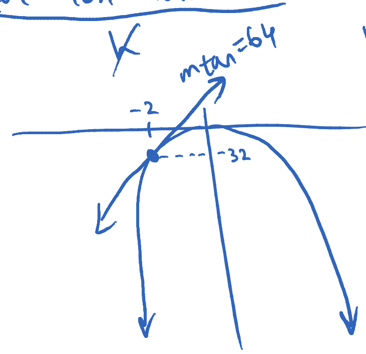
$$\text{aroc} = \frac{-2\left[(-2)^4+4(-2)^3h+6(-2)^2h^2+4(-2)^1h^3+h^4\right] - \left[-2(-2)^4\right]}{h}$$

$$\text{aroc} = \frac{-2\left[16-32h+24h^2-8h^3+h^4\right]+32}{h}$$

$$\text{aroc} = \frac{\cancel{-32}+64h-48h^2+16h^3-2h^4+\cancel{32}}{h}$$

$$\text{aroc} = \frac{\cancel{h}\left(64-48h+16h^2-2h^3\right)}{\cancel{h}} \xrightarrow[\text{as } h\to 0]{} \text{iroc} = 64-0+0\ldots$$

∴ Slope at pt. $(-2,-32)$ is 64



4. Find a point on the graph of $g(x)=\frac{1}{x}$ where the rate of change of the function is -9. $=$ iroc

find a

$$aroc = \frac{f(a+h)-f(a)}{h}$$

$$aroc = \frac{\frac{1}{a+h}-\frac{1}{a}}{h} = \frac{1}{h}\left[\frac{1(a)-1(a+h)}{a(a+h)}\right]$$

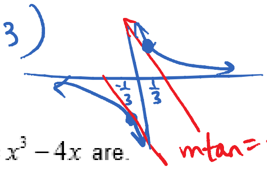
$$aroc = \frac{1}{h}\left[\frac{\cancel{a}-\cancel{a}-h}{a(a+h)}\right] = \frac{-\cancel{h}}{\cancel{h}a(a+h)} \xrightarrow[\text{as } h\to 0]{} iroc = \frac{-1}{a(a+0)}$$

$$iroc = \frac{-1}{a^2}$$

$$-9 = \frac{-1}{a^2}$$

$$9a^2 = 1 \quad a^2 = \frac{1}{9} \quad a = \pm\frac{1}{3}$$

$\therefore$ at pt. $(\frac{1}{3}, 3)$ and $(-\frac{1}{3}, -3)$ the slope is -9



5. a. Find where the turning points of the graph $y=x^3-4x$ are.
b. Discuss of a way to determine if each of the turning points would be a local maximum or minimum.

(a) iroc $= 0$ find x.

$$aroc = \frac{f(x+h)-f(x)}{h} = \frac{\left[(x+h)^3-4(x+h)\right]-\left[x^3-4x\right]}{h}$$

1
1 1
1 2 1
1 3 3 1

$$aroc = \frac{\cancel{x^3}+3x^2h+3xh^2+h^3-\cancel{4x}-4h-\cancel{x^3}+\cancel{4x}}{h}$$

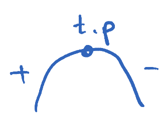
$$aroc = \frac{\cancel{h}\left[3x^2+3xh+h^2-4\right]}{\cancel{h}} \xrightarrow[\text{as } h\to 0]{} iroc = 3x^2-4$$

$$0 = 3x^2-4$$

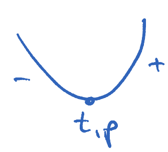
$\therefore \pm\sqrt{\frac{4}{3}} = x$ are the turning points

$\pm 1.15 \doteq x$

(b) find aroc on left and right side of t.p



$\therefore$ MAX



$\therefore$ MiN

ex. for $x=-1.15$ t.p.

find aroc on $[-1.2, -1.15]$ on left of -1.15

and on $[-1.15, -1]$ on right of -1.15

1
1 1
1 2 1
1 3 3 1

6. As a large snowball melts, its size changes with respect to its radius. The volume, V, in cubic centimeters, is given by $V=\frac{4}{3}\pi r^3$ where r is the radius, in centimeters, and $r\in[0,30]$. Find the instantaneous rate of change of the volume when the radius of the snowball is 10 cm.

domain

i.r.o.c = ? at r = 10 = a

common factor

$$a.r.o.c = \frac{V(10+h)-V(10)}{h} = \frac{\frac{4}{3}\pi(10+h)^3 - \frac{4}{3}\pi(10)^3}{h} = \frac{\frac{4}{3}\pi\left[(10+h)^3 - 10^3\right]}{h}$$

$$a.r.o.c = \frac{\frac{4}{3}\pi\left[\cancel{1(10)^3} + 3(10)^2h + 3(10)h^2 + h^3 \cancel{-10^3}\right]}{h}$$

common

$$aroc = \frac{4}{3}\pi\left[\frac{300h + 30h^2 + h^3}{h}\right] = \frac{\frac{4}{3}\pi(\cancel{h})(300+30h+h^2)}{\cancel{h}} \xrightarrow[\text{as } h\to 0]{} \text{i.r.o.c}$$

$$= \frac{4}{3}\pi(\overset{100}{\cancel{300}}+0+0)$$

$$= 400\pi \ \frac{\text{cm}^3 \text{ of Vol}}{\text{cm of radius}}$$

7. The surface area of the snowball, S, in square centimeters, is given by $S=4\pi r^2$. Find the instantaneous rate of change of the surface area when the volume of the snowball is 1766.25 cm^3

$V=\frac{4}{3}\pi r^3$

$1766.25=\frac{4}{3}\pi r^3$

$421.876=r^3$

$7.5=r$

$$m_{sec} = \frac{S(7.5+h)-S(7.5)}{h} = \frac{4\pi(7.5+h)^2 - 4\pi(7.5)^2}{h}$$

$$m_{sec} = \frac{4\pi\left[\cancel{7.5^2} + 2(7.5)h + h^2 - \cancel{7.5^2}\right]}{h}$$

$$m_{sec} = \frac{4\pi\cancel{h}[15+h]}{\cancel{h}} \rightarrow m_{tan} = 4\pi(15)$$

$$= 60\pi \ \text{cm}^2/\text{cm}$$

8. Mark purchased a new car for \$31 000. The yearly depreciation of the value of the car can be modeled by the equation $V(t)=31000(0.87)^t$ where $V(t)$ is the value of the car and t is the number of years he has owned the car. What is the approximate instantaneous rate of change after 3 years?

difference quotient method #4 will not work for exponentials, sin, cos, log → power (polyn), rational, sq. root.

i.r.o.c can still be found by other 3 methods ...

10

Graphical Models

CBR Set-up: (get program to work between calc and ranger) Calc: 2^{nd}, Link, -> , Enter
ranger: 82/83

(get into program)	Program, Ranger, Enter
(get sample)	Set up, up, start now, ... repeat sample
(matching graphs)	Applications, meters, dist.match

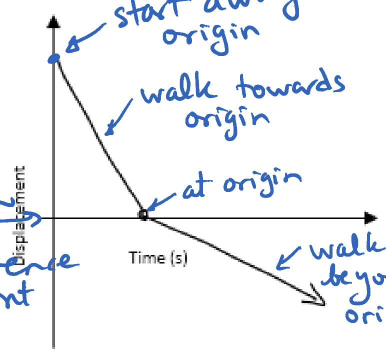
Instructions: create a clear area for walking in front of CBR, hold a flat surface(ie book) in front yourself as you walk, walk steady, ignore jumps in the graph, the instrument is not very precise.

1. What is the difference between **velocity** and **speed** or **displacement** and **distance**? These concepts are very important in physics. Unfortunately, the textbook we are using often does not distinguish between these concepts and treats the terms as the same.

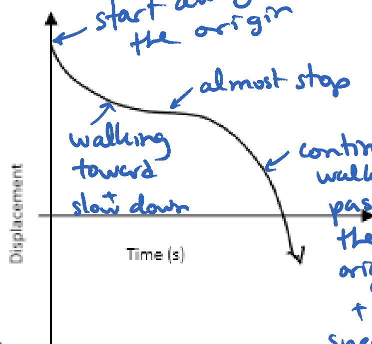
have direction and can be negative.

Describe a scenario that would yield the following d-t graphs.

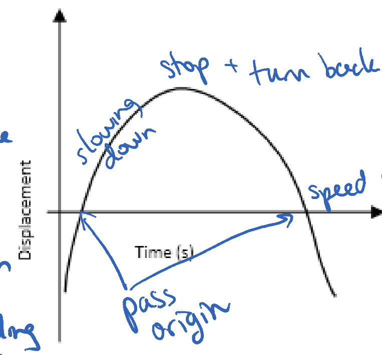
2.



3.



4.



* straight lines mean constant speed

5. Summarize what each of the following implies for the d-t graphs. displacement - time graphs

a. Direction

pos. slope → away

neg. slope → toward

* if displacement is neg. (below x-axis) the above is switched

b. Concavity

Concave up
positive acceleration
→ speed up in pos. dir.
or → slow down in neg. dir.

Concave down
neg. acceleration
→ slow down in pos. dir
→ speed up in neg. dir.

c. Slope

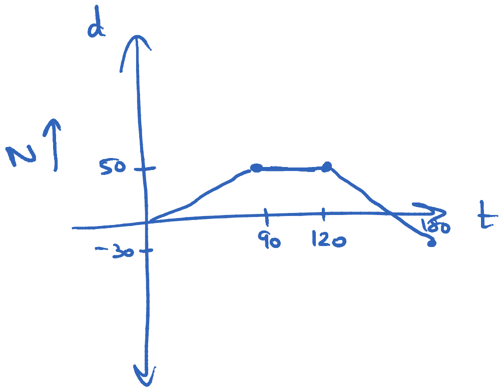
pos. slope
pos. speed
→ go away from pt. of reference
OR → go toward a pt. of reference

neg. slope

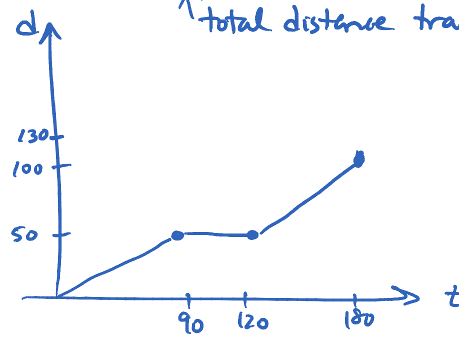
11

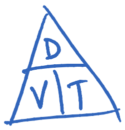
6. A scout moves north in the forest for a distance of 50 m in 90 sec, stops for 30 sec, then moves south 80m in 60 sec.

a. Sketch a displacement-time graph



b. Sketch a distance-time graph can't be negative
total distance travelled.

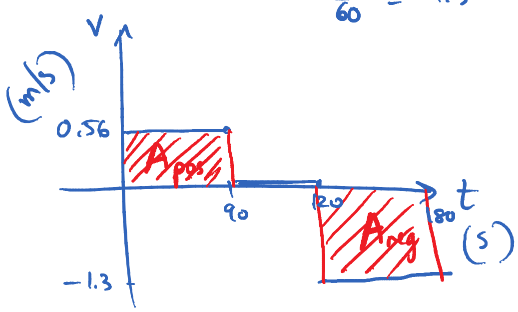




c. Sketch a speed-time graph

$V = \frac{D}{T} = \frac{50}{90} = \frac{rise}{run} = 0.56$ = slope of d-t is speed.

$\frac{-80}{60} = -1.\bar{3}$



d. Find the area under the speed-time graph.

$A = lw$

$A_{pos} = (\cancel{90s})\left(\frac{50\,m}{\cancel{90\,s}}\right)$

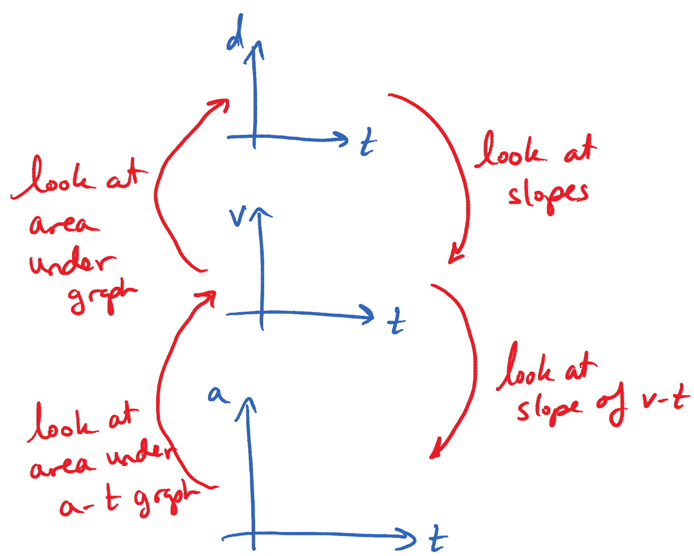
$= 50\,m$

lw

$A_{neg} = (\cancel{60s})\left(\frac{-80\,m}{\cancel{60\,s}}\right)$

$= -80\,m$

displacement = area under the v-t graph



e. What does the slope of d-t graph represent?

Speed

f. What does the area under the s-t graph represent?

displacement

g. What would the slope of s-t graph represent?

acceleration

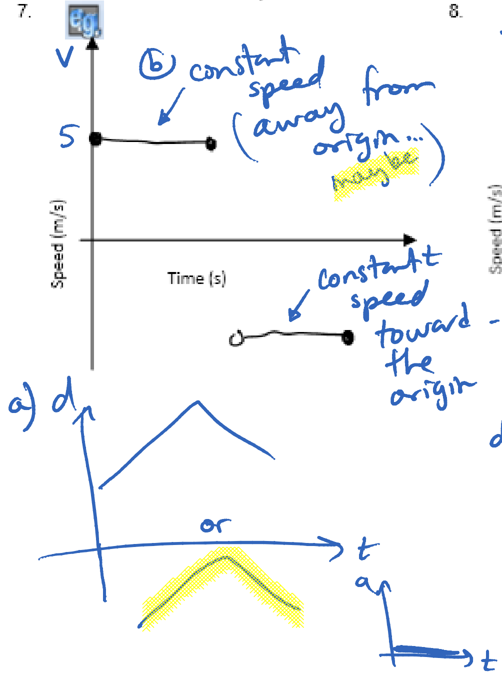
h. What would the area under the a-t graph represent?

speed.

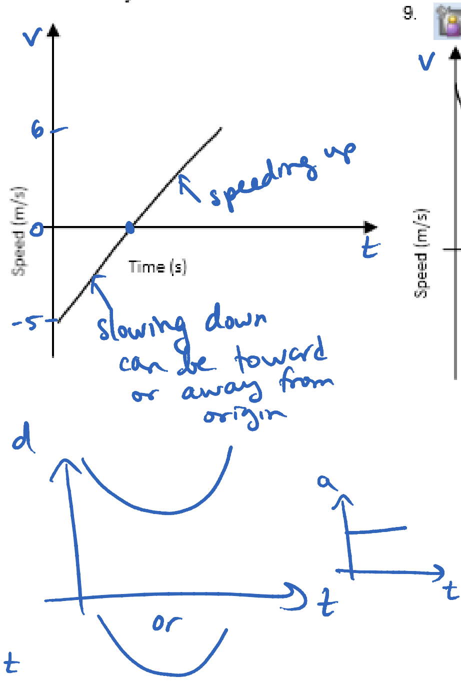
For each of the following v-t graphs

a) sketch a possible d-t graph and a-t graph
b) describe a scenario that would yeild these.

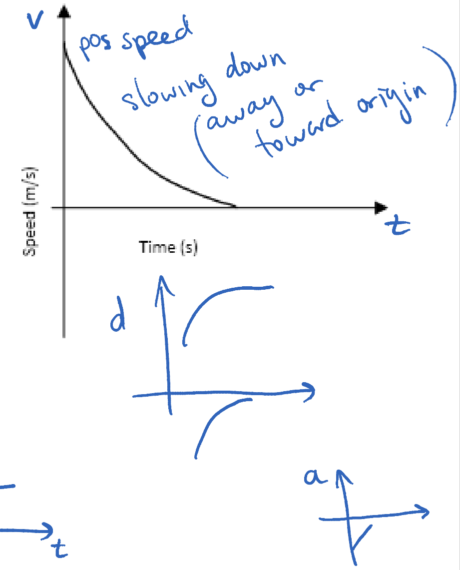
7.



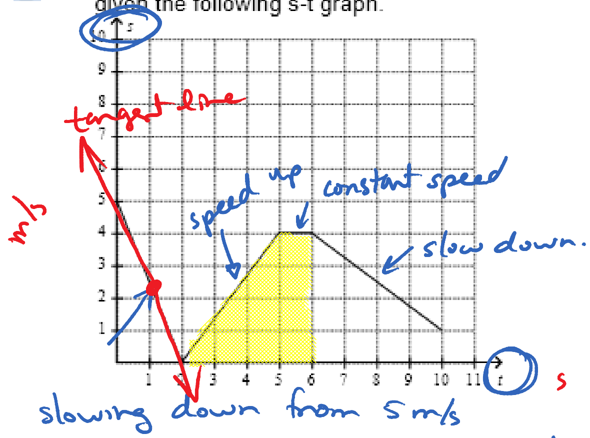
8.

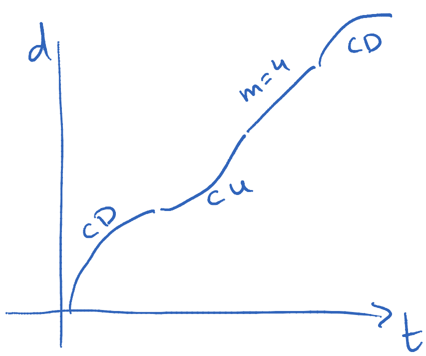


9.



10. Create a story to describe what could be happening given the following s-t graph.





"story" be creative

a. Find the instantaneous rate of change at t=1. What does it represent?

sketch tangent line at t=1

$$m_{tan} = \frac{rise}{run} = \frac{-5}{2} = \frac{m/s}{s} = m/s^2$$

it represent accelation.

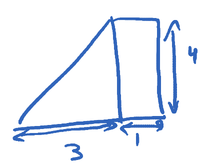
b. Find the average acceleration from t=2 to t=6

average slope of s-t

$$a.r.o.c = \frac{f(6)-f(2)}{6-2} = \frac{4-0}{4} = 1\ \frac{m/s}{s}$$

c. Find the distance travelled from t=2 to t=6

area under s-t graph



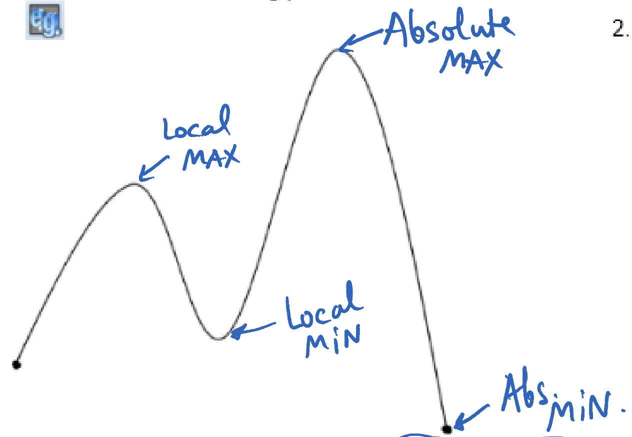
$$A_\triangle = \frac{bh}{2} = \frac{(3)(4)}{2} = 6 \quad + \quad A_\square = lw = (1)(4) = 4$$

∴ distance travelled from t=2 to 6 sec is 10m

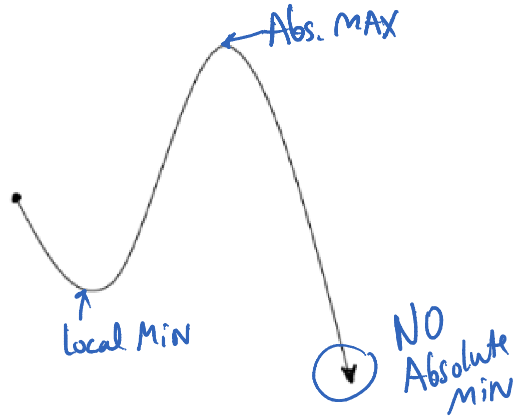
Absolute and Local Extreme points

There are different types of optimal values that you can study. Local Max/Min and Absolute Max/Min. **Absolute Max** is the highest the graph EVER reaches on the given domain, it can occur at the turning points as well as at the endpoints of domain interval. **Local Max** occurs at the turning points of the curve, it may or may not be the highest point overall. On the following pictures label the local and absolute optimal values.

1.



2.



3. Summarize the steps of finding the range or the Absolute optimal values:

(1.) Find any turning points of the function on domain $x \in [a, b]$

(2.) Evaluate all endpts and turning points in the function $f(a) =$

endpts of given interval $f(b) =$

$f(t.p.) =$

(3) Pick the lowest and highest output for Range or MAX/MIN

4. Find the range given the domain

a. $f(x) = -2x + 12$ on $x \in (-5, 11]$ b. $g(x) = -3x^2 + 8$ on $x \in (-2, 3]$ c. $g(x) = 4(0.5)^{-x}$ on $x \in [-5, -1]$

linear

not include -5

include "

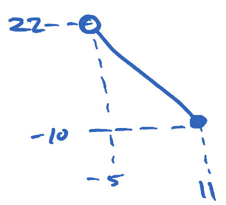
$-3(x-0)^2+8$

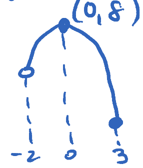
quadratic

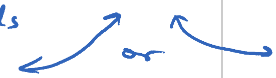
$(0, 8)$

exponentials

no. t.p.







$g(-5) = 4(0.5)^{--5} = 0.125$

$g(-1) = 4(0.5)^{--1} = 2$

$\therefore$ Range

$y \in [0.125, 2]$

or

$0.125 \leq y \leq 2$

$g(-2) = -3(-2)^2+8 = -3(4)+8 = -4$

$g(3) = -3(3)^2+8 = -27+8 = -19$

$g(t.p.) = g(0) = -3(0)^2+8 = 8$

abs. MIN

abs. MAX

Range:

$f(-5) = -2(-5)+12 = 10+12 = 22$

$f(11) = -2(11)+12 = -22+12 = -10$

abs. MIN

$\therefore$ Range: $\{y \in \mathbb{R}, -10 \leq y < 22\}$

OR

$y \in [-10, 22)$

order matters

no Abs MAX

$\therefore$ Range $y \in [-19, 8]$

5. On a merry-go-round, each horse moves up and down in a periodic motion modelled by the function:

$h(t)=\frac{1}{2}\cos\frac{\pi}{15}(t)+1$, where $h(t)$ is the height in meters from the ground and t is the time in seconds

$\pi = 180°$

a. If the merry-go-round does only 5 turns, find the domain and range of this function.

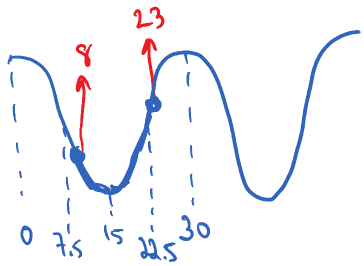
b. If the domain is shrunk to only $8 \le t \le 23$ what are the ABSOLUTE maximum and ABSOLUTE minimum values of the function on this interval?

ⓐ period $= \frac{360°}{k} = 360° \div \frac{180}{15} = 30$

$\therefore$ 5 cycles $\times 30 = 150$

$D = \{t \in \mathbb{R},\ 0 \le t \le 150\}$

$R = \{h \in \mathbb{R},\ 0.5 \le h \le 1.5\}$



b) ABS MAX at $h(23) \doteq 1.05$

ABS MIN at $h(15) = 0.5$

6. Find the ABSOLUTE Max and Min of

a. $f(x)=-(x+3)^2+6$ on x ∈ (– 4, 10)

turning pt. at $x=-3$
check if within interval !!

$f(-4) = -(-4+3)^2+6 = -1+6 = 5$

$f(10) = -(10+3)^2+6 = -169+6 = -163$

$f(t.p) = f(-3) = 6$

$\therefore$ Range $y \in (-163, 6]$

no Abs MiN

abs. MAX at $x=-3$, $y=6$

b. $g(x)=\sqrt{2-x}+10$ on x ∈ [– 14, 2)

no t.p.

$g(-14) = 14$

$g(2) = 10$

$\therefore y \in (10, 14]$

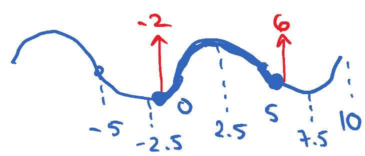
no Abs MiN

Abs MAX at $x=-14$, $y=14$

c. $h(x) = 2\sin\frac{\pi}{5}x+1$ on x ∈ [– 2, 6]

$\pi = 180°$

period $= \frac{360°}{k} = 360° \div \frac{180°}{5} = 10$



t.p at $x = 2.5$

$h(6) \doteq -0.18$

$h(-2) \doteq -0.90$

$h(t.p) = h(2.5) = 3$

$\therefore y \in [-0.90, 3]$

abs MiN at $x=-2$, $y=-0.90$

abs. MAX at $x=2.5$, $y=3$

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7. A golf shot is modeled by the function $H(t) = -5t^2 + 40t$, where $H(t)$ is the height of the golf ball in meters and t is time that it is in the air in seconds.
   a. Find at what time the golf ball will be at a maximum height above the ground.
   b. What is the domain and range that makes sense for this real life situation?
   c. What sign (+/-) do you expect slope will have on the left side/right side/at the maximum?
   d. Prove your prediction from c. by finding the average speed of the ball a.r.o.c.
      i. 0.5 seconds before it reaches maximum height $x \in [3.5, 4]$
      ii. 1.5 seconds after it reaches maximum $x \in [4, 5.5]$
   e. Use the difference quotient to show that instantaneous rate at the answer found in a. is zero.

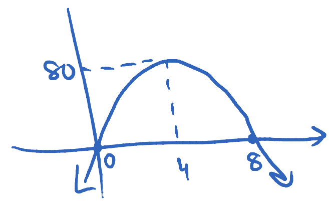
$x = 4$ answer is i.r.o.c $= 0$

a) can't see turning pt. → complete sq. to see vertex $(4, 80)$

$-5(t^2 - 8t + 16 - 16)$

$-5(t-4)^2 + 80$

$\therefore$ at $x = 4$ ball at MAX height



b) $D = \{t \in \mathbb{R},\ 0 \le t \le 8\}$

$R = \{h \in \mathbb{R},\ 0 \le h \le 80\}$

c) on left slope is +

on Right slope is −

d) $\text{a.r.o.c.} = \dfrac{f(4) - f(3.5)}{4 - 3.5} = \dfrac{80 - 78.75}{0.5} = 2.5$ ⊕ on left of $x = 4$

$\text{aroc} = \dfrac{f(5.5) - f(4)}{5.5 - 4} = \dfrac{68.75 - 80}{1.5} = -7.\dot{6}$ ⊖ on right of $x = 4$

e) $\text{a.r.o.c.} = \dfrac{f(4+h) - f(4)}{h} = \dfrac{[-5(4+h)^2 + 40(4+h)] - [-5(4)^2 + 40(4)]}{h}$

$\boxed{1\ \ 2\ \ 1}$ $= \dfrac{-5(16 + 2(4)h + h^2) + 160 + 40h - 80}{h}$

$\text{a.r.o.c} = \dfrac{\cancel{-80} - 40h - 5h^2 + \cancel{160} \cancel{+40h} \cancel{-80}}{h}$

$\text{a.r.o.c} = -5h^2$

$\text{a.r.o.c.} = -5\overline{h}^{h}$ $\xrightarrow{h \to 0}$ $\text{i.r.o.c} = -5(0) = 0$