COMBINATION OF FUNCTIONS – review

NAME:____________________

1.

What is the domain of $f - g$, where $f(x) = \sqrt{x+1}$ and $g(x) = 2\log[-(x+1)]$?

2.

a) If $f(x) = \frac{1}{3x+4}$ and $g(x) = \frac{1}{x-2}$, what is $f + g$?

b) What is the domain of $f + g$?

c) What is $(f + g)(8)$?

3.

Describe or give an example of

a) two odd functions whose sum is an even function

b) two functions whose sum represents a vertical stretch applied to one of the functions

c) two rational functions whose difference is a constant function

4.

Let $f(x) = x^2 - nx + 5$ and $g(x) = mx^2 + x - 3$. The functions are combined to form the new function $h(x) = f(x) + g(x)$. Points $(1, 3)$ and $(-2, 18)$ satisfy the new function. Determine the values of m and n.

5.

If $f(x) = \sqrt{1+x}$ and $g(x) = \sqrt{1-x}$, determine the domain of $y = (f \times g)(x)$.

6.

Is the following statement true or false? "If $f(x) \times g(x)$ is an odd function, then both $f(x)$ and $g(x)$ are odd functions." Justify your answer.

7.

$f(x) = x^2$, $g(x) = \log(x)$. State the domain of $f \div g$.

8.

Given $f = \{(0, 1), (1, 2), (2, 5), (3, 10)\}$ and $g = \{(2, 0), (3, 1), (4, 2), (5, 3), (6, 4)\}$, determine the following values.

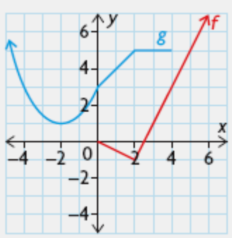
a) $(g \circ f)(2)$ d) $(f \circ g)(0)$

b) $(f \circ f)(1)$ e) $(f \circ f^{-1})(2)$

9.

Use the graphs of f and g to evaluate each expression.

a) $f(g(2))$ c) $(g \circ g)(-2)$

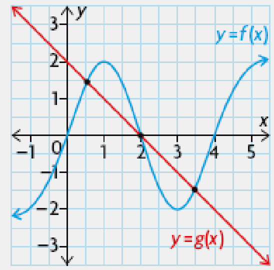


10.

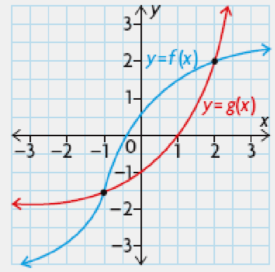
For each graph shown below, state the solution to each of the following:

a) $f(x) = g(x)$ c) $f(x) \le g(x)$

i)



ii)



11.

Find the number of solutions of the following by showing sketches

a) $\cos x = x$, when $x \in \left[0, \frac{\pi}{2}\right]$

b) $\sin(2\pi x) = -4x^2 + 16x - 12$, $0 \le x \le 5$

12.

Give an example of two functions, f and g, such that $f(x) \geq g(x)$ when $x \in [-4, -2]$ or $x \in [1, \infty)$.

13.

Give an example of two functions, f and g, such that $f(x) \geq 0$ when $x \in [-5, 5]$ and $f(x) \geq g(x)$ when $x \in [-4, 5]$.

14. For $f(x) = -4x + 5$ on domain of $[-2, 12]$

$g(x) = -x^2 + 8x$ on domain of $[-5, 10]$

a. Find the ranges of $f(x)$ and $g(x)$ on the provided domains

b. Find domain of $(f - g)(x)$

c. Find domain of $(f \div g)(x)$

d. Find domain and range of $(f \circ g)(x)$

1. Domain of $f(x) = \sqrt{x+1}$

$$x+1 \geq 0$$
$$x \geq -1$$

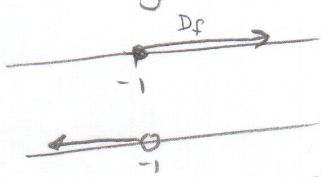
$\therefore D_f = \{x \geq -1, x \in \mathbb{R}\}$

Domain of $g(x) = 2\log[-(x+1)]$

$$-(x+1) > 0$$
$$-x-1 > 0$$
$$-1 > x$$

$\therefore D_g = \{x < -1, x \in \mathbb{R}\}$

$\therefore$ Domain of $(f - g)$ is the intersection of D_f and D_g



intersection is null - nothing!

$\therefore$ there is no graph for $(f-g)$

2. ⓐ $f+g = \dfrac{1}{3x+4} + \dfrac{1}{x-2}$

$$= \frac{x-2+3x+4}{(3x+4)(x-2)}$$

$$= \frac{4x+2}{(3x+4)(x-2)}$$

$$= \frac{2(2x+1)}{(3x+4)(x-2)}$$

2ⓑ domain of $f-g$

$$\{x \in \mathbb{R}, x \neq -4/3, 2\}$$

ⓒ $(f+g)(8) = \dfrac{2(16+1)}{(24+4)(8-2)}$

$$= \frac{34}{(28)(6)} = \frac{34}{168}$$

$$= \frac{17}{84}$$

3. Reminder

- <u>Even function</u>
  if $f(-x) = f(x)$
  or reflect in y-axis gives the same function back
- <u>Odd function</u>
  if $f(-x) = -f(x)$
  or reflect in both axes get back same thing

ⓐ assume f and g are odd

so $f(-x) = -f(x)$
$g(-x) = -g(x)$

then $f+g$ to be even need:

$$f(-x) + g(-x) = f(x) + g(x)$$

sub in

$$-f(x) - g(x) = f(x) + g(x)$$
$$-2g(x) = 2f(x)$$
$$-g(x) = f(x)$$

$\therefore$ pick f and g to be odd functions that are reflections of each other

ex. $f = x^3$ is odd
$g = -x^3$ is odd

$f+g=0$ is a horizontal line at x-axis which is both even + odd.

3(b) many solutions

ex. $f=x$

$g=x$

then $f+g=x+x=2x$

vertical stretch of f or g.

OR

$f=x^2$

$g=3x^2$

then $f+g=x^2+3x^2=4x^2$

vertical stretch of f.

3(c) many solutions

$f=\frac{1}{x}$

$g=\frac{1}{x}$

then $f-g=\frac{1}{x}-\frac{1}{x}=0$ constant

OR

$f=\frac{1}{x+1}+2$

$g=\frac{1}{x+1}$

then $f-g=\frac{1}{x+1}+2-\frac{1}{x+1}=2$ constant.

4. $f(x)=x^2-nx+5$

$g(x)=mx^2+x-3$

$f+g=x^2+mx^2+x-nx+2$

sub pt. $(1,3)$

$3=1+m+1-n+2$

$3=m-n+4$

$\boxed{-1=m-n}$ (1)

sub pt. $(-2,18)$

$18=4+4m-2+2n+2$

$18=4+4m+2n$

$\boxed{14=4m+2n}$ (2)

use elimination (or sub.) method

2×(1) $-2=2m-2n$

(2) $14=4m+2n$

add

$12=6m$

$2=m$

$\therefore -1=m-n$

$-1=2-n$

$n=2+1$

$n=3$

5. $f(x) = \sqrt{1+x}$ domain:

$$1 + x \geq 0$$
$$x \geq -1$$

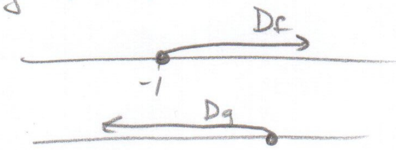
$\therefore D_f = \{x \geq -1, x \in \mathbb{R}\}$

$g(x) = \sqrt{1-x}$ domain:

$$1 - x \geq 0$$
$$1 \geq x$$

$\therefore D_g = \{x \leq 1, x \in \mathbb{R}\}$

$f \times g$ domain is the intersection of both



$\therefore D_{f \times g}$ is $\{-1 \leq x \leq 1\}$

6. $f \times g$ is odd means $f \times g(-x) = -f \times g$

ie. $f(-x) \times g(-x) = -f(x) \times g(x)$

but if f is odd then $f(-x) = -f(x)$

and if g is odd then $g(-x) = -g(x)$

$$f(-x) \times g(-x) = -f(x) \times -g(x)$$
$$= + f(x) \times g(x)$$

not neg.

$\therefore$ False unless f or g or both are zero functions

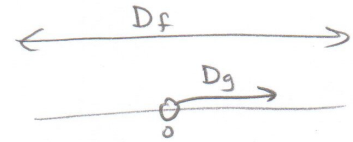
7. $f(x) = x^2$ domain:

$D_f = \{x \in \mathbb{R}\}$

$g(x) = \log x$ domain:

$D_g = \{x > 0, x \in \mathbb{R}\}$

$f \div g$ domain is the intersection PLUS exclude restrictions



$\therefore$ intersection is $\{x > 0\}$

restrictions:

$$\frac{f}{g} = \frac{x^2}{\log x}$$

$\log x \neq 0$ switch form

$10^0 \neq x$

$1 \neq x$

$\therefore D_{\frac{f}{g}} = \{x > 0, x \neq 1, x \in \mathbb{R}\}$

8. $f = \{(0,1)\ (1,2)(2,5)(3,10)\}$

$g = \{(2,0)\ (3,1)\ (4,2)(5,3)\ (6,4)\}$

ⓐ $(g \circ f)(2) = g(f(2))$
$= g(5)$
$= 3$

ⓓ $(f \circ g)(0) = f(g(0))$
$= f(\text{can't})$
N/A

ⓑ $(f \circ f)(1) = f(f(1))$
$= f(2)$
$= 5$

ⓔ $f^{-1} = \{(1,0)\ (2,1)\ (5,2)(10,3)\}$

$f \circ f^{-1}(2) = f(f^{-1}(2))$
$= f(1)$
$= 2$

explain long way too!!

or f and f^{-1} cancel since inverses get input

9. ⓐ $f(g(2))$
$= f(5)$
$= 5$

ⓒ $g \circ g(-2) = g(g(-2))$
$= g(1)$
$= 4$

10. i) graph

ⓐ $f = g$ when $x = 0.5, 2, 3.5$

ⓒ $f \leq g$ when

$(-\infty, 0.5]\ [2, 3.5]$

ii) graph

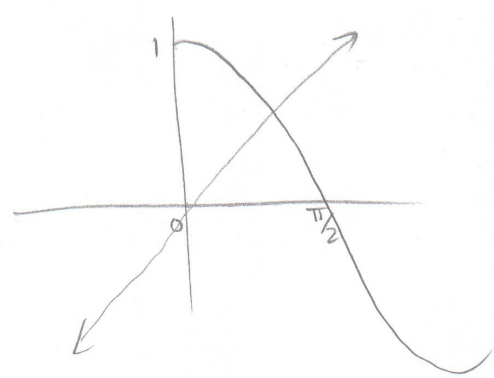
ⓐ $f = g$ when $x = -1, 2$

ⓒ $f \leq g$ when

$(-\infty, -1]\ [2, \infty)$

11. ⓐ $\cos x = x$ on $[0, \pi/2]$

sketch both sides and see how many times they will meet



meet only once

11 (b) $\sin(2\pi x) = -4x^2 + 16x - 12$ on $[0, 5]$

period $= \frac{2\pi}{2\pi} = 1$

vertex: $-4(x^2 - 4x + 4 - 4) - 12$

$-4(x-2)^2 + 16 - 12$

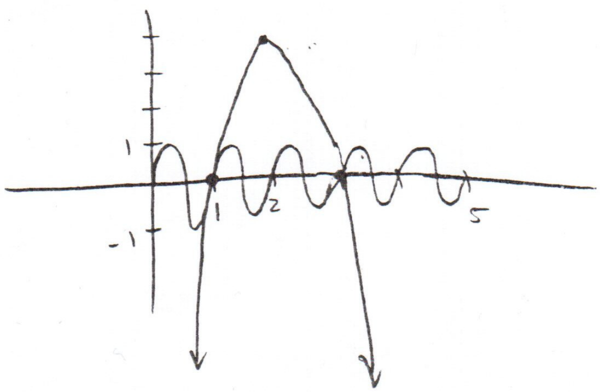
$-4(x-2)^2 + 4$

vertex $(2, 4)$

zeros: $-4(x^2 - 4x + 3)$

$-4(x-3)(x-1)$

at $x = 3, 1$

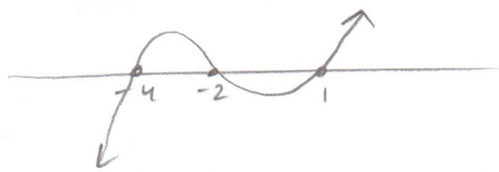


$\therefore$ two solutions on $[0, 5]$

12. $f \geq g$

$f - g \geq 0$ on $[-4,-2]$ $[1, \infty)$

above x' axis on these intervals



$f - g = +(x+4)(x+2)(x-1)$

$= (+x+4)(x^2+x-2)$

$= +x^3 + x^2 - 2x + 4x^2 + 4x - 8$

$= +x^3 + 5x^2 + 2x - 8$

thanks Kiya for catching this mistake

$\therefore$ let $f = +x^3 - 8$

$g = -5x^2 - 2x$

} take any combination like $f = x^3$, $g = -5x^2 - 2x + 8$

13. $f \geq 0$ on $[-5, 5]$ $f(x) = -(x+5)(x-5)$

$g(x) = -(x+4)(x-5)$ $f \geq g$

14 a) range is $y \in$ [abs. MIN, abs. MAX]

recall you must find output values of endpoints and t.p.

$f(x) = -4x + 5$ on $x \in [-2, 12]$

$f(-2) = 13$ ← abs. MAX

$f(12) = -43$ ← abs. MIN

and no t.p to check

$\therefore$ Range of $f = \{y \in \mathbb{R}, y \in [-43, 13]\}$

$g(x) = -x^2 + 8x$ on $x \in [-5, 10]$

$= -x(x-8)$

zeros at $x = 0, 8$

t.p. at $x = 4$

$g(-5) = -65$ ← abs. MIN

$g(10) = -20$

$g(4) = 16$ ← abs. MAX

$\therefore$ Range of $g(x) = \{y \in \mathbb{R}, y \in [-65, 16]\}$

14 b) $f - g = -4x + 5 - (-x^2 + 8x)$

$= x^2 - 12x + 5$

domain of $f-g$ is the intersection of two given domains

$x \in [-2, 12] \cap x \in [-5, 10] = x \in [-2, 10]$

↑ intersection symbol

c) $\dfrac{f}{g} = \dfrac{-4x+5}{-x^2+8x}$

$= \dfrac{-4x+5}{-x(x-8)}$ $x \neq 0, 8$

domain of $\frac{f}{g}$ is the intersection of two given domains AND exclude new restrictions

$\therefore$ $x \in [-2, 10]$, $x \neq 0, 8$

d) $f \circ g = f(g(x)) = -4(-x^2 + 8x) + 5$

$= 4x^2 - 32x + 5$

domain of $f \circ g$ = domain of input $g(x)$ except any restrictions of $f(x)$

range of $f \circ g$ = range of $f(x)$ with input values of range of $g(x)$

input of $g \xrightarrow{g}$ output of g / input of $f \xrightarrow{f}$ output of f

$[-5, 10] \xrightarrow{g} [-65, 16] \xrightarrow{f} [-59, 265]$

Domain of $f \circ g$

Range of $f \circ g$

But only $[-2, 12]$ is allowed in f

$\therefore$ $[-5, 10] \xrightarrow{g} [-65, 16] \xrightarrow{\text{restricted to}} [-2, 12] \xrightarrow{f} [-43, 13]$