2013NOTES

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See below

Polynomial Functions Unit 3

Tentative TEST date ~~Fri. Oct. 18~~ Mon. Oct. 21

parent x^3

Big idea

In this unit will you extend your knowledge from linear, quadratic and cubic functions (learned in gr9-11) to general polynomial functions. Recall that the quadratic function can be written in 3 forms – factored, expanded/standard and vertex forms. When you study polynomials you will also use all three of these forms. The vertex form will just be called the transformed form since a polynomial function may have many turning points, not just one vertex as a parabola did. You will learn that polynomial functions can be described by their end-behaviour, symmetry, number of zeros and number of turning points. You will be introduced to how to sketch the polynomial function from factored form as well as how to find the equation from a given graph. The main part of this unit will be to learn how to factor polynomials, which can be a long process involving long division or synthetic division. You will not learn how to find where exactly the turning points will occur – that is part of calculus.

Corrections for the textbook answers:
Sec 3.2 #12 get rid of the word range
Sec 3.3 #14 graph is wrong in answer
Sec 3.4 #6f) (-11, -3), (-4, -2), (10, 6)

Success Criteria

☐ I understand the new topics for this unit if I can do the practice questions in the textbook/handouts

Date	pg	Topics	# of quest. done? You may be asked to show them	Questions I had difficulty with ask teacher before test!
Oct. 3+4	1-5	INVESTIGATIONS Polynomials, Expanded and Factored Forms Section 3.1 & 3.2 & TWO Handouts	?	
Oct. 7	6-7	Graphing and Finding Equations Section 3.3 & TWO Handouts	?	
Oct. 8	8-10	Transformed Form of Polynomial Functions Section 3.4 & Handout		
Oct. 9 mass / Oct. 10	11-12	Long Division & Synthetic Division Section 3.5 & TWO Handouts		
Oct. 15	13-15	Theorems and Applications THREE Handouts		
Oct. 16	16-18	Factoring Polynomials Section 3.6 & TWO Handouts		
Oct. 17	19-20	Factoring Sum & Difference of Cubes Section 3.7 & Handout		
Oct. 18		REVIEW		

Oct. 21 - TEST

Reflect – previous TEST mark __________, Overall mark now__________.

What is a Polynomial Function?

1. How can you tell if the equation or graph or table that is given is of a polynomial function?

Equation: expanded version of polynomial equation has every term in a form of ax^n, $a \in \mathbb{R}$, $n \in W$ 0, 1, 2, ...

Graphs: Continuous + smooth graph with infinite end behaviour.

Table: polyn of "degree" n will have n^{th} constant difference

2. Identify whether the folowing are considered to be polynomial functions.

a. $y = 2x^1 - bc^0$ yes

b. $f(x) = 3x^{\frac{1}{2}} - x$ not a whole #. NO.

c. $h(x) = \sqrt[3]{x} = x^{1/3}$, not a whole # exp.

d. $y = x^3 + 2x^2 - x + 11$ YES

e. $y = -0.2(4x-3)(x+3)$ YES.

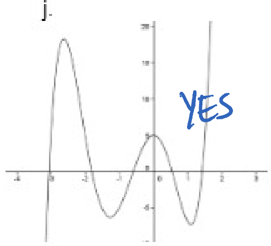
f. $y = x^{-3} + 2x^2 - x^{-1} + 11$ NO.

g. $x = -6$ vertical line not a function

h. $y = 2x^0$ YES.

i. $y = \frac{1}{x} = x^{-1}$ NO $\frac{1}{x}$

j.



YES

k.

x	y
-3	-27
-2	-8
-1	-1
0	0
1	1
2	8

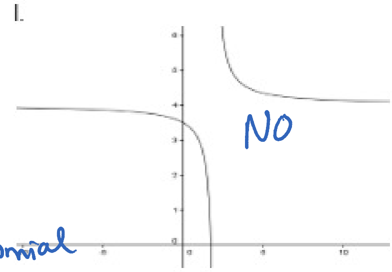
Δy: +19, +7, +1, +1, +7

$\Delta\Delta y$: -12, -6, 0, +6

$\Delta\Delta\Delta y$: +6, +6, +6

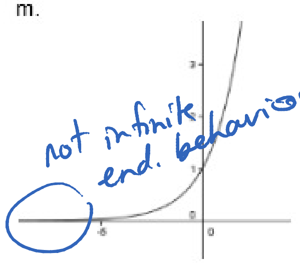
∴ cubic polynomial

l.



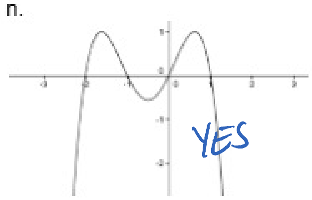
NO

m.



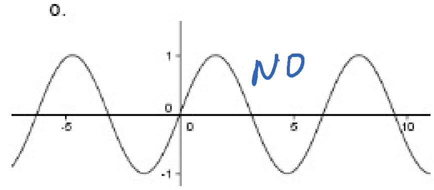
not infinite end. behaviour

n.



YES

o.



NO

3. How can you find the degree and the leading coefficient of a polynomial? Explain using these examples

$y = 3x - 9x^3 + 10x^6 - x^2 + 8$

$y = 10x^6 - 9x^3 - x^2 + 3x + 8$

Leading Coefficient = L.C. = 10

Degree = Deg = 6

$y = -2(x-4)(3x^3 - x^2)(1 - x^4)$

rough: $-2(x)(3x^3)(-x^4) = 6x^8$

∴ LC = 6

Deg = 8

transformed (x appears once!)

factored

standard/expanded

$5(-3x)^6$ $-1(-4x)^3$

4. Identify the form of the polynomial, the degree and the leading coefficient.

a. $y=(x+1)^3(1-2x)(5x^4+x^3+1)$ — $-10x^8$
Form: factored
L.C = −10
Deg = 8

b. $y=x^3-3x+7x^4-9x^5+1$
standard
L.C = −9
Deg = 5

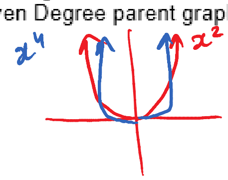
c. $y=5(1-3x)^6+8$
transformed
L.C = 3645
Deg = 6

d. $y=-(1-4x)^3$
transf or factored
L.C = 64
Deg = 3

INVESTIGATE Transformed Form of Polynomials.

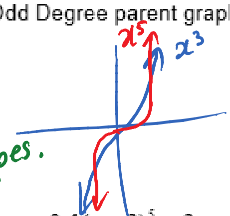
5. Investigate how does the degree affect the shape of the graph?

a. Even Degree parent graphs:



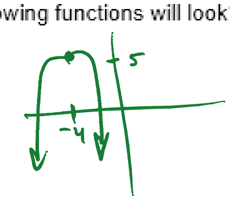
* if x appears once the graph will resemble these parent shapes.

b. Odd Degree parent graphs:

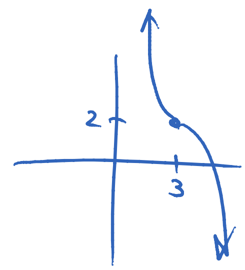


6. How do you think the following functions will look?

a. $y=-(x+4)^4+5$
reflect in x-axis
left
up



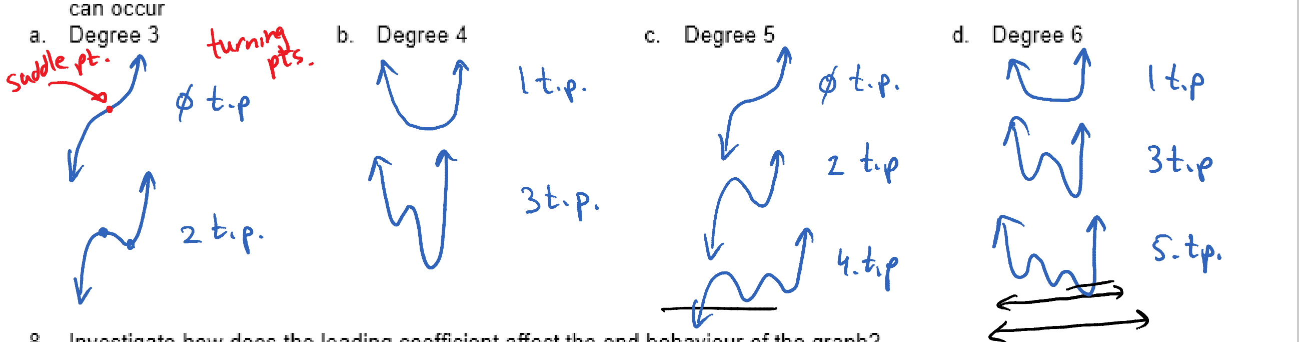
b. $y=-0.1(x-3)^5+2$
- reflect in x-axis
- vertical compress.
right
up



INVESTIGATE Standard/Expanded Form of Polynomials.

7. Using technology, investigate how does having many terms affect the shape of the graph. Draw all possible twists that can occur

a. Degree 3 b. Degree 4 c. Degree 5 d. Degree 6



8. Investigate how does the leading coefficient affect the end behaviour of the graph?

ODD degree + pos. L.C. as $x\to\infty, y\to\infty$; as $x\to-\infty, y\to-\infty$
neg. L.C. as $x\to\infty, y\to-\infty$; $x\to-\infty, y\to\infty$

EVEN. pos L.C. as $x\to\pm\infty, y\to\infty$
neg L.C. as $x\to\pm\infty, y\to-\infty$

9. What conclusions can you make about the range of polynomial graphs?

Odd deg. range: $y\in\mathbb{R}$ $(-\infty,\infty)$

Even. Deg. range: $y\in\mathbb{R}$, $y\geq$ Abs. Min or $y\leq$ Abs. MAX.

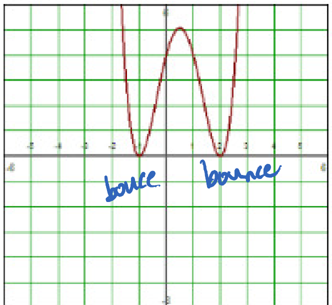
10. Summarize
Absolute maximum/minimum can only occur for ___EVEN.___ degree polynomials.
A polynomial of EVEN degree n can have at least ___none___, and at most ___n___ zeros.
A polynomial of EVEN degree 8 can have ___7, 5, 3, 1___ turning points
A polynomial of ODD degree n can have at least ___one___, and at most ___n___ zeros.
A polynomial of ODD degree 11 can have ___10, 8, 6, 4, 2, ø___ turning points.

INVESTIGATE Factored Form of Polynomials.

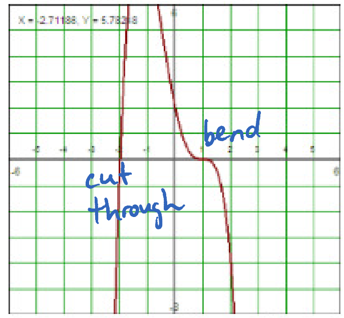
As you may have noticed from above, polynomial graphs behave near the x-axis in several different ways: The graph can "bounce off", "cut through", or "bend" near the roots (zeros).

11. For each graph and equation given find the relationship between the power of the factor in the equation and whether the graph would "bounce", "cut" or "bend" through zeros.

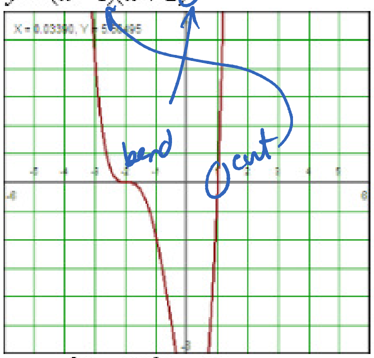
1. $y = (x+1)^2(x-2)^4$



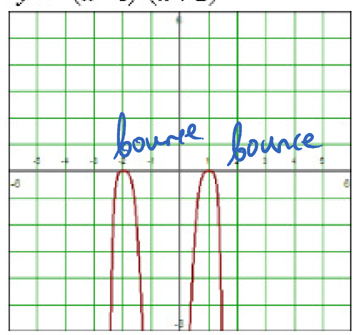
2. $y = -(x-1)^3(x+2)$



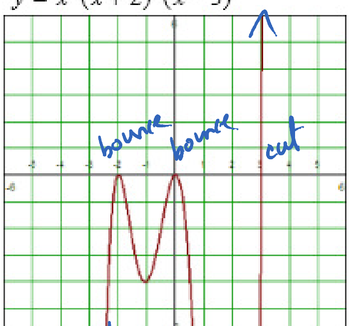
3. $y = (x-1)(x+2)^3$



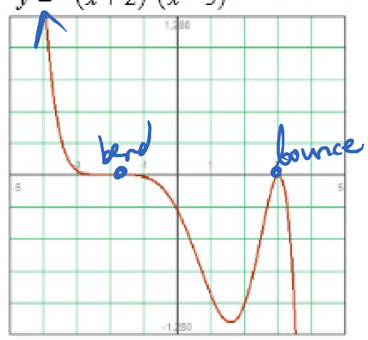
4. $y = -(x-1)^4(x+2)^4$



5. $y = x^2(x+2)^2(x-3)$



6. $y = -(x+2)^5(x-3)^2$



12. Summarize:

$(x+1)^2 = (x+1)(x+1)$

When a polynomial function has a repeated **even** root, the graph of the function ____bounce____ at that root ie. looks quadratic near the zero

When a polynomial function has a repeated **odd** root greater than 1, the graph of the function ____bend____ at that root. ie. looks cubic near the zero

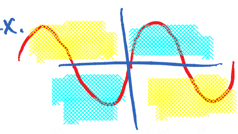
When a polynomial function has a root of order 1, the graph of the function ____cut through____ at that root. ie. look linear near the zero.

INVESTIGATE Symmetry of Polynomials.

13. You just finished looking at Odd and Even Degree Polynomials. Please don't confuse the words Odd Function with Odd Degree Function. These are totally different concepts. Recall what you learned about symmetry: summarize here how something can have odd symmetry, or even symmetry or neither symmetry – graphically and algebraically.

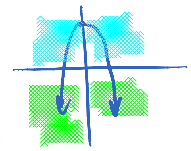
Odd function : $-f(-x) = f(x)$ or $f(-x) = -f(x)$

ex.



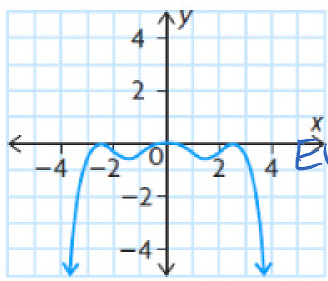
Even function : $f(-x) = f(x)$

ex.



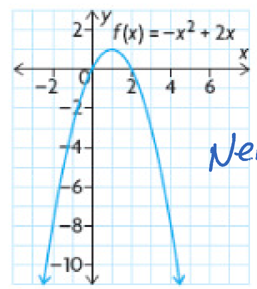
14. For each graph or equation given determine the degree & symmetry.

a. $f(x) = -0.01(x-0.5)(x+0.5)(x-2.5)^2(x+2.5)^2$



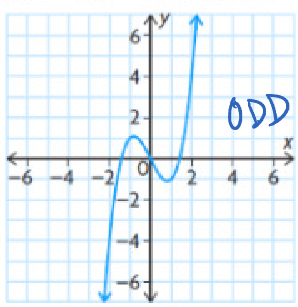
EVEN

b.



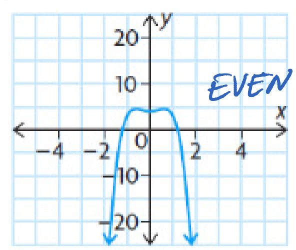
Neither

c. $f(x) = x(x-\sqrt{2})(x+\sqrt{2})$



ODD

d. $y = -4x^4 + 3x^2 + 4$



EVEN

e. $f(x) = x^4 - 2x^2 + 1x^0$

even: $f(-x) = (-x)^4 - 2(-x)^2 + 1 = x^4 - 2x^2 + 1$

odd: $-f(x) =$

$\therefore$ EVEN

f. $f(x) = x^2 - 3x + 4$

$f(-x) = (-x)^2 - 3(-x) + 4 = x^2 + 3x + 4$

$-f(x) = -(x^2 - 3x + 4) = -x^2 + 3x - 4$

$\therefore$ Neither.

g. $f(x) = x^5 - 3x^7 + 9x^0$

$f(-x) = (-x)^5 - 3(-x)^7 + 9 = -x^5 + 3x^7 + 9$

$-f(x) = -(x^5 - 3x^7 + 9) = -x^5 + 3x^7 - 9$

$\therefore$ neither

h. $f(x) = x^5 - 3x^1$

$f(-x) = (-x)^5 - 3(-x) = -x^5 + 3x$

$-f(x) = -(x^5 - 3x) = -x^5 + 3x$

$\therefore$ odd

15. Summarize:

Odd degree polynomial can have ___odd___ or ___neither___ symmetry. It will be symmetric only if expanded form has ___all odd powers on each x___

Even degree polynomial can have ___even___ or ___neither___ symmetry. It will be symmetric only if expanded form has ___all even powers on each x___, constant #'s have x^0 on them.

Graphing and Finding Equations

1. Summarize the steps of sketching polynomials given expanded (standard) form. Explain what details you still don't know about the graph.

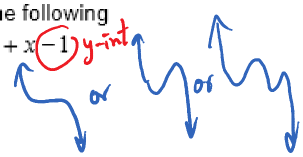
(1.) Find degree + L.C. this will give you the end behaviour

(2.) Sketch all possible ways the polyn can "twist" in the middle.

From standard form we can't know exactly how many "twists"/t.p/zeros

2. Sketch each of the following

a. $f(x)=4x^2-5x^5+x-1$ y-int

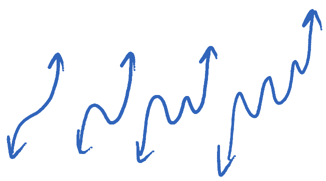


Deg = 5
L.C = -5
t.p: 4, 2, ∅

b. $f(x)=3-7x+8x^4-x^3$

Deg = 4
L.C = 8
t.p = 3, 1

c. $f(x)=11x-9x^4+10x^7$



Deg = 7
LC = 10
t.p. 6, 4, 2, ∅

d. $f(x)=-6x^6+8x^2-x^3+9$

Deg = 6
L.C = -6
t.p: 5, 3, 1

3. Summarize the steps of sketching polynomials given factored form. Explain what details you still don't know about the graph.

(1.) Find deg + L.C. to figure out + sketch end behaviour

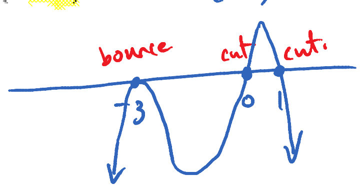
(2.) Find the zeros and look at the powers on each factor to determine whether it bounces/cut/bends near that zero.

Factored form does not tell you how high/low t.p. are.

4. Sketch each of the following

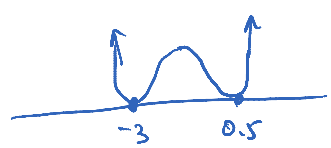
a. $y=(x)(x+3)^2(2-2x)$ $\quad x\cdot x^2\cdot(-2x)$

Deg: 4
L.C. = -2



b. $y=(x+3)^2(2-4x)^2$

Deg: 4
L.C = 16

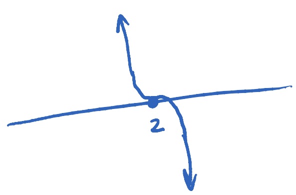


c. $y=(2-x)^3$

Deg = 3
L.C = -1

$(-x+2)^3$

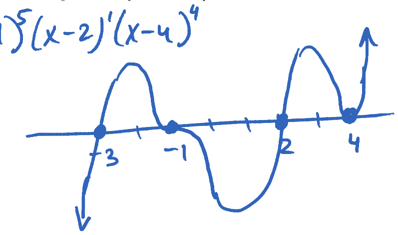
$1(-1(x-2))^3+0$ (a, k, d, c)



d. positive leading coefficient,
zeros at -3, -1 (order 5), 2, 4 (order 4)

$+\ (x+3)^1(x+1)^5(x-2)^1(x-4)^4$

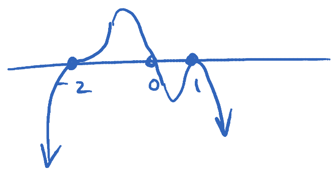
Deg 11
L.C (+)



Sketch each of the following

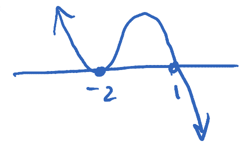
e. $y = -x(x+2)^3(1-x)^4$ $\quad -x(x)^3(-x)^4 = -x^8$

Deg = 8
LC = −1



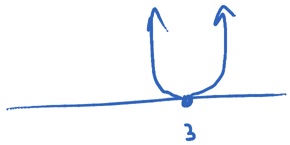
f. $y = -3(2x+4)^2(x-1)$

Deg = 3
LC = −12



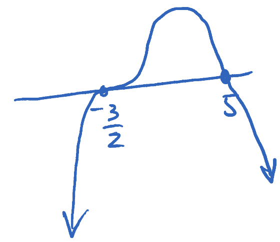
g. $y = (6-2x)^4$

Deg = 4
L.C = 16



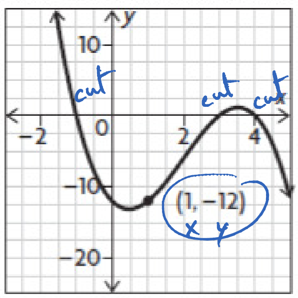
h. $y = (2x+3)^3(5-x)$

Deg = 4
LC = −8



5. Find the equations for the given graphs

a. polynomial of degree 3 with the following graph



$y = a(x+1)^1(x-3)^1(x-4)^1$

$-12 = a(1+1)(1-3)(1-4)$

$-12 = a(2)(-2)(-3)$

$-12 = +12a$

$-1 = a$

$\therefore y = -1(x+1)(x-3)(x-4)$

b. The function is cubic with zeros at 4(order 2) and -1 and with a y-intercept of 10.

$y = a(x-4)^2(x+1)$ $\quad$ pt. $(0, 10)$ $\quad x, y$

$10 = a(0-4)^2(0+1)$

$10 = 16a$

$\frac{5}{8} = a$

$\therefore y = \frac{5}{8}(x-4)^2(x+1)$

6. The function $f(x) = kx^3 + 20x^2 - 8x - 96$ has a zero when x= – 3. Determine the value of *k*.

$0 = k(-3)^3 + 20(-3)^2 - 8(-3) - 96$

$0 = -27k + 108$

$-108 = -27k$

$4 = k$

Transformed Form of Polynomials

1. Write out the transformed form of a polynomial of degree n. Summarize what type of transformations the letters control and recap the steps of transforming a function.

$y = a\left[k(x-d)\right]^n + c$

* Factor out "k" to see "d" (there should be a bracket between k and x.)

a → reflection in x-axis $a<0$
a → vertical stretch $|a|>1$ / compress $|a|<1$

k → reflect in y-axis $k<0$
k → horiz. stretch $|k|<1$ / compress $|k|>1$

d → shift left/right
c → shift up down

Accurate graphs :

x	y=parent	$\frac{x}{k} \pm d$	$y \cdot a \pm c$

Sketch : can ignore stretch/compressions.

2. Sketch the following

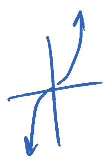
$1 \div \frac{1}{4} = 1 \times \frac{4}{1}$

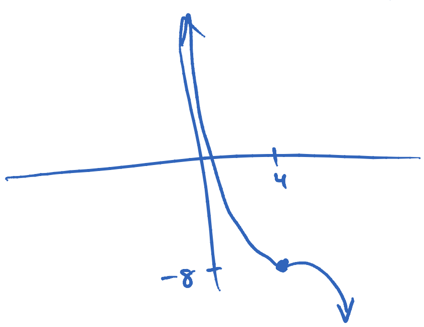
a. $f(x) = -2(\frac{1}{4}x - 1)^3 - 8$

$= -2\left[\frac{1}{4}(x-4)\right]^3 - 8$

a: reflect in x-axis, vertical stretch
k: horiz stretch
d: right shift
c: down shift.

parent : $y = x^3$



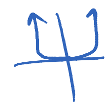


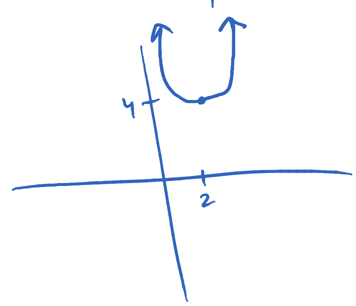
b. $f(x) = 3(10 - 5x)^4 + 4$

$= 3\left[-5(x-2)\right]^4 + 4$

3: vertical stretch
−5: reflect in y-axis, horiz. compress
d: right
+4: up

parent $y = x^4$

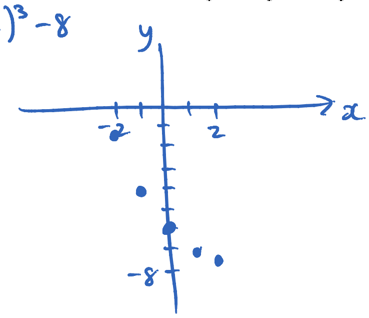




3. Explain why it is important to follow the steps outlined above instead of just taking the x values below and just plugging them into the equation. Use question a. as an example in your explanation.

x	y
-2	-1.25
-1	-4.1
0	-6
1	-7.2
2	-7.8

$y = -2(\frac{1}{4}x - 1)^3 - 8$

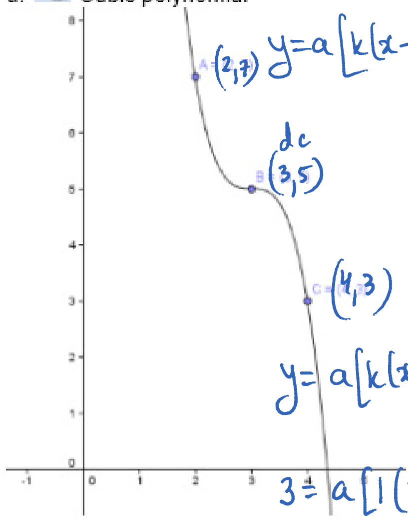


Transformations are important since they provide a better picture of the "interesting" part of the graph.

when finding "a" or "k" – you can let one of them be ONE, and solve for the other (except for sinusoidals)

4. Find the equations for the following graphs. Be careful not all pictures represent a transformed form of a parent!!!

a. Cubic polynomial



$y=a[k(x-d)]^3+c$

$y=a[k(x-3)]^3+5$ let $k=1$

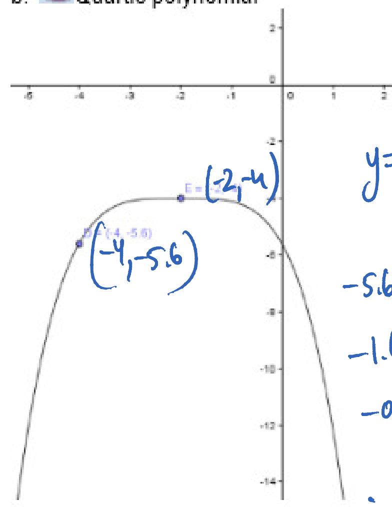
$3=a[1(4-3)]^3+5$

$3=a+5$

$-2=a$

$\therefore y=-2[1(x-3)]^3+5$

b. Quartic polynomial



$y=a[k(x+2)]^4-4$

let $k=1$

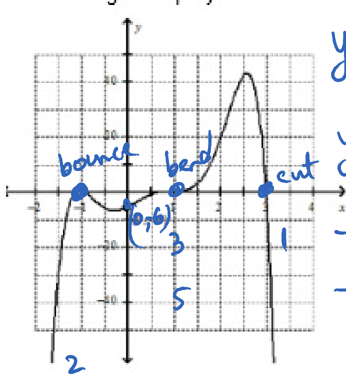
$-5.6=a[(-4+2)]^4-4$

$-1.6=16a$

$-0.1=a$

$\therefore y=-0.1(x+2)^4-4$

c. Degree 6 polynomial



$y=a(x+1)^?(x-1)^?(x-3)^?$

$y=a(x+1)^2(x-1)^3(x-3)^1$

$-6=a(0+1)^2(0-1)^3(0-3)^1$

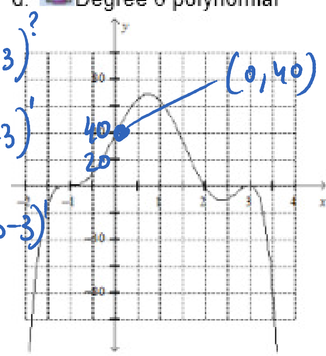
$-6=a(1)(-1)(-3)$

$-6=3a$

$-2=a$

$\therefore y=-2(x+1)^2(x-1)^3(x-3)$

d. Degree 6 polynomial



$y=a(x+1)^3(x-2)(x-3)^2$

$40=a(0+1)^3(0-2)(0-3)^2$

$40=a(1)(-2)(9)$

$\frac{40}{-18}=a$

$\therefore y=-\frac{20}{9}(x+1)^3(x-2)(x-3)^2$

crosses x-axis?

5. Determine the zeros of the following polynomials

a. $y = 3(4x-8)^4 - 243$

sub $y=0$

~~SAMDEB~~

$0 = 3(4x-8)^4 - 243$

$\frac{243}{3} = \frac{3(4x-8)^4}{3}$

$\pm\sqrt[4]{81} = \sqrt[4]{(4x-8)^4}$

$\pm 3 = 4x - 8$

$\frac{\pm 3 + 8}{4} = \frac{4x}{4}$

$x = \frac{11}{4}$ or $x = \frac{5}{4}$

b. $y = 2(2x-10)^3 + 128$

c. Why can't the above method be used on the following $y = -5x^4 - 4x^2 + 1$?

Impossible to isolate for x using SAMDEB when x appears many times.

d. What method do you know of that could help you find the roots of $y = -5x^4 - 4x^2 + 1$?

let $a = x^2$

"pseudo-quadratic"

$y = -5a^2 - 4a + 1$

$0 = (-5a + 1)(a + 1)$

-5 $\quad$ 1

1 $\quad$ 1

one neg

$0 = (-5x^2 + 1)(x^2 + 1)$

$0 = (1 - 5x^2)(x^2 + 1)$

$0 = (1 + \sqrt{5}x)(1 - \sqrt{5}x)(x^2 + 1)$

$x\sqrt{5}$

$1 + \sqrt{5}x = 0$ $\quad$ $1 - \sqrt{5}x = 0$ $\quad$ $x^2 + 1 = 0$

$x = -\frac{1}{\sqrt{5}}$ $\quad$ $x = \frac{1}{\sqrt{5}}$ $\quad$ N/A.

e. Would the method you used for d. work on $y = -5x^4 - 4x^3 + 1$?

this is not a pseudo-quadratic so we don't know how to factor this

Wait 2 more lessons.

Long Division and Synthetic Division of Polynomials

1. Remind yourself how to do long division with numbers, since the same approach will be used for polynomial division.

a. Identify where the following terms would be located on the long division table: Quotient, Divisor, Dividend, Remainder, then find the quotient and remainder.

Divisor → 4) 25697 ← Dividend

06424 ← Quotient

$$
\begin{array}{r}
25697 \\
-24 \\
\hline
16 \\
-16 \\
\hline
09 \\
-8 \\
\hline
17 \\
-16 \\
\hline
1
\end{array}
$$

1 ← Remainder

b. State the formula that relates all of these terms together and modify it to the one that has dividend – which will always be the function, f(x) – isolated.

$$\boxed{\text{Divisor} \times \frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} \times \text{Divisor} + \frac{\text{Remainder}}{\text{Divisor}} \times \text{Divisor}}$$

$$\frac{25697}{4} = 6424 + \frac{1}{4}$$

$$\boxed{\text{Dividend} = (\text{Quotient})(\text{Divisor}) + \text{Remainder}}$$

2. Provide an explanation of how to do long division.

1.) Fill in any missing terms with zeros.
2.) Divide only 1st terms you see
3.) Multiply the result of step 2 with whole divisor
4.) Subtract
5.) Repeat after dropping down next term.

3. Provide an explanation of how to do synthetic division.

1.) Record the "root" of divisor outside and only coefficients of dividend inside for missing terms use 0
2.) Carry down 1st coefficient.
3.) Multiply ...
4.) Add ...

} see an example

4. Divide using long division, write down the result in TWO different ways

a. $(2x^4 - 5x^2 + 3) \div (x + 4)$

$$
\begin{array}{r|l}
 & 2x^3 - 8x^2 + 27x - 108 \quad \leftarrow \text{quotient} \\
x+4 & 2x^4 + 0x^3 - 5x^2 + 0x + 3 \\
 & -(2x^4 + 8x^3) \\
\hline
 & \cancel{0} \; -8x^3 - 5x^2 \\
 & -(-8x^3 - 32x^2) \\
\hline
 & \cancel{0} \; + 27x^2 + 0x \\
 & -(27x^2 + 108x) \\
\hline
 & -108x + 3 \\
 & -108x - 432 \\
\hline
 & +435 \leftarrow \text{Remainder}
\end{array}
$$

$$\therefore \frac{2x^4 - 5x^2 + 3}{x+4} = 2x^3 - 8x^2 + 27x - 108 + \frac{435}{x+4}$$

$$\therefore 2x^4 - 5x^2 + 3 = (2x^3 - 8x^2 + 27x - 108)(x+4) + 435$$

b. $(12x^4 - 6x^3 - x + 81) \div (3x^2 - 9)$

$$
\begin{array}{r|l}
 & 4x^2 - 2x + 12 \\
3x^2 + 0x - 9 & 12x^4 - 6x^3 + 0x^2 - x + 81 \\
\ominus & -12x^4 - 0x^3 + 36x^2 \\
\hline
 & -6x^3 + 36x^2 - x \\
\ominus & +6x^3 + 0x^2 - 18x \\
\hline
 & 36x^2 - 19x + 81 \\
\ominus & -36x^2 + 0x + 108 \\
\hline
 & -19x + 189
\end{array}
$$

$$\therefore \frac{12x^4 - 6x^3 - x + 81}{3x^2 - 9} = 4x^2 - 2x + 12 + \frac{-19x + 189}{3x^2 - 9}$$

OR

$$12x^4 - 6x^3 - x + 81 = (4x^2 - 2x + 12)(3x^2 - 9) - 19x + 189$$

c. $(3x^3 - x^2 + 10x) \div (x-2)$

$f(x) = (3x^2 + 5x + 20)(x-2) + 40$

d. $(10x^3 - 14x + 5) \div (x+1)$

$f(x) = (10x^2 - 10x - 4)(x+1) + 9$

5. Divide using synthetic division, don't forget to write down the result.

a. $(4x^5 - 2x^4 + 3x - 9) \div (x-2)$

$f(x) =$

dividend

root of divisor

$$\begin{array}{c|cccccc} 2 & 4 & -2 & 0 & 0 & 3 & -9 \\ & & 8 & 12 & 24 & 48 & 102 \\ \hline & 4 & 6 & 12 & 24 & 51 & \boxed{93} \end{array}$$

carry down ×2 add

Remainder

Quotient

Dividend

$f(x) = (4x^4 + 6x^3 + 12x^2 + 24x + 51)(x-2) + 93$

b. $(2x^3 + 5x^2 - 4x - 5) \div (2x+1)$ ← f(x)

$$\begin{array}{c|cccc} -\frac{1}{2} & 2 & 5 & -4 & -5 \\ & & -1 & -2 & 3 \\ \hline & 2 & 4 & -6 & \boxed{-2} \end{array} = R$$

Careful!!

$\therefore f(x) = (2x^2 + 4x - 6)(x + \frac{1}{2}) - 2$

c. $(x^3 - 8x^2 - 4x - 7) \div (x^2 + 1)$ non-Linear divisor $\therefore$ synthetic doesn't work.

f(x)

$$\begin{array}{r} x - 8 \\ x^2 + 0x + 1 \overline{\big) x^3 - 8x^2 - 4x - 7} \\ -\underline{(x^3 + 0x^2 + 1x) } \\ -8x^2 - 5x - 7 \\ \underline{-8x^2 + 0x - 8} \\ -5x + 1 \end{array}$$

$\therefore f(x) = (x-8)(x^2+1) \quad -5x+1$

d. $(12x^4 - 20x^3 - 4x + 24) \div (4x-8)$

f(x)

$$\begin{array}{c|ccccc} 2 & 12 & -20 & 0 & -4 & 24 \\ & & 24 & 8 & 16 & 24 \\ \hline & 12 & 4 & 8 & 12 & \boxed{48} \end{array} = R$$

$\therefore f(x) = (12x^3 + 4x^2 + 8x + 12)(x-2) + 48$

6. What are the disadvantage(s) of long division?

- lots of writing
- subtract instead of add.

7. What are the disadvantages of synthetic division?

- doesn't work for non-Linear divisors
- when divisor has a coefficient on x you should be careful to record (x − root) instead of original divisor.

Theorems and Applications

Remainder Theorem:

When a polynomial $f(x)$ is divided by a linear divisor $(x-a)$, then the remainder is equal to $f(a)$.

(If the divisor is not linear then the remainder might not be just one constant number but a polynomial – and this theorem will not work)

Proof:

divident

$$\boxed{f(x)=(quotient)(divisor)+remainder}$$

divisor is $x-a$

$f(x)=(quotient)(x-a)+remainder$

sub in the zero of the divisor $x=a$

$f(a)=(quotient)(0)+remainder$

$f(a)=remainder$

1. Find the remainder (without doing the division) of $f(x)=x^3-12x^2-42$ when it is divided by $x-3$

$R=f(3)=3^3-12(3)^2-42=-123$

check with synthetic:

3	1	−12	0	−42
	↓	3	−27	−81
	1	−9	−27	$\boxed{-123}$ = R

2. Determine the remainder, r, to make this multiplication statement true

$(x^2-4)(3x^2+1)+r=3x^4-10x^2+5$

quot. divisor dividend

thm. won't work here.

FOIL, isolate r

$\cancel{3x^4}+x^2-12x^2-4+r=\cancel{3x^4}-10x^2+5$

$-3x^4$

$r=x^2+9$

3. If the dividend is $x^4+x^3-7x^2+6x-2$, quotient is $x^3-3x^2+5x-14$ and remainder is 54, find the divisor by using comparing coefficients method.

dividend = (quot.)(divisor) + rem.

given 3 of 4 things in the formula

$x^4+x^3-7x^2+6x-2=(x^3-3x^2+5x-14)(ax+b)+54$

linear? quad? ?

∴ Divisor is $ax+b$
$=1x+4$

compare x^4 term

LS = RS

$1x^4=ax^4$

$1=a$

compare x^3

LS = RS

$x^3=bx^3+-3ax^3$

$1x^3=(b-3a)x^3$

$1=b-3a$

$1=b-3(1)$

$4=b$

faster to do 1st + Last

compare constant term

$-2=54-14b$

$-56=-14b$

$4=b$

13

TEAM.

4. Find the remainder (without doing the division) of $f(x)=2x^5+4x^4-5x^3-6x^2+2x-4$ when it is divided by $x+3$

$\therefore R=-91$ ✓

Team

5. If the divisor is x^2+5x+8, the dividend is $x^4-5x^3+3x^2-7x+11$. Find the remainder.

$\therefore R=-152x-349$ ✓ ✓

6. If the dividend is $-2x^4-5x^3+2x^2+3x-1$, quotient is $-2x^3-7x^2-5x-2$ and remainder is – 3, find the divisor by using comparing coefficients method.

$$-2x^4-5x^3+2x^2+3x-1=(-2x^3-7x^2-5x-2)(ax+b)-3$$

compare x^4

$-2x^4=-2ax^4$

$\div -2x^4 \quad \div -2x^4$ isolate

$1=a$ ✓

compare constant term

$-1=-2b-3$

$+3$

$2=-2b$

$-1=b$ ✓

$\therefore$ Divisor is $(x-1)$ ✓

7. Remind yourself what is the definition of the word "factor"

↳ divides evenly

or remainder is zero.

by remainder theorem $f(a)=0$ ✱ see this in factor th.

many polynomials have irrational zeros

Rational Root Theorem:

If a polynomial $y = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$ has any rational (integer or fractional) zeros then the zeros are from the list of all combinations of $\pm \frac{p}{q}$, where p is an integer factor of a_0 and q is an integer factor of a_n.

(Keep in mind that zeros can also be irrational – these can only be found using technology. In university you'll study how the technology is programmed to find the zeros using different approximation methods)

8. Find the list of possible zeros of $f(x) = 3x^3 - 5x^2 + 5x - 2$

q p

$\pm \frac{p}{q}: \pm\frac{1}{1}, \pm\frac{2}{1}$ $\pm\frac{1}{3}$ $\pm\frac{2}{3}$

9. Find the list of possible zeros of $f(x) = 2x^3 + 3x^2 - 18x + 8$

$\pm\frac{p}{q}: \pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{4}{1}, \pm\frac{8}{1}$

$\pm\frac{1}{2}, \cancel{\pm\frac{2}{2}}, \cancel{\pm\frac{4}{2}}, \cancel{\pm\frac{8}{2}}$

Factor Theorem:

The polynomial $f(x)$ has a factor $(x-k)$ if and only if (this means the implication goes both ways) $f(k) = 0$

Proof - "if" implication:

if $x-k$ is a factor of $f(x)$
then it divides into $f(x)$ evenly
This means the remainder is zero.
By Remainder theorem $f(k) = remainder$
Therefore $f(k) = 0$, since remainder is zero.

Proof - "only if" implication:

if $f(k) = 0$
then by Remainder theorem $remainder = 0$
but that implies the divisor $x-k$ goes into $f(x)$ evenly
which means the divisor $x-k$ is a factor.

10. Which of the numbers from the list you've made for $f(x) = 3x^3 - 5x^2 + 5x - 2$ above, are actual zeros?

$f(1) = 1$
$f(-1) = -15$
$f(2) = 12$
$f(-2) = -56$
$f(\frac{1}{3}) = -7/9$
$f(-\frac{1}{3}) = -4\frac{1}{3}$
$\boxed{f(2/3) = 0}$
$f(-2/3) = -8\frac{4}{9}$

$f(x) = (x - 2/3)(ax^2 + bx + c)$

11. Determine if the binomial $2x - 3$ is a factor of divides evenly.

$f(x) = 2x^5 + 3x^3 + 6$

$f\left(\frac{3}{2}\right) \overset{?}{\neq} 0$ $\therefore (2x-3)$ is not a factor of $f(x)$

12. Which of the numbers from the list you've made for $f(x) = 2x^3 + 3x^2 - 18x + 8$ above, are actual zeros? Use your answers to factor the polynomial completely.

$f(1) =$
$f(-1) =$
$\boxed{f(2) = 0}$
$f(-2) =$
$f(4) =$
$\boxed{f(-4) = 0}$
$f(8) =$
$f(-8) =$
$\boxed{f(\frac{1}{2}) = 0}$
$f(-\frac{1}{2}) = 17.5$

$f(x) = a(x-r)(x-t)(x-q)$
$f(x) = 2(x-2)(x+4)(x-\frac{1}{2})$

13. Determine if the binomial $x + 4$ is a factor of

$f(x) = x^3 + 5x^2 + 2x - 8$

$f(-4) \overset{?}{=} 0$ ✓ $\therefore (x+4)$ is a factor of $f(x)$

$f(x) = (x+4)(ax^2 + bx + c)$

Factoring Polynomials

1. Find the prime factorization of 324. How does this help find the factors of polynomial such as in question a. below?

$324 \to (2),\ 162$; $162 \to (2),\ 81$; $81 \to (3),\ 27$; $27 \to (3),\ 9$; $9 \to (3),\ (3)$

$324 = (2)^2(3)^4$

factors: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm9, \pm12, \ldots$

2. How do you think you can factor a cubic polynomial by just finding one zero? What about a quartic?

(1.) Use trial + error to find which $\pm p/q$ combination makes the function $= \emptyset$

(2.) Use synthetic division to find quotient

(3.) Record result: $f(x) = (\text{quotient})(\text{divisor}) + R$ ← zero.

(4.) Factor quotient more (if quadratic use gr. 10 math; if cubic, repeat the steps)

3. Factor completely by first finding one zero

a. $x^5 - 45x^3 + 324x$ long way:

$f(x)$

$f(x) = x(x^4 - 45x^2 + 324)$

$\pm\frac{p}{q}$: $\pm\frac{324}{1}, \pm1, \pm2, \pm3, \pm4, \pm6, \pm9$

try # between 0 and 10

$f(6) = 0 \therefore (x-6)$ is a factor by factor th. ✓

$$\begin{array}{c|ccccc} 6 & 1 & 0 & -45 & 0 & 324 \\ & \downarrow & 6 & 36 & -54 & -324 \\ \hline & 1 & 6 & -9 & -54 & \boxed{0} \leftarrow R \end{array}$$

$f(x) = x(x-6)(x^3 + 6x^2 - 9x - 54) + \emptyset$

$f(3) = 0 \therefore (x-3)$ is a factor

$$\begin{array}{c|cccc} 3 & 1 & 6 & -9 & -54 \\ & \downarrow & 3 & 27 & 54 \\ \hline & 1 & 9 & 18 & \boxed{0} \leftarrow R \end{array}$$

$f(x) = x(x-6)(x-3)(x^2 + 9x + 18)$

1 6
1 3

$$\boxed{\therefore f(x) = x(x-6)(x-3)(x+6)(x+3)}$$

b. $x^5 - 45x^3 + 324x$ shortcut:

only works if variable pattern is like criss cross factoring

"pseudo" quadratic

$f(x) = x(x^4 - 45x^2 + 324)$

$a = x^2$

$a^2 - 45a + 324$

1 −9
1 −36

$f(x) = x(x^2 - 9)(x^2 - 36)$

$= x(x+3)(x-3)(x+6)(x-6)$

c. $12x^3+8x^2-3x-2 = f(x)$

q p

$\frac{p}{q}$: $\pm\frac{1}{1}$, $\pm\frac{2}{1}$ $\pm\frac{1}{2}$ $\cancel{\pm\frac{2}{2}}$ $\pm\frac{1}{3}$ $\pm\frac{2}{3}$

$\frac{1}{6}$ $\cancel{\frac{2}{6}}$ $\frac{1}{12}$ $\cancel{\frac{2}{12}}$ $\frac{1}{4}$ $\cancel{\frac{2}{4}}$

try # between -1 and 1

$f(\quad)=0$ $\therefore$ ( ) is a factor

$(2)(x+\frac{1}{2})$ $(2x-1)(3x+2)$

$f(x)=(2x+1)(2x-1)(3x+2)$

or

$=12(x+\frac{1}{2})(x-\frac{1}{2})(x+\frac{2}{3})$

d. $3x^3+8x^2-21x+6$

$f(x)\doteq 3(x-1.4)(x+4.4)(x-\frac{1}{3})$

or

$=(x-1.4)(x+4.4)(3x-1)$

When completing practice questions, you may use the following online calculator to help you check your answers, or to use to find zeros of polynomials for which you get stuck on or the ones that have only irrational solutions – textbook sometimes does give questions that are unfactorable over rationals.
http://www.mathportal.org/calculators/polynomials-solvers/polynomial-roots-calculator.php

desmos.com is better

4. When ax^3+3x^2-2x-b is divided by $x-1$, the remainder is 1. When it is divided by $x-2$, the remainder is 40. Find $a+b$

$f(x)$

by remainder theorem $f(1)=1$ pt $(1,1)$

$f(2)=40$ pt $(2,40)$

pt. $1=a(1)^3+3(1)^2-2(1)-b$

$1=a+3-2-b$

$\boxed{0=a-b}$ ①

pt. $40=8a+12-2(2)-b$

$\boxed{32=8a-b}$ ②

$0=a-b$

Subtract

$32=7a$

$\frac{32}{7}=a$

solve by elimination or substitution.

$0=\frac{32}{7}-b$

$b=\frac{32}{7}$

$\therefore a+b$

$=\frac{32}{7}+\frac{32}{7}$

$=\frac{64}{7}$

5. For the function $f(x)=x^3-5x^2+kx-16$, the remainder from $f(x)\div(x+1)$ is twice the remainder from $f(x)\div(x-1)$. Determine the value of k.

$f(\ )=R_1$

$f(\ )=R_2$

$R_1=2R_2$

$f(\ )=2f(\ )$

Factoring a Sum and a Difference of Cubes

1. Develop the SUM of CUBES formula by factoring the following polynomial x^3+k^3: $(-k)^3+k^3=0$

$\therefore (x+k)$ is a factor

$$\begin{array}{r|l} & x^2-kx+k^2 \\ x+k & x^3+0x^2+0x+k^3 \\ - & x^3+kx^2 \\ & -kx^2+0x \\ & -kx^2-k^2x \\ & k^2x+k^3 \\ & k^2x+k^3 \\ & 0 \end{array}$$

dividend = (divisor)(quot) + R

$\therefore x^3+k^3=(x+k)(x^2-kx+k^2)$

$$\boxed{a^3+b^3=(a+b)(a^2-ab+b^2)}$$

This will never factor.

2. The DIFFERENCE of CUBES formula is similarly derived. The formula is: x^3-k^3

$$x-k \overline{)\,x^3+0x^2+0x-k^3}$$

$R=0$

$$\boxed{a^3-b^3=(a-b)(a^2+ab+b^2)}$$

3. Factor the following by using the formulas developed above

a. $\frac{8}{343}x^3-\frac{1}{216}$

$=\left(\frac{2x}{7}\right)^3-\left(\frac{1}{6}\right)^3$

$=\left(\frac{2}{7}x-\frac{1}{6}\right)\left(\frac{4}{49}x^2+\frac{1}{21}x+\frac{1}{36}\right)$

$\left(\frac{2}{7}x\right)\left(\frac{1}{6}\right)$ 3

b. $-54x^4-250x$ common factor 1st.

$=-2x(27x^3+125)$

$(3x)^3 \quad 5^3$

$=-2x(3x+5)(9x^2-15x+25)$

squared

sign

SOAP — always positive

same sign as the question. opposite sign c.

$\underbrace{(x-5)}_{a}{}^3 - \underbrace{(5x-1)}_{b}{}^3$

$= \left[(x-5) - (5x-1)\right]\left[(x-5)^2 + (x-5)(5x-1) + (5x-1)^2\right]$

$= \left[x-5-5x+1\right]\left[x^2-10x+25 + 5x^2-26x+5 + 25x^2-10x+1\right]$

$= \left[-4x-4\right]\left[31x^2-46x+31\right]$

$= -4(x+1)(31x^2-46x+31)$

d.

$(2a+b)^3 + (2a-b)^3$

$(\quad + \quad)(\quad - \quad + \quad)$ ✓sign

$(2a+b+2a-b)\left((2a+b)^2 - (2a+b)(2a-b) + (2a-b)^2\right)$

$(4a)\left(\underbrace{4a^2+4ab+b^2}_{✓} - (4a^2-b^2) + 4a^2-4ab+b^2\right)$

✓

$\underbrace{-4a^2+b^2}_{✓}$

$(4a)(4a^2+3b^2)$ ✓