 12AdvF Date:___________ Name: _______________________

Polynomial Functions Unit 3

Tentative TEST date_______________

Big idea

In this unit will you extend your knowledge from linear, quadratic and cubic functions (learned in gr9-11) to general polynomial functions. Recall that the quadratic function can be written in 3 forms – factored, expanded/standard and vertex forms. When you study polynomials you will also use all three of these forms. The vertex form will just be called the transformed form since a polynomial function may have many turning points, not just one vertex as a parabola did. You will learn that polynomial functions can be described by their end-behaviour, symmetry, number of zeros and number of turning points. You will be introduced to how to sketch the polynomial function from factored form as well as how to find the equation from a given graph. The main part of this unit will be to learn how to factor polynomials, which can be a long process involving long division or synthetic division. You will not learn how to find where exactly the turning points will occur – that is part of calculus.

<u>Corrections for the textbook answers:</u>
Sec 3.2 #12 get rid of the word range
Sec 3.3 #14 graph is wrong in answer
Sec 3.4 #6f) (-11, -3), (-4, -2), (10, 6)

Success Criteria

☐ I <u>understand the new topics</u> for this unit if I can do the practice questions in the textbook/handouts

Date	pg	Topics	# of quest. done? You may be asked to show them	**Questions I had difficulty with** ask teacher before test!
	1-5	INVESTIGATIONS Polynomials, Expanded and Factored Forms Section 3.1 & 3.2 & TWO Handouts		
	6-7	Graphing and Finding Equations Section 3.3 & TWO Handouts		
	8-10	Transformed Form of Polynomial Functions Section 3.4 & Handout		
	11-12	Long Division & Synthetic Division Section 3.5 & TWO Handouts		
	13-15	Theorems and Applications THREE Handouts		
	16-18	Factoring Polynomials Section 3.6 & TWO Handouts		
	19-20	Factoring Sum & Difference of Cubes Section 3.7 & Handout		
		REVIEW		

Reflect – previous TEST mark ___________, Overall mark now___________.

What is a Polynomial Function?

1. How can you tell if the equation or graph or table that is given is of a polynomial function?

2. Identify whether the folowing are considered to be polynomial functions.

a. $y = 2x - 1$

b. $f(x) = 3x^{\frac{1}{2}} - x$

c. $h(x) = \sqrt[3]{x}$

d. $y = x^3 + 2x^2 - x + 11$

e. $y = -0.2(4x - 3)(x + 3)$

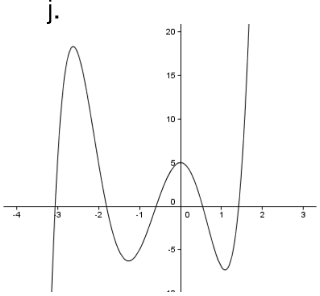
f. $y = x^{-3} + 2x^2 - x^{-1} + 11$

g. $x = -6$

h. $y = 2$

i. $y = \dfrac{1}{x}$

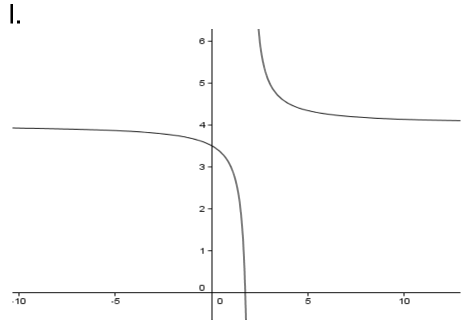
j.



k.

x	y
-3	-27
-2	-8
-1	-1
0	0
1	1
2	8

l.



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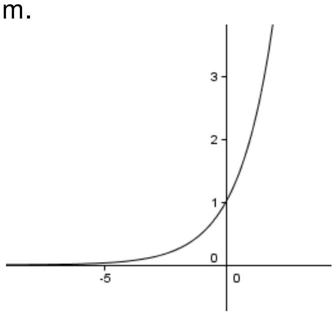
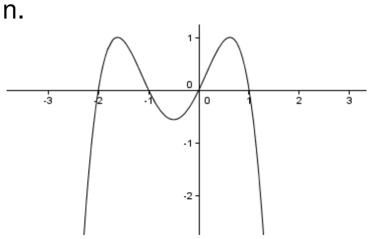
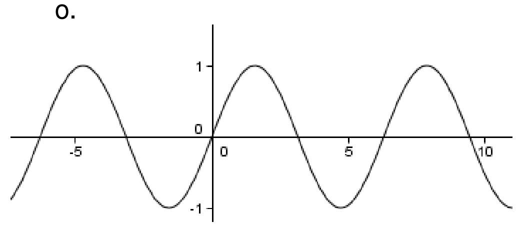


n.



o.



3. How can you find the degree and the leading coefficient of a polynomial? Explain using these examples

$y = 3x - 9x^3 + 10x^6 - x^2 + 8$ $\qquad$ $y = -2(x-4)(3x^3 - x^2)(1 - x^4)$

4. Identify the form of the polynomial, the degree and the leading coefficient.

a. $y=(x+1)^3(1-2x)(5x^4+x^3+1)$ b. $y=x^3-3x+7x^4-9x^5+1$ c. $y=5(1-3x)^6+8$ d. $y=-(1-4x)^3$

INVESTIGATE Transformed Form of Polynomials.

5. Investigate how does the degree affect the shape of the graph?

a. Even Degree parent graphs: b. Odd Degree parent graphs:

6. How do you think the following functions will look?

a. $y=-(x+4)^4+5$ b. $y=-0.1(x-3)^5+2$

INVESTIGATE Standard/Expanded Form of Polynomials.

7. Using technology, investigate how does having many terms affect the shape of the graph. Draw all possible twists that can occur

a. Degree 3 b. Degree 4 c. Degree 5 d. Degree 6

8. Investigate how does the leading coefficient affect the end behaviour of the graph?

9. What conclusions can you make about the range of polynomial graphs?

10. Summarize

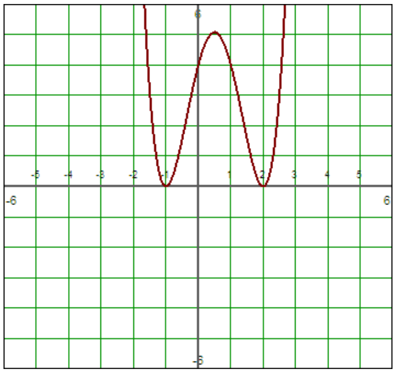
Absolute maximum/minimum can only occur for _____________ degree polynomials.
A polynomial of EVEN degree *n* can have at least ____________, and at most __________ zeros.
A polynomial of EVEN degree 8 can have ____ turning points
A polynomial of ODD degree *n* can have at least ____________, and at most __________ zeros.
A polynomial of ODD degree 11 can have ____ turning points.

INVESTIGATE Factored Form of Polynomials.

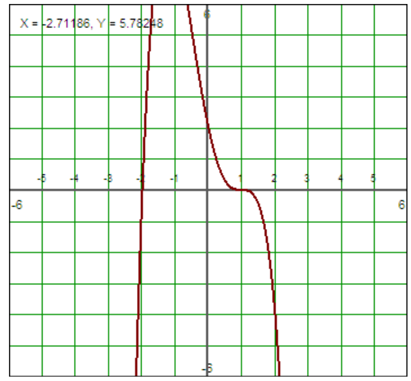
As you may have noticed from above, polynomial graphs behave near the x-axis in several different ways: The graph can "bounce off", "cut through", or "bend" near the roots (zeros).

11. For each graph and equation given find the relationship between the power of the factor in the equation and whether the graph would "bounce", "cut" or "bend" through zeros.

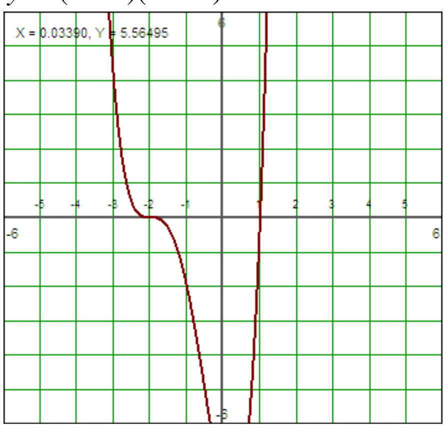
1. $y = (x+1)^2(x-2)^4$



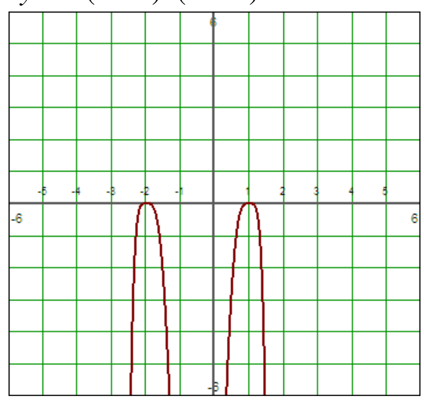
2. $y = -(x-1)^3(x+2)$



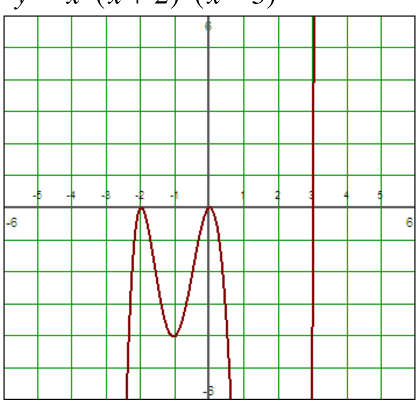
3. $y = (x-1)(x+2)^3$



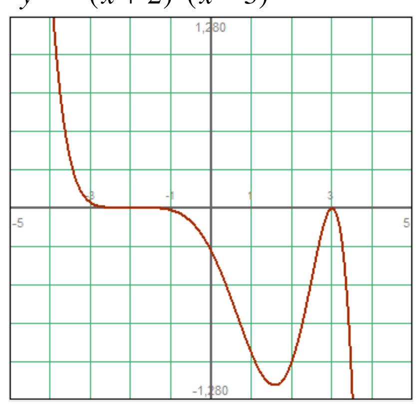
4. $y = -(x-1)^4(x+2)^4$



5. $y = x^2(x+2)^2(x-3)$



6. $y = -(x+2)^5(x-3)^2$



12. Summarize:

When a polynomial function has a repeated **even** root, the graph of the function ______________________________ at that root

When a polynomial function has a repeated **odd** root greater than 1, the graph of the function _____________________________ at that root.

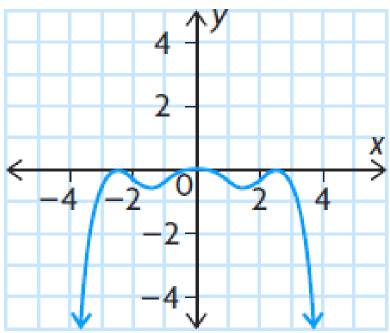
When a polynomial function has a root of order 1, the graph of the function ______________________________ at that root.

INVESTIGATE Symmetry of Polynomials.

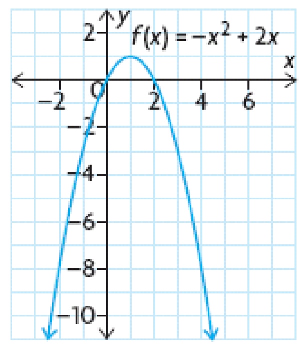
13. You just finished looking at Odd and Even Degree Polynomials. Please don't confuse the words Odd Function with Odd Degree Function. These are totally different concepts. Recall what you learned about symmetry: summarize here how something can have odd symmetry, or even symmetry or neither symmetry – graphically and algebraically.

14. For each graph or equation given determine the degree & symmetry.

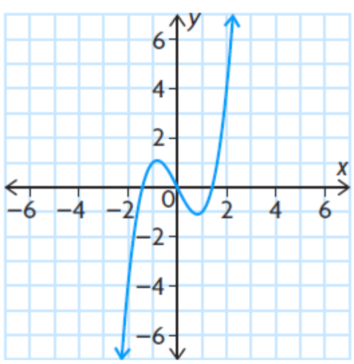
a. $f(x) = -0.01(x-0.5)(x+0.5)(x-2.5)^2(x+2.5)^2$



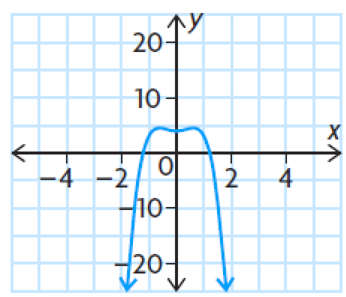
b.



c. $f(x) = x(x-\sqrt{2})(x+\sqrt{2})$



d. $y = -4x^4 + 3x^2 + 4$



e. $f(x) = x^4 - 2x^2 + 1$

f. $f(x) = x^2 - 3x + 4$

g. $f(x) = x^5 - 3x^7 + 9$

h. $f(x) = x^5 - 3x$

15. Summarize:

Odd degree polynomial can have ____________________ or __________________ symmetry. It will be symmetric only if expanded form has ____

Even degree polynomial can have ____________________ or __________________ symmetry. It will be symmetric only if expanded form has ____

Graphing and Finding Equations

1. Summarize the steps of sketching polynomials given expanded form. Explain what details you still don't know about the graph.

2. Sketch each of the following

a. $f(x) = 4x^2 - 5x^5 + x - 1$

b. $f(x) = 3 - 7x + 8x^4 - x^3$

c. $f(x) = 11x - 9x^4 + 10x^7$

d. $f(x) = -6x^6 + 8x^2 - x^3 + 9$

3. Summarize the steps of sketching polynomials given factored form. Explain what details you still don't know about the graph.

4. Sketch each of the following

a. $y = x(x+3)^2(2-2x)$

b. $y = (x+3)^2(2-4x)^2$

c. $y = (2-x)^3$

d. positive leading coefficient,
zeros at -3, -1 (order 5), 2, 4 (order 4)

Sketch each of the following

e. $y=-x(x+2)^3(1-x)^4$

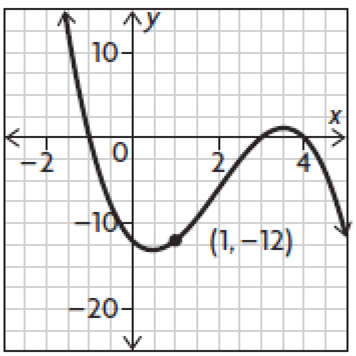
f. $y=-3(2x+4)^2(x-1)$

g. $y=(6-2x)^4$

h. $y=(2x+3)^3(5-x)$

5. Find the equations for the given graphs

a. polynomial of degree 3 with the following graph



b. The function is cubic with zeros at 4(order 2) and -1 and with a y-intercept of 10.

6. The function $f(x)=kx^3+20x^2-8x-96$ has a zero when x= – 3. Determine the value of *k*.

Transformed Form of Polynomials

1. Write out the transformed form of a polynomial of degree n. Summarize what type of transformations the letters control and recap the steps of transforming a function.

2. Sketch the following

a. $f(x) = -2(\frac{1}{4}x - 1)^3 - 8$

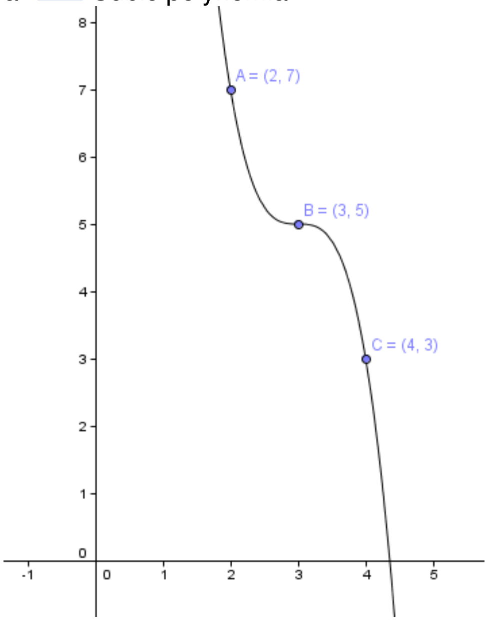
b. $f(x) = 3(10 - 5x)^4 + 4$

3. Explain why it is important to follow the steps outlined above instead of just taking the x values below and just plugging them into the equation. Use question a. as an example in your explanation.

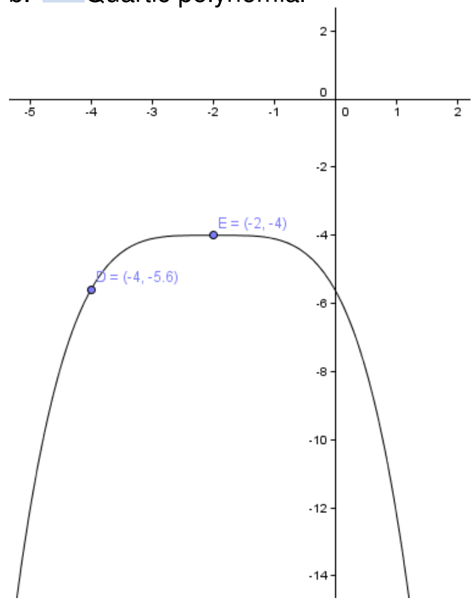
x	y
-2	
-1	
0	
1	
2	

4. Find the equations for the following graphs. Be careful not all pictures represent a transformed form of a parent!!!

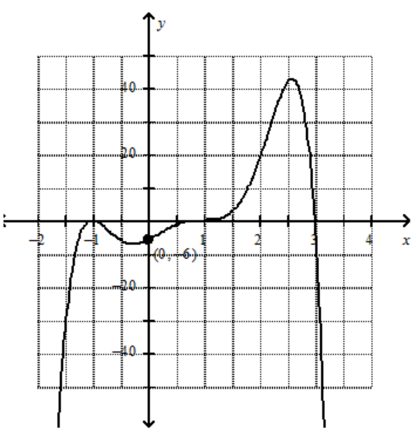
a. Cubic polynomial



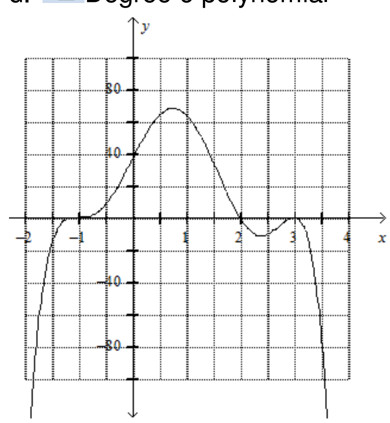
b. Quartic polynomial



c. Degree 6 polynomial



d. Degree 6 polynomial



5. Determine the zeros of the following polynomials

a. $y = 3(4x - 8)^4 - 243$

b. $y = 2(2x - 10)^3 + 128$

c. Why can't the above method be used on the following $y = -5x^4 - 4x^2 + 1$?

d. What method do you know of that could help you find the roots of $y = -5x^4 - 4x^2 + 1$?

e. Would the method you used for d. work on $y = -5x^4 - 4x^3 + 1$?

Long Division and Synthetic Division of Polynomials

1. Remind yourself how to do long division with numbers, since the same approach will be used for polynomial division.

a. Identify where the following terms would be located on the long division table: Quotient, Divisor, Dividend, Remainder, then find the quotient and remainder.

$$4\overline{)25697}$$

b. State the formula that relates all of these terms together and modify it to the one that has dividend – which will always be the function, f(x) – isolated.

2. Provide an explanation of how to do long division.

3. Provide an explanation of how to do synthetic division.

4. Divide using long division, write down the result in TWO different ways

a. $(2x^4 - 5x^2 + 3) \div (x + 4)$

b. $(12x^4 - 6x^3 - x + 81) \div (3x^2 - 9)$

5. Divide using synthetic division, don't forget to write down the result.

a. $(4x^5 - 2x^4 + 3x - 9) \div (x - 2)$

b. $(2x^3 + 5x^2 - 4x - 5) \div (2x + 1)$

6. What are the disadvantage(s) of long division?

7. What are the disadvantages of synthetic division?

8. Practice both methods

a. $(x^3 - 8x^2 - 4x - 7) \div (x^2 + 1)$

b. $(12x^4 - 20x^3 - 4x + 24) \div (4x - 8)$

c. $(3x^3 - x^2 + 10x) \div (x - 2)$

d. $(10x^3 - 14x + 5) \div (x + 1)$

Theorems and Applications

Remainder Theorem:

When a polynomial $f(x)$ is divided by a linear divisor $(x-a)$, then the remainder is equal to $f(a)$.

(If the divisor is not linear then the remainder might not be just one constant number but a polynomial – and this theorem will not work)

Proof:

$$\boxed{f(x)=(quotient)(divisor)+remainder}$$

divisor is $x-a$

$f(x)=(quotient)(x-a)+remainder$

sub in the zero of the divisor $x=a$

$f(a)=(quotient)(0)+remainder$

$f(a)=remainder$

1. Find the remainder (without doing the division) of $f(x)=x^3-12x^2-42$ when it is divided by $x-3$

2. Determine the remainder, r, to make this multiplication statement true
$(x^2-4)(3x^2+1)+r=3x^4-10x^2+5$

3. If the dividend is $x^4+x^3-7x^2+6x-2$, quotient is $x^3-3x^2+5x-14$ and remainder is 54, find the divisor by using comparing coefficients method.

4. Find the remainder (without doing the division) of $f(x)=2x^5+4x^4-5x^3-6x^2+2x-4$ when it is divided by $x+3$

5. If the divisor is x^2+5x+8 , the dividend is $x^4-5x^3+3x^2-7x+11$. Find the remainder.

6. If the dividend is $-2x^4-5x^3+2x^2+3x-1$, quotient is $-2x^3-7x^2-5x-2$ and remainder is – 3, find the divisor by using comparing coefficients method.

7. Remind yourself what is the definition of the word "factor"

Rational Root Theorem:

If a polynomial $y = a_n x^n + a_{n-1}x^{n-1} + ... + a_2 x^2 + a_1 x + a_0$ has any rational (integer or fractional) zeros then the zeros are from the list of all combinations of $\pm\frac{p}{q}$, where p is an integer factor of a_0 and q is an integer factor of a_n.

(Keep in mind that zeros can also be irrational – these can only be found using technology. In university you'll study how the technology is programmed to find the zeros using different approximation methods)

8. Find the list of possible zeros of $f(x) = 3x^3 - 5x^2 + 5x - 2$

9. Find the list of possible zeros of $f(x) = 2x^3 + 3x^2 - 18x + 8$

Factor Theorem:

The polynomial $f(x)$ has a factor $(x-k)$ if and only if (this means the implication goes both ways) $f(k) = 0$

Proof - "if" implication:

if $x-k$ is a factor of $f(x)$
then it divides into $f(x)$ evenly
This means the remainder is zero.
By Remainder theorem $f(k) = remainder$
Therefore $f(k) = 0,$ since remainder is zero.

Proof - "only if" implication:

if $f(k) = 0$
then by Remainder theorem $remainder = 0$
but that implies the divisor $x-k$ goes into $f(x)$ evenly
which means the divisor $x-k$ is a factor.

10. Which of the numbers from the list you've made for $f(x) = 3x^3 - 5x^2 + 5x - 2$ above, are actual zeros?

11. Determine if the binomial $2x-3$ is a factor of $2x^5 + 3x^3 + 6$

12. Which of the numbers from the list you've made for $f(x) = 2x^3 + 3x^2 - 18x + 8$ above, are actual zeros? Use your answers to factor the polynomial completely.

13. Determine if the binomial $x+4$ is a factor of $x^3 + 5x^2 + 2x - 8$

Factoring Polynomials

1. Find the prime factorization of 324. How does this help find the factors of polynomial such as in question a. below?

2. How do you think you can factor a cubic polynomial by just finding one zero? What about a quartic?

3. Factor completely by first finding one zero

a. $x^5 - 45x^3 + 324x$ long way:

b. $x^5 - 45x^3 + 324x$ shortcut :
only works if variable pattern is like criss cross factoring

c. $12x^3+8x^2-3x-2$

d. $3x^3+8x^2-21x+6$

When completing practice questions, you may use the following online calculator to help you check your answers, or to use to find zeros of polynomials for which you get stuck on or the ones that have only irrational solutions – textbook sometimes does give questions that are unfactorable over rationals.
http://www.mathportal.org/calculators/polynomials-solvers/polynomial-roots-calculator.php

4. When $ax^3 + 3x^2 - 2x - b$ is divided by $x-1$, the remainder is 1. When it is divided by $x-2$, the remainder is 40. Find $a+b$

5. For the function $f(x) = x^3 - 5x^2 + kx - 16$, the remainder from $f(x) \div (x+1)$ is twice the remainder from $f(x) \div (x-1)$. Determine the value of k.

Factoring a Sum and a Difference of Cubes

1. Develop the SUM of CUBES formula by factoring the following polynomial $x^3 + k^3$:

2. The DIFFERENCE of CUBES formula is similarly derived. The formula is:

3. Factor the following by using the formulas developed above

a.
$$\frac{8}{343}x^3 - \frac{1}{216}$$

b.
$$-54x^4 - 250x$$

c.

$(x-5)^3-(5x-1)^3$

d.

$(2a+b)^3+(2a-b)^3$