Date: ______________________ Name: ___________________________

Solve Polynomial Equations

1. Solve the following equations, then sketch the function that corresponds to the one with all the terms moved to the **left** hand side.

a. $6x^3+49x^2+8x-12=2x+4$

let $f(x)=6x^3+49x^2+6x-16=0$

* zeros of this function are roots of original equation

possible zeros $\pm 16 \pm 8 \pm 4 \pm 2 \pm 1$

$\pm\frac{16}{6}, \pm\frac{8}{6}, \pm\frac{4}{6}, \pm\frac{2}{6}, \pm\frac{1}{6}$

$\pm\frac{16}{3}, \pm\frac{8}{3}, \pm\frac{4}{3}, \pm\frac{2}{3}, \pm\frac{1}{3}$

$\pm\frac{16}{2}, \pm\frac{8}{2}, \pm\frac{4}{2}, \pm\frac{2}{2}, \pm\frac{1}{2}$

$f(-8)=0$ (or $f(\frac{1}{2})=0$ or $f(-\frac{2}{3})=0$)

$\therefore (x+8)$ is a factor

$$\begin{array}{r|rrrr} -8 & 6 & 49 & 6 & -16 \\ & \downarrow & -48 & -8 & 16 \\ \hline & 6 & 1 & -2 & 0 \end{array}$$

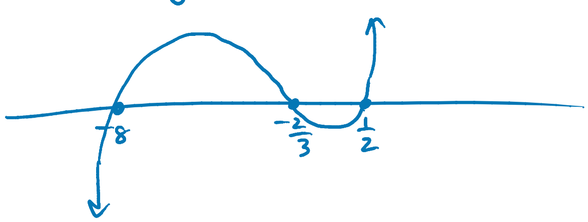
$\therefore f(x)=(x+8)(6x^2+x-2)$

$=(x+8)(2x-1)(3x+2)$

one neg

$\therefore$ solutions are $x=-8,\ x=\frac{1}{2},\ x=-\frac{2}{3}$

Sketch of $f(x)$:



b. $113x-30=8x^3-30x^2$

let $f(x)=-8x^3+30x^2+113x-30$

possible zeros $\frac{\pm 30}{8}$ and their factors

$f(6)=0 \quad \therefore (x-6)$ is a factor of $f(x)$

$$\begin{array}{r|rrrr} 6 & -8 & 30 & 113 & -30 \\ & \downarrow & -48 & -108 & 30 \\ \hline & -8 & -18 & 5 & 0 \end{array}$$

$\therefore f(x)=(x-6)(-8x^2-18x+5)$

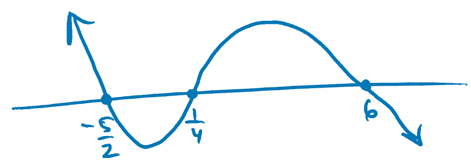
one neg

$f(x)=(x-6)(2x+5)(-4x+1)$

$\therefore$ solutions are

$x=6,\ x=-\frac{5}{2},\ x=\frac{1}{4}$

sketch of $f(x)$:



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Solve the following equations, then sketch the function that corresponds to the one with all the terms moved to the **left** hand side.

c. $-6x^2 - 6x = 5x^3 + 1$

let $f(x) = -5x^3 - 6x^2 - 6x - 1 = 0$

possible zeros $\pm 1 \quad \pm \frac{1}{5}$

$f(-\frac{1}{5}) = 0 \quad \therefore (x + \frac{1}{5})$ is a factor

$$\begin{array}{r|rrrr} -\frac{1}{5} & -5 & -6 & -6 & -1 \\ & \downarrow & 1 & +1 & 1 \\ \hline & -5 & -5 & -5 & 0 \end{array}$$

$\therefore f(x) = (x + \frac{1}{5})(-5x^2 - 5x - 5)$

or

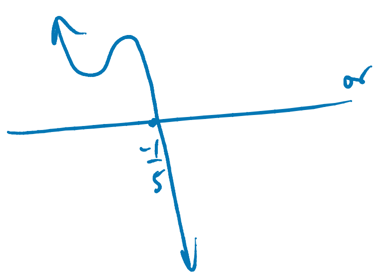
$= (x + \frac{1}{5})(-5)(x^2 + x + 1)$

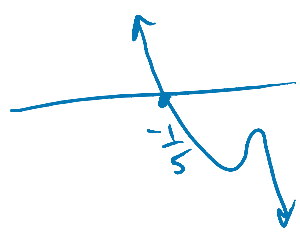
$= -(5x+1)\underbrace{(x^2 + x + 1)}_{}$

can't factor more since $b^2 - 4ac$ is neg.

$\therefore$ only solution $x = -\frac{1}{5}$

can't draw with all details





* in calculus you'll be able to find the t.p. fast

* in this course you CAN find where t.p. occur by solving for $x = a$ when $m_{tan} = 0$ (LONG :( )

* to see if MAX/MIN can look at slopes before t.p and after
  if pos. then neg. — max
  if neg. then pos — min
  if pos then pos — neither

d. $8x^3 + 22x^2 - 3x - 2 = 0$

$f(x) =$

possible zeros $\frac{\pm 2}{8}$ and their factors

$f(-\frac{1}{4}) = 0 \quad \therefore (x + \frac{1}{4})$ is a factor of $f(x)$

$$\begin{array}{r|rrrr} -\frac{1}{4} & 8 & 22 & -3 & -2 \\ & \downarrow & -2 & -5 & 2 \\ \hline & 8 & 20 & -8 & 0 \end{array}$$

$\therefore f(x) = (x + \frac{1}{4})(8x^2 + 20x - 8)$

or

$= (4x+1)(2x^2 + 5x - 2)$

doesn't factor

use quad.

$$x = \frac{-5 \pm \sqrt{5^2 - 4(2)(-2)}}{2(2)}$$

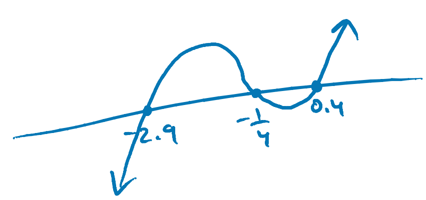
$x \doteq -2.9$ or $x \doteq 0.4$

$\therefore f(x) \doteq (4x+1)(2)(x+2.9)(x-0.4)$

need this for coefficients to match !!

$\therefore$ solutions $x = -\frac{1}{4}$ $x \doteq -2.9$ $x \doteq 0.4$

sketch of $f(x)$:



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Families of Polynomials

1. Recall that if you are given a set of conditions you may have many answers (or a **family** of) equations that will satisfy them. There will be one unique equation only if enough information is provided. Come up with the polynomial family that satisfies the following conditions.

a.

i. A cubic polynomial with end behaviour $x \to -\infty, y \to \infty$ and a y-intercept of 6.

ii. The above polynomial has a double root at 4 and a root at 8.

iii. Sketch this function.

i) $y = ax^3 + bx^2 + cx^2 + d$ y-int

$y = ax^3 + bx^2 + cx^2 + 6$, $a<0$, $b, c, \in \mathbb{R}$

a is neg for end. beh. to be

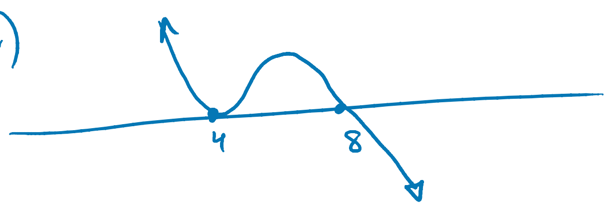
ii) $y = a(x-4)^2(x-8)$ sub pt. (0,6)

$6 = a(0-4)^2(0-8)$

$6 = -128a$

$\frac{-3}{64} = a$ $\therefore y = -\frac{3}{64}(x-4)^2(x-8)$

iii)



b.

i. Polynomial of degree 6, with zeros at -2, 4 (order2), 8 (order 3)

ii. The above polynomial goes through the point (-4, 80)

iii. Sketch this function

i) $y = a(x+2)(x-4)^2(x-8)^3$

$a \in \mathbb{R}$

ii) $80 = a(-4+2)(-4-4)^2(-4-8)^3$

$80 = a(-2)(-8)^2(-12)^3$

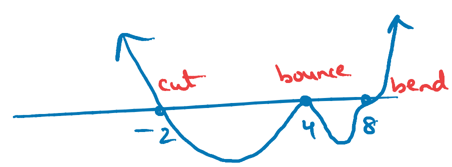
$80 = 221184a$

$\frac{80}{221184} = a$

$\frac{5}{13824} = a$

$\therefore y = \frac{5}{13824}(x+2)(x-4)^2(x-8)^3$

iii)



2. To find the equation for a given table of values (show the interactive MME from eLearningOntario course) to see what is the relationship between the constant differences, the degree of the function and the leading coefficient. Record the important information here:

Find all differences until you have a constant column then to find Lead. Coefficient = $\boxed{a = \frac{\text{Difference \#}}{n!\,(\Delta x)^n}}$ if polyn is of Degree = n

Degree n polynomials will have n^{th} constant difference.

3. Find the equations for the given tables

Back track to x=0 to find y-int

a.

X 0	Y 1
1	0
2	1
3	10
4	33
5	76
6	145
7	246

-1, 2, 6

1, 8, 6

9, 14, 6

23, 20, 6

43, 26, 6

69, 32, 6

101

$d = 1$

Constant 3^{rd} difference

$\therefore$ polyn Degree = 3

$y = ax^3 + bx^2 + cx + d$

$a = \frac{\text{Diff \#}}{3!} = \frac{6}{3\times2\times1} = \frac{6}{6} = 1$ $a=1$

$y = 1x^3 + bx^2 + cx + 1$

sub pt. (1,0)

$0 = 1^3 + b(1)^2 + c(1) + 1$

$\boxed{-2 = b + c}$

sub pt. (2,1)

$1 = 2^3 + b(2)^2 + c(2) + 1$

$\boxed{-8 = 4b + 2c}$

sub in

$b = -2 - c$

$-8 = 4(-2-c) + 2c$

$-8 = -8 - 4c + 2c$

$0 = -2c$

$0 = c$

$b = -2 - 0$

$b = -2$

$\boxed{\therefore y = 1x^3 - 2x + 0x + 1}$

b.

X 0	Y -3
1	4
2	15
3	30
4	49
5	72
6	99

7, +11, +15, +19, +23, +27

4, 4, 4, 4, 4, 4

Δx is change in x if there is a skip count!!

$\therefore$ Degree = 2

$y = ax^2 + bx + c$

$c = -3$

$a = \frac{4}{2!(1)^2} = \frac{4}{2} = 2$

$y = 2x^2 + bx - 3$

sub pt. (1,4)

$4 = 2 + b - 3$

$5 = b$

$\boxed{\therefore y = 2x^2 + 5x - 3}$

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Sometimes it is imposible to find the equation algebraically for two reasons.

- The given data is from a real life situation and the values will not yeild exact constants for any difference column. This is mainly due to error or to the fact that you can only approximate values for real life data.
- The x values are not sequential – they skip count

In both cases you must use technology to find the regression equation. Ask your teacher for instructions of how to do this with either Excel, TI-83, or CurveExpert. (* this was in the extra assignment of unit #2 – instructions online)

4. A certain business company spends money of advertising to get specific revenues back. The data is provided below.

Advertising (in \$10 000's)	Revenue (in \$100 000's)
50	34.375
100	125.000
even 150	253.125
spacing 200	400.000
can 250	546.875
check 300	675.000
differences 350	765.625
400	800.000
can't 450	759.375
figure out 500	625.000

lead. coeff.

actually $\boxed{a = \frac{\text{Difference}}{n!(\Delta x)^n}}$

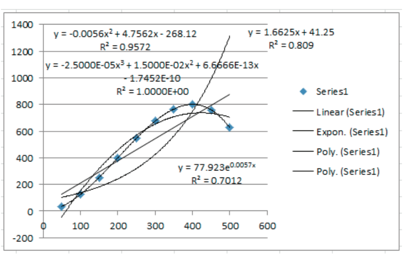
First differences: 90.625, 128.125, 146.875, 146.875, 128.125, 90.625, 34.375, −40.625, −134.375

Second differences: 37.5, 18.75, 0, −18.75, −37.5, −56.25, −75, −93.75

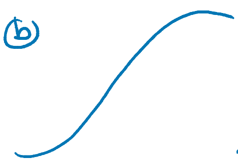
Third differences: −18.75, −18.75

$a = \frac{-18.75}{3!(50)^3} = 0.000025$

a. Use techology to determine the equation that best fits the data.
b. Use techology to create the sketch
c. What is the domain of this function for this real life situation?
d. For what amount of advertising does the company get an absolute maximum in revenue?



(b)

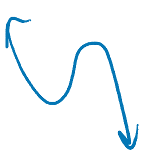


(a) $y = -0.000025x^3 + 0.015x^2 + (\text{almost zero})x + (\text{almost zero})$

$y \doteq -0.000025x^3 + 0.015x^2$

(c) $D = \{\text{advertising money} \geq 0\}$

(d) end behaviour is ↑ ↓ ∴ (400, 800) is the local MAX. on $x \in (-\infty, \infty)$ but on the domain $x \in [0, \infty)$ it is the absolute MAX.



6

Solve Polynomial Inequalities by Graphing

1. Summarize the following terms, use examples in the explanation

FUNCTION – has both input + output variables

ex. $f(x) = x^2 - 4$

EQUATION – has only one variable

ie. if $y = 12$
$12 = x^2 - 4$

INEQUALITIES – uses $<, \leq, >, \geq$ symbols

ex. $12 < x^2 - 4$

ZEROS/x-intercepts of a function

where the function cross the x-axis

ex. above f(x) has zeros (−2, 0) (2, 0)

root = zeros if equation was made equal zero

ROOTS of an equation

values of x that make the equation valid / true

ex. $12 = x^2 - 4$
$0 = x^2 - 16$
$x = 4$ or $x = -4$

SOLUTIONS of inequalities

intervals / region of x values that make the inequality valid.

2. Here are some examples that show that some strategies used to solve equations may not work when solving inequalities.

ex1. $5 - 3x = 14$ -5
$-3x = 9$
$x = -3$

$5 - 3x < 14$ -5
$\frac{-3x}{-3} < \frac{9}{-3}$
$x > -3$

NEW RULES

* when mult/dividing by a negative switch sign < to >

ex2. $\frac{1}{x} = \frac{2}{3}$
$3 = 2x$
$\frac{3}{2} = x$

$\frac{1}{x} > \frac{2}{3}$

* x may be pos or neg. sign may or may not flip.

* NEVER cross multiply inequalities!

ex3. $2x^2 - 1 = 7$ $+1$
$2x^2 = 8$
$x^2 = 4$
$x = \pm 2$

$2x^2 - 1 \geq 7$
$x^2 \geq 4$
$x \geq 2$ or ~~$x \geq -2$~~
not true if you try $x = -1$

* NEVER sq. root, 4th/6th ... inequalities

ex4. $2x^2 - 9x = 5$ $2x^2 - 9x \leq 5$

can complete sq. then isolate x
OR
make equation = 0 then factor to find zeros
$2x^2 - 9x - 5 = 0$
$(2x + 1)(x - 5)$
$x = -\frac{1}{2}$ $x = 5$

* Will create another inequality with zero on one side then factor then (1) sketch (2) +/− table

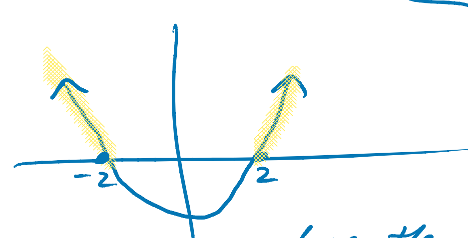
3. Solve the following inequalities using:
   i. Algebraic solution, if possible
   ii. Graphical solutions of the related functions with one side of each inequality as zero

i) not possible since can't sq. root

a. $-4 < x^2 - 8$ $+4$

ii) $0 < x^2 - 4$
$0 < (x + 2)(x - 2) = f(x)$

sketch f(x)



where $f(x) > 0$
look at where the graph is above the x-axis

$\therefore$ Solution is $x \in (-\infty, -2), (2, \infty)$
OR $\{x \in \mathbb{R}, x < -2, x > 2\}$

Solve using: i. Algebraic solution if possible OR ii. Graphical solution of the related function

b. $18 \le (3x+4)-(6x+2) < 21$ Double inequality

$18 \le -3x+2$ OR $-3x+2 < 21$

$16 \le -3x$ $\quad -3x < 19$

$\frac{16}{-3} \ge \frac{-3x}{-3}$ $\quad x > \frac{-19}{3}$

$-\frac{16}{3} \ge x$

$-\frac{19}{3}$ $\quad -\frac{16}{3}$ $\quad \therefore x \in \left(\frac{-19}{3}, \frac{-16}{3}\right]$

c. $3\left(x+\frac{1}{5}\right) < \frac{18}{5}$

$3x + \frac{3}{5} < \frac{18}{5}$

$3x < \frac{18}{5} - \frac{3}{5}$

$3x < \frac{15}{5}$

$x < 1$ $\quad \therefore x \in (-\infty, 1)$

d. $2x^2 + 9x - 3 \ge 2$ ← Quad make equation $=0$ factor sketch

$2x^2 + 9x - 5 \ge 0$

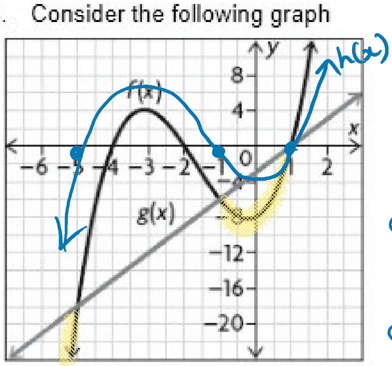
$f(x) = (2x-1)(x+5) \ge 0$

$f(x) \ge 0$ above x-axis

-5 $\quad \frac{1}{2}$

ⓓ $x \in (-\infty, -5]$ or $\left[\frac{1}{2}, \infty\right)$

4. Consider the following graph



a) Determine when $f(x) \le g(x)$, write answer in interval notation and set notation

b) Come up with equations for $f(x)$ and $g(x)$

c) Graph $f(x) - g(x)$ by finding its zeros and compare your results.

ⓐ $x \in (-\infty, -5] \cup [-1, 1]$

$x \le -5, \ -1 \le x \le 1$

ⓑ Zeros of $f(x)$ $-4, -2, 1$ $\quad y = a(x+4)(x+2)(x-1)$ sub pt. $(0,-8)$

$-8 = a(4)(2)(-1)$

$-8 = -8a$

$1 = a$ $\quad \therefore f(x) = (x+4)(x+2)(x-1)$

Zeros of $g(x)$ 1 $\quad y = a(x-1)$ sub pt. $(-1,-6)$

$-6 = a(-1-1)$

$-6 = -2a$

$3 = a$ $\quad \therefore g(x) = 3(x-1)$

ⓒ $f(x) - g(x)$

$= (x+4)(x+2)(x-1) - 3(x-1)$

$= (x+4)(x^2+x-2) - 3x + 3$

$= x^3 + x^2 - 2x + 4x^2 + 4x - 8 - 3x + 3$

$= x^3 + 5x^2 - x - 5 = h(x)$

$h(1) = 0 \quad \therefore (x-1)$ is a factor of $h(x)$

$$\begin{array}{r|rrrr} 1 & 1 & 5 & -1 & -5 \\ & \downarrow & 1 & 6 & 5 \\ \hline & 1 & 6 & 5 & 0 \end{array}$$

$h(x) = (x-1)(x^2+6x+5)$

$= (x-1)(x+5)(x+1)$

Zeros of $h(x)$ coincide with where $f(x) = g(x)$

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Solve Polynomial Inequalities with +/- Table

1. As you have already seen, it is not always possible to solve an inequality by simply isolating the variable. Summarize the steps you've learned about solving any polynomial inequality by a graphing method. Then use the steps to show a solution for the given question.

STEPS

1.) Move all terms to one side

2.) Factor the related function

3.) Graph the function + look for solutions:

$f(x) > 0$ above x-axis

$f(x) < 0$ below x-axis

if equals, include zeros too.

eg. $6x^3 - 41x > 11x^2 - 60$

$6x^3 - 41x - 11x^2 + 60 > 0$

let $f(x) = 6x^3 - 11x^2 - 41x + 60$

possible zeros $\frac{\pm 60}{6}$ and their factors

$f(3) = 0 \therefore (x-3)$ is a factor of $f(x)$

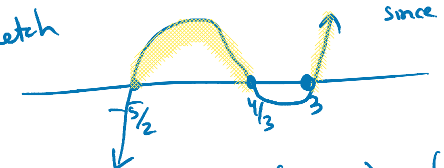
$$\begin{array}{r|rrrr} 3 & 6 & -11 & -41 & 60 \\ & \downarrow & 18 & 21 & -60 \\ \hline & 6 & 7 & -20 & 0 \end{array}$$

$\therefore f(x) = (x-3)(6x^2 + 7x - 20)$

$6\binom{2}{1}\binom{2}{3}$ $\quad 4\binom{5}{5}\binom{-4}{-4}$ 10 2 2 10 one neg

$f(x) = (x-3)(2x+5)(3x-4)$

Sketch



since $f(x) > 0$

$\therefore x \in \left(-\frac{5}{2}, \frac{4}{3}\right) \cup (3, \infty)$

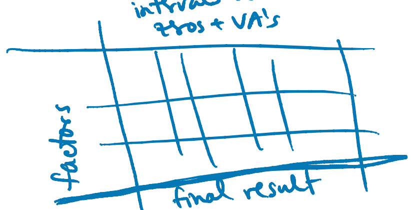
2. In the next unit you will study rational functions, which are not so easily graphed as the polynomial functions are, so you should learn another method of solving inequalities that does not involve graphing. Outline the new steps and solve the same equation with the +/- table.

STEPS

1.) Move all terms to one side

2.) Factor the related function

3.) Create a chart



4.) Multiply the # of + and - in the chart to find the overall result.

eg. $6x^3 - 41x > 11x^2 - 60$

$f(x) = 6x^3 - 41x - 11x^2 + 60 > 0$

$f(x) = (x-3)(2x+5)(3x-4) > 0$

too much to write do: $-\infty \quad -5/2 \quad 4/3 \quad 3 \quad \infty$

	$x < -5/2$	$-5/2 < x < 4/3$	$4/3 < x < 3$	$x > 3$
$x-3$	−	−	−	+
$2x+5$	−	+	+	+
$3x-4$	−	−	+	+
$f(x)$	−	+	−	+

since $f(x) > 0$ look at positives

$\therefore x \in \left(-\frac{5}{2}, 4/3\right) \cup (3, \infty)$

9

$x = -1$

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3. Practice using the +/- table to solve the following

check if can factor more!

a. $-3x(x^2+1)(2x+1)(5-x) \leq 0$

	$-\infty$	$-\frac{1}{2}$	0	5	∞
-3	$-$	$-$	$-$	$-$	
x	$-$	$-$	$+$	$+$	
x^2+1	$+$	$+$	$+$	$+$	
$2x+1$	$-$	$+$	$+$	$+$	
$5-x$	$+$	$+$	$+$	$-$	
$f(x)$	$-$	$+$	$-$	$+$	

$\therefore x \in (-\infty, -\frac{1}{2}] \cup [0, 5]$

include since also equals zero!

b. $2x(1-x) + x(1-x^2) > 0$

can't use +/- chart right away → must have everything being multiplied → factor!!

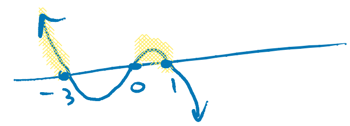
$2x(1-x) + x(1+x)(1-x) > 0$

$(1-x)[2x + x(1+x)] > 0$

$(1-x)(2x + x + x^2) > 0$

$(1-x)(3x + x^2) > 0$

$x(1-x)(3+x) > 0$



	$-\infty$	-3	0	1	∞
x	$-$	$-$	$+$	$+$	
$1-x$	$+$	$+$	$+$	$-$	
$3+x$	$-$	$+$	$+$	$+$	
$f(x)$	$+$	$-$	$+$	$-$	

$\therefore x \in (-\infty, -3) \cup (0, 1)$

c. $(6-3x)(x+2)(1-x) \leq 0$

	$-\infty$	-2	1	2	∞
$6-3x$	$+$	$+$	$+$	$-$	
$x+2$	$-$	$+$	$+$	$+$	
$1-x$	$+$	$+$	$-$	$-$	
$f(x)$	$-$	$+$	$-$	$+$	

$\therefore x \in (-\infty, -2] \cup [1, 2]$

d. $16x^2 + 36x + 18 \leq 2x^4 + 4x^3$

$0 \leq 2x^4 + 4x^3 - 16x^2 - 36x - 18$

$0 \leq 2(x^4 + 2x^3 - 8x^2 - 18x - 9) = f(x)$

$f(3) = 0 \quad \therefore (x-3)$ is a factor of $f(x)$

$$\begin{array}{c|ccccc} 3 & 1 & 2 & -8 & -18 & -9 \\ & \downarrow & 3 & 15 & 21 & 9 \\ \hline & 1 & 5 & 7 & 3 & 0 \end{array}$$

$f(x) = 2(x-3)\underbrace{(x^3+5x^2+7x+3)}_{g(x)} \geq 0$

$g(-3) = 0 \quad \therefore (x+3)$ is a factor of $g(x)$

$$\begin{array}{c|cccc} -3 & 1 & 5 & 7 & 3 \\ & \downarrow & -3 & -6 & -3 \\ \hline & 1 & 2 & 1 & 0 \end{array}$$

$\therefore f(x) = 2(x-3)(x+3)(x^2+2x+1) \geq 0$

$2(x-3)(x+3)(x+1)^2 \geq 0$

	$-\infty$	-3	-1	3	∞
2	$+$	$+$	$+$	$+$	
$x-3$	$-$	$-$	$-$	$+$	
$x+3$	$-$	$+$	$+$	$+$	
$(x+1)^2$	$+$	$+$	$+$	$+$	
$f(x)$	$+$	$-$	$-$	$+$	

$\therefore x \in (-\infty, -3] \cup [3, \infty)$

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Problem Solve

1.

There are two roller coasters that have cars travelling at the same time. The heights of the coasters are modelled during 5 seconds. The height of the first coaster in metres is modelled by $h_1(t) = t^3 - 6t^2 + 28t - 10$. The height of the second coaster in metres is modelled by $h_2(t) = t^2 + 14t - 2$.

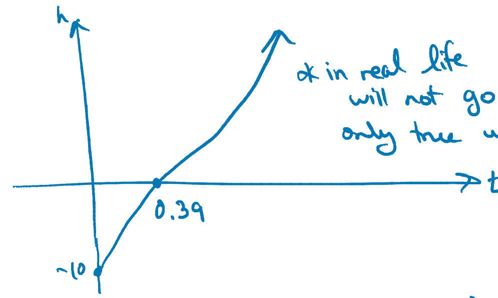
a) Determine the real roots of the function for the height of the first coaster.
b) Determine the real roots of the function for the height of the second coaster.
c) At what times are the two coasters at the same height?
d) What are the heights that correspond to the times in part c)?
e) What is the maximum height of the first coaster during the 5-second interval?
f) What is the maximum height of the second coaster during the 5-second interval?

ⓐ $0 = t^3 - 6t^2 + 28t - 10$ try $\pm 10, \pm 5, \pm 2, \pm 1$

none work ☹ doesn't mean no zeros since cubic must have at least one zero!!

*just like not all quadratics factor nicely other polynomials also may not factor (show cubic formula - too crazy to help find solutions)

∴ can use technology

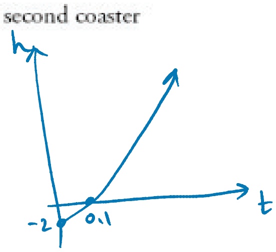


* in real life will not go to ∞! only true up to 5 sec.

ⓑ $0 = t^2 + 14t - 2$

$t = \dfrac{-14 \pm \sqrt{14^2 - 4(1)(-2)}}{2(1)}$

$t \doteq -14.1$ or $t \doteq 0.1$



ⓒ $h_1 = h_2$

$t^3 - 6t^2 + 28t - 10 = t^2 + 14t - 2$

let $f(t) = t^3 - 7t^2 + 14t - 8 = 0$

$f(1) = 0$ ∴ $(t-1)$ is a factor of $f(t)$

1	1	-7	14	-8
	↓	1	-6	8
	1	-6	8	0

∴ $f(t) = (t-1)(t^2 - 6t + 8)$

$f(t) = (t-1)(t-2)(t-4)$

∴ the coasters are at the same height at $t = 1$ sec, 2 sec, 4 sec

ⓓ heights are 13 m, 30 m, 70 m respectively

SHOW with technology!

ⓔ Abs. MAX on $t \in [0, 5]$

no turning points for this cubic

$h_1(0) = -10$ m

$h_1(5) = 105$ m ← abs. max at $(5, 105)$

ⓕ parabola has a turning pt. but not in the interval $t \in [0, 5]$

∴ $h_2(0) = -2$ m

$h_2(5) = 93$ m ← abs. max at pt. $(5, 93)$

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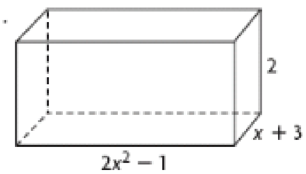
2.



A box has a length of $2x^2 - 1$ units, a width of $x + 3$ units, and a height of 2 units.

a. Write a function for the volume of the box in terms of x.
b. Determine the values of x that will produce a zero volume
c. What is the domain for this real life situation?
d. What are the dimensions of the box if the volume is $204 cm^3$?

(a) $V = lwh$

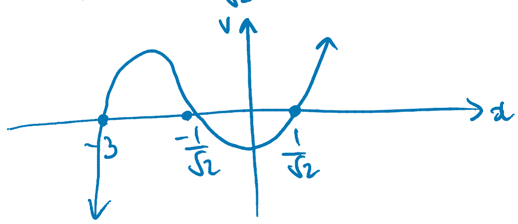
$V = (2x^2-1)(x+3)(2)$

(b) $0 = (2)(x^2 - \frac{1}{2})(x+3)(2)$

$0 = 4(x + \frac{1}{\sqrt{2}})(x - \frac{1}{\sqrt{2}})(x+3)$

$\therefore$ at $x = \pm\frac{1}{\sqrt{2}}$ and $x = -3$ Volume $= 0$

(c)



$D = \{x \in \mathbb{R},\ -3 < x < -\frac{1}{\sqrt{2}},\ x > \frac{1}{\sqrt{2}}\}$

(d) $204 = 2(2x^2-1)(x+3)$

$204 = 2(2x^3 + 6x^2 - x - 3)$

$0 = 4x^3 + 12x^2 - 2x - 6 - 204 = f(x)$

$0 = 2(2x^3 + 6x^2 - x - 105) = f(x)$

$f(3) = 0$ $\therefore (x-3)$ is a factor of $f(x)$

3	2	6	-1	-105
	↓	6	36	105
	2	12	35	0

$\therefore 0 = 2(x-3)(2x^2 + 12x + 35)$

$b^2 - 4ac$ is neg.

$\therefore$ no more solutions only $x = 3$

$\therefore$ length $= 2x^2 - 1 = 2(3)^2 - 1 = 17cm$

width $= x+3 = 3+3 = 6cm$

height $= 2cm$

give volume of $204 cm^3$

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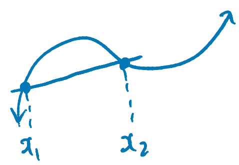
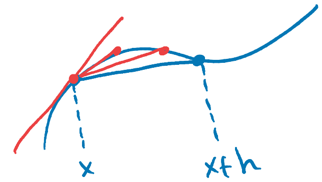
Rates of Change of Polynomial Functions

1. Recall how to find the average rates of change and instantaneous rates of change.

$$a.r.o.c = \frac{f(x_2)-f(x_1)}{x_2-x_1}$$

$$i.r.o.c \xleftarrow[h\to 0]{as} a.r.o.c = \frac{f(x+h)-f(x)}{h}$$





2. For $y = x^3 + 7x - 1$
   a. Find the slope of the tangent line at x=4
   b. Determine the equation of the tangent line in part a.

3. For $f(x) = 2x^2 + 7x - 1$
   a. Determine the ave.r.o.c. of f(x) on the interval $-1 \le x \le 1$
   b. Determine the equation of the secant line in part a.

1
1 1
1 2 1
1 3 3 1

ⓐ $m_{sec} = \frac{f(4+h)-f(4)}{h}$

$= \frac{[(4+h)^3 + 7(4+h) - 1] - [4^3 + 7(4) - 1]}{h}$

$= \frac{\cancel{4^3} + 3(4)^2h + 3(4)h^2 + h^3 + \cancel{7(4)} + 7h \cancel{-1} \cancel{-4^3} \cancel{-7(4)} \cancel{+1}}{h}$

$= \frac{\cancel{h}[48 + 12h + h^2 + 7]}{\cancel{h}}$

$\xrightarrow[h\to 0]{as}$ $m_{tan} = 48 + 12(0) + 0^2 + 7 = 55$

$\therefore m_{tan} = 55$ at $x = 4$

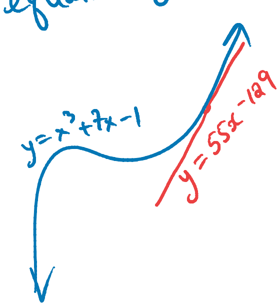
ⓑ $y = mx + b$ $m = 55$ pt. $(4, 91)$

$91 = 55(4) + b$

$91 = 220 + b$

$-129 = b$

$\therefore y = 55x - 129$ is the equation of tangent line



ⓐ $a.r.o.c = \frac{f(1) - f(-1)}{1 - -1}$

$= \frac{8 - -6}{2}$

$= \frac{14}{2}$

$= 7$

ⓑ $y = mx + b$

$y = 7x + b$ pt. $(1, 8)$

$8 = 7(1) + b$

$8 = 7 + b$

$1 = b$

$\therefore$ secant line is

$y = 7x + 1$

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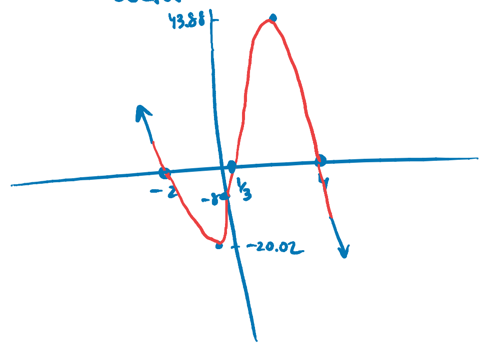
4. Sketch the polynomial $f(x) = -3x^3 + 7x^2 + 22x - 8$ by showing all the details, follow the following steps.

a. Find the y-intercept and sketch it on the graph
b. Factor the polynomial to find zeros
c. Sketch the zeros and the end behaviour
d. Find the equation for the slope of the tangent at any point x (use the difference quotient)
e. Find where the tangent slope is zero – ie. Find where the turning points are
f. Find the y-coordinates of the turning points
g. Sketch the turning points and connect all the sketched information with a smooth curve.

If you are not sure if t.p. are MAX or MIN look at slopes before and after.



ⓐ y-int $= (0, -8)$

ⓑ $f(-2) = 0 \quad \therefore (x+2)$ is a factor of $f(x)$

$$\begin{array}{r|rrrr} -2 & -3 & 7 & 22 & -8 \\ & \downarrow & 6 & -26 & 8 \\ \hline & -3 & 13 & -4 & 0 \end{array}$$

$f(x) = (x+2)(-3x^2+13x-4)$

$= (x+2)(-3x+1)(x-4)$

zeros $(-2, 0)$ $(\frac{1}{3}, 0)$ $(4, 0)$

ⓓ $m_{sec} = \dfrac{f(x+h) - f(x)}{h} = \dfrac{[-3(x+h)^3 + 7(x+h)^2 + 22(x+h) - 8] - [-3x^3 + 7x^2 + 22x - 8]}{h}$

$= \frac{1}{h}[-3(x^3 + 3x^2h + 3xh^2 + h^3) + 7(x^2 + 2xh + h^2) + \cancel{22x} + 22h \cancel{-8} + 3x^3 - 7x^2 \cancel{- 22x} \cancel{+8}]$

$= \frac{1}{h}[\cancel{-3x^3} - 9x^2h - 9xh^2 - 3h^3 \cancel{+7x^2} + 14xh + 7h^2 + 22h + \cancel{3x^3} \cancel{-7x^2}]$

$= \frac{\cancel{h}}{\cancel{h}}[-9x^2 - 9xh - 3h^2 + 14x + 7h + 22] \xrightarrow[h \to 0]{as} m_{tan} = -9x^2 + 14x + 22$

ⓔ $0 = -9x^2 + 14x + 22$

$x = \dfrac{-14 \pm \sqrt{14^2 - 4(-9)(22)}}{2(-9)}$

$x \doteq \dfrac{-14 \pm \sqrt{988}}{-18}$ → $x \doteq 2.52$ → $x \doteq -0.97$

ⓕ Turning pts are approx $(2.52, 43.88)$ $(-0.97, -20.02)$