Polynomial Equations & Inequalities Unit 4

Tentative TEST date_____

Big idea

This unit is a continuation of the last unit, where you have learned about characteristics of polynomial functions as well as how to factor polynomial functions. Keep in mind that not all polynomials can be factored over the rational numbers. Recall that some quadratic equations have irrational solutions that could only be found by the quadratic formula – this is also true for higher degree polynomials. When you cannot factor the given polynomial you can resort to graphing it by using technology to find the solutions of those that do not factor nicely. (Of course on the test you will be provided with questions that DO work out algebraically.) Once you are comfortable with factoring any polynomial you will use that knowledge in problem solving as well as in solving equations and inequalities. At the end you will revisit rates of change once again.

Corrections for the textbook answers:

Section 4.1 #12 text has extra negative sign in the question – ignore one of them #16 2(x+2)(x-5)(x+3)

Section 4.2 #1c) x< – 5 #13 x< 8 min #17b) -4<x<2.8 approx

Section 4.3 #6e) x<=-3/2, x>=3

Section 4.4 #4a answer is not 3 but 6 or 7

Success Criteria

☐ I understand the new topics for this unit if I can do the practice questions in the textbook/handouts

Date	pg	Topics	# of quest. done? You may be asked to show them	**Questions I had difficulty with** ask teacher before test!
	2-3	Solve Polynomial Equations TWO Handouts		
	4-6	Families of Polynomials Handout & TWO Handouts on REGRESSION		
	7-8	Solve Polynomial Inequalities - Graphing Section 4.2 & 4.3 & EXTRA Handout with Technology		
	9-10	Solve Polynomial Inequalities - +/- Table Section 4.2 & 4.3 & Handout		
	11-12	Problem Solve Section 4.1 & Handout		
	13-14	Rates of Change of Polynomial Functions Section 4.4		
		REVIEW		

Reflect – previous TEST mark _____, Overall mark now_____.

Solve Polynomial Equations

1. Solve the following equations, then sketch the function that corresponds to the one with all the terms moved to the **left** hand side.

a. $6x^3 + 49x^2 + 8x - 12 = 2x + 4$

b. $113x - 30 = 8x^3 - 30x^2$

Solve the following equations, then sketch the function that corresponds to the one with all the terms moved to the **left** hand side.

c. $-6x^2 - 6x = 5x^3 + 1$

d. $8x^3 + 22x^2 - 3x - 2 = 0$

Families of Polynomials

1. Recall that if you are given a set of conditions you may have many answers (or a **family** of) equations that will satisfy them. There will be one unique equation only if enough information is provided. Come up with the polynomial family that satisfies the following conditions.

a. 

i. A cubic polynomial with end behaviour $x \to -\infty, y \to \infty$ and a y-intercept of 6.

ii. The above polynomial has a double root at 4 and a root at 8.

iii. Sketch this function.

b. 

i. Polynomial of degree 6, with zeros at -2, 4 (order2), 8 (order 3)

ii. The above polynomial goes through the point (-4, 80)

iii. Sketch this function

2. To find the equation for a given table of values (show the interactive MME from eLearningOntario course) to see what is the relationship between the constant differences, the degree of the function and the leading coefficient. Record the important information here:

3. Find the equations for the given tables

a.

X	Y
1	0
2	1
3	10
4	33
5	76
6	145
7	246

b.

X	Y
1	4
2	15
3	30
4	49
5	72
6	99

Sometimes it is imposible to find the equation algebraically for two reasons.

- The given data is from a real life situation and the values will not yeild exact constants for any difference column. This is mainly due to error or to the fact that you can only approximate values for real life data.
- The x values are not sequential – they skip count

In both cases you must use technology to find the regression equation. Ask your teacher for instructions of how to do this with either Excel, TI-83, or CurveExpert.

4. A certain business company spends money of advertising to get specific revenues back. The data is provided below.

Advertising (in $10 000's)	Revenue (in $100 000's)
50	34.375
100	125.000
150	253.125
200	400.000
250	546.875
300	675.000
350	765.625
400	800.000
450	759.375
500	625.000

a. Use techology to determine the equation that best fits the data.
b. Use techology to create the sketch
c. What is the domain of this function for this real life situation?
d. For what amount of advertising does the company get an absolute maximum in revenue?

Solve Polynomial Inequalities by Graphing

1. RULES FOR INEQUALITIES and reasons for them:

2. $5-3x<14$

3. $2x<10 \text{ or } \frac{x}{2}\geq 3$

4. $x+8\geq 9 \text{ and } \frac{x}{7}\leq 1$

5. $2x^2-1\geq 7$ show both algebraic and graphical solution to help understand why can't just isolate the x

6. $2x^2-3x\leq 5$

7. $3|x+5|\leq 6$

8. $\left|\frac{m}{2}\right|+5\geq 6$

9. $9x^2+31x\leq -12$

10. $4x^2+4x+1>0$

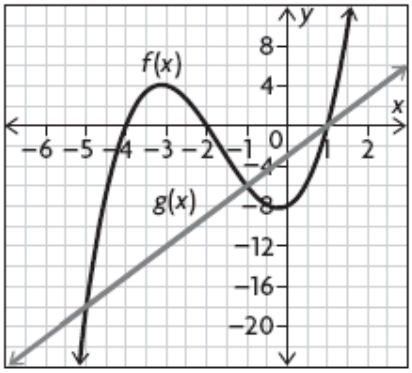
11. Solve.

a. $-4 < x^2 - 8$ show both algebraic and graphical solution for this one

b. $-3\left|x+\frac{1}{5}\right| < \frac{18}{5}$

c. $2x^2 + 9x - 3 \geq 2$

12. Consider the following graph



a) Determine when $f(x) \leq g(x)$, write answer in interval notation and set notation

b) Come up with equations for $f(x)$ and $g(x)$

c) Graph $f(x) - g(x)$ by finding its zeros and compare your results.

Solve Polynomial Inequalities with +/- Table

1. As you have already seen, it is not always possible to solve an inequality by simply isolating the variable. Summarize the steps you`ve learned about solving any polynomial inequality by a graphing method. Then use the steps to show a solution for the given question.

STEPS

Eg. $6x^3 - 41x > 11x^2 - 60$

2. In the next unit you will study rational functions, which are not so easily graphed as the polynomial functions are, so you should learn another method of solving inequalities that does not involve graphing. Outline the new steps and solve the same equation with the +/- table.

STEPS

Eg. $6x^3 - 41x > 11x^2 - 60$

3. Practice using the +/- table to solve the following

a. $-3x(x^2+1)(2x+1)(5-x) \le 0$

b. $2x(1-x)+x(1-x^2) > 0$

c. $(6-3x)(x+2)(1-x) \le 0$

d. $16x^2+36x+18 \le 2x^4+4x^3$

Problem Solve

1.

There are two roller coasters that have cars travelling at the same time. The heights of the coasters are modelled during 5 seconds. The height of the first coaster in metres is modelled by $h(t) = t^3 - 6t^2 + 28t - 10$. The height of the second coaster in metres is modelled by $h(t) = t^2 + 14t - 2$.

a) Determine the real roots of the function for the height of the first coaster.

b) Determine the real roots of the function for the height of the second coaster.

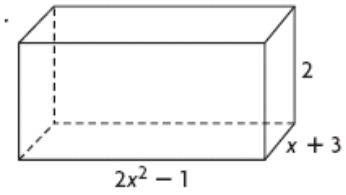
c) At what times are the two coasters at the same height?

d) What are the heights that correspond to the times in part c)?

e) What is the maximum height of the first coaster during the 5-second interval?

f) What is the maximum height of the second coaster during the 5-second interval?

2.



A box has a length of $2x^2 - 1$ units, a width of $x + 3$ units, and a height of 2 units.

a. Write a function for the volume of the box in terms of x.
b. Determine the values of x that will produce a zero volume
c. What is the domain for this real life situation
d. What are the dimensions of the box if the volume is $204cm^3$?

Rates of Change of Polynomial Functions

1. Recall how to find the average rates of change and instantaneous rates of change.

2. For $y = x^3 + 7x - 1$
   a. Find the slope of the tangent line at x=4
   b. Determine the equation of the tangent line in part a.

3. For $f(x) = 2x^2 + 7x - 1$
   a. Determine the ave.r.o.c. of f(x) on the interval $-1 \le x \le 1$
   b. Determine the equation of the secant line in part a.

4. Sketch the polynomial $f(x) = -3x^3 + 7x^2 + 22x - 8$ by showing all the details, follow the following steps.

   a. Find the y-intercept and sketch it on the graph
   b. Factor the polynomial to find zeros
   c. Sketch the zeros and the end behaviour
   d. Find the equation for the slope of the tangent at any point x (use the difference quotient)
   e. Find where the tangent slope is zero – ie. Find where the turning points are
   f. Find the y-coordinates of the turning points
   g. Sketch the turning points and connect all the sketched information with a smooth curve.