

Unit 3: Exponential and Logarithmic Functions

Activity 7: Applications of Exponential and Logarithmic Functions

Homework/Formative Assignment

1. There are now 400 insects in a colony. If the population of the colony triples every two months:
 - a. How many insects will there be in 6 months?
 - b. How long does it take the population to grow to 100,000?
2. "Radium-226" is a radioactive element with a half-life of 1600 years.
 - a. How much of a 1000 g sample of the element will be present after 6400 years?
 - b. How long will it take for the 1000 gram sample to decay to 1 gram?
3. On September 26, 2001, an earthquake in North Bay measured 5.0 on the Richter scale. What is the magnitude of an earthquake 3 times as intense as North Bay's earthquake?
4. How long will it take, to the nearest month, for \$2500 to grow to \$4000, if it is invested at 7%, compounded monthly?
5. How much louder is a MP3 player (100dB) to a crying baby (80dB)?
6. A liquid has a pH of 3.46. Find the hydrogen ion concentration $[H^+]$

Homework/Formative Assignment SOLUTIONS

1. There are now 400 insects in a colony. If the population of the colony triples every two months:

a. How many insects will there be in 6 months?

$$p(x) = 400(3)^{\frac{x}{2}}$$

$$p(6) = 400(3)^{\frac{6}{2}} \quad \text{There would be approximately 10800 insects.}$$

$$p(6) = 10800$$

b. How long does it take the population to grow to 100,000?

$$100000 = 400(3)^{\frac{x}{2}}$$

$$250 = 3^{\frac{x}{2}}$$

There would be 100000 insects in approximately 10 months.

$$\log 250 = \log 3^{\frac{x}{2}}$$

$$\log 250 = \frac{x}{2} \log 3$$

$$\frac{\log 250}{\log 3} = \frac{x}{2}$$

$$\frac{2 \log 250}{\log 3} = x$$

$$10.05 = x$$

2. "Radium-226" is a radioactive element with a half-life of 1600 years.

a. How much of a 1000 g sample of the element will be present after 6400 years?

$$y = 1000 \left(\frac{1}{2}\right)^{\frac{x}{1600}}$$

$$y = 1000 \left(\frac{1}{2}\right)^{\frac{6400}{1600}}$$

$$y = 62.5$$

There would be approximately 62.5 g remaining.

b. How long will it take for the 1000 gram sample to decay to 1 gram?

$$1 = 1000 \left(\frac{1}{2}\right)^{\frac{x}{1600}}$$

$$0.001 = \left(\frac{1}{2}\right)^{\frac{x}{1600}}$$

$$\log 0.001 = \log \left(\frac{1}{2}\right)^{\frac{x}{1600}}$$

$$\log 0.001 = \frac{x}{1600} \log \left(\frac{1}{2}\right)$$

$$\frac{\log 0.001}{\log \left(\frac{1}{2}\right)} = \frac{x}{1600}$$

$$\frac{1600 \log 0.001}{\log \left(\frac{1}{2}\right)} = x$$

$$15945.23 = x$$

It would take about 15945 years for the 1000 g sample to decay to 1 g.

3. On September 26, 2001, an earthquake in North Bay measured 5.0 on the Richter scale. What is the magnitude of an earthquake 3 times as intense as North Bay's earthquake?

$$M = \log\left(\frac{I}{I_0}\right)$$

Let N represent the magnitude of earthquake in North Bay, that is: $N = \log\left(\frac{I}{I_0}\right) = 5.0$.

If the new earthquake is 3 times as intense, you have:

$$M = \log\left(\frac{3I}{I_0}\right)$$

$$M = \log 3 + \log\left(\frac{I}{I_0}\right)$$

$$M = \log 3 + N$$

$$M = \log 3 + 5.0$$

$$M \approx 5.5$$

4. How long will it take, to the nearest month, for \$2500 to grow to \$4000, if it is invested at 7%, compounded monthly?

The 7% annual interest rate must be converted to a monthly rate. To do this, divide

$$\frac{0.07}{12} = 0.005833. \text{ Now you have:}$$

$$4000 = 2500(1.005833)^x$$

$$1.6 = 1.005833^x$$

$$\log 1.6 = \log 1.005833^x$$

$$\log 1.6 = x \log 1.005833$$

$$\frac{\log 1.6}{\log 1.005833} = x$$

$$80.8 = x$$

Where x is the number of growth periods which is months here. It will take 81 months or 6.75 years for the investment to grow from \$2500 to \$4000.

5. How much louder is a MP3 player (100dB) to a crying baby (80dB)?

$$L = 10 \log \left(\frac{I}{I_o} \right)$$

For the MP3 player, you have: $100 = 10 \log \left(\frac{I_m}{I_o} \right)$

and the crying baby: $80 = 10 \log \left(\frac{I_b}{I_o} \right)$.

$$100 = 10 \log \left(\frac{I_m}{I_o} \right)$$

$$80 = 10 \log \left(\frac{I_b}{I_o} \right)$$

MP3: $10 = \log \left(\frac{I_m}{I_o} \right)$

Crying Baby: $8 = \log \left(\frac{I_b}{I_o} \right)$

$$10^{10} = \left(\frac{I_m}{I_o} \right)$$

$$10^8 = \frac{I_b}{I_o}$$

$$10^{10} I_o = I_m$$

$$10^8 I_o = I_b$$

Now, comparing the two sounds,

$$\frac{I_m}{I_b} = \frac{10^{10} I_o}{10^8 I_o}$$

$$\frac{I_m}{I_b} = 10^2 = 100$$

So the MP3 player is 100 times louder than the crying baby.

6. A liquid has a pH of 3.46. Find the hydrogen ion concentration $[H^+]$

$$pH = -\log(H^+)$$

$$3.46 = -\log(H^+)$$

$$-3.46 = \log(H^+)$$

$$10^{-3.46} = H^+$$

$$0.000346 = H^+$$

The hydrogen ion concentration is 0.00346 mol/L