

**Logarithmic Equations****Solve each equation.**

1)  $\log(n + 9) = \log 4n$

2)  $\log -5x = \log(10 - 3x)$

3)  $\log(-3m - 1) = \log(-4m - 6)$

4)  $\log a = \log(4a - 9)$

5)  $-4 \log_3 -9m = -4$

6)  $7 \log_9 (x + 8) = 7$

7)  $-8 + \log_9(m + 1) = -8$

8)  $-2 \log_8(a + 1) = -8$

9)  $\log_2(a^2 - 6a) = \log_2(10 + 3a)$

10)  $\log_{15}(x^2 + 13) = \log_{15}(-9x - 1)$

11)  $\log_{19}(x^2 + 17) = \log_{19}(8x + 2)$

12)  $\log_{12}(m^2 + 73) = \log_{12}(17m + 3)$

$$13) \log x - \log 6 = \log 15$$

$$14) \log 7 + \log x = 2$$

$$15) \log x + \log 2 = \log 2$$

$$16) \log x + \log 8 = 1$$

$$17) \log_4 (x^2 - 3) + \log_4 10 = 1$$

$$18) \log_7 2 + \log_7 (x - 5) = 2$$

$$19) \log_5 3 - \log_5 5x = 2$$

$$20) \log_3 (x^2 + 8) - \log_3 4 = 3$$

$$21) \ln (x + 7) + \ln (x + 3) = \ln 77$$

$$22) \ln (x + 1) - \ln (x - 1) = 3$$

$$23) \ln (x + 2) - \ln (x - 1) = 1$$

$$24) \ln (x + 3) - \ln (x + 2) = 5$$

## Logarithmic Equations

**Solve each equation.**

1)  $\log(n+9) = \log 4n$

{3}

2)  $\log -5x = \log(10 - 3x)$

{-5}

3)  $\log(-3m-1) = \log(-4m-6)$

{-5}

4)  $\log a = \log(4a-9)$

{3}

5)  $-4\log_3 -9m = -4$

\left\{-\frac{1}{3}\right\}

6)  $7\log_9(x+8) = 7$

{1}

7)  $-8 + \log_9(m+1) = -8$

{0}

8)  $-2\log_8(a+1) = -8$

{4095}

9)  $\log_2(a^2 - 6a) = \log_2(10 + 3a)$

{-1, 10}

10)  $\log_{15}(x^2 + 13) = \log_{15}(-9x - 1)$

{-7, -2}

11)  $\log_{19}(x^2 + 17) = \log_{19}(8x + 2)$

{5, 3}

12)  $\log_{12}(m^2 + 73) = \log_{12}(17m + 3)$

{7, 10}

$$13) \log x - \log 6 = \log 15$$

$$\{90\}$$

$$14) \log 7 + \log x = 2$$

$$\left\{ \frac{100}{7} \right\}$$

$$15) \log x + \log 2 = \log 2$$

$$\{1\}$$

$$16) \log x + \log 8 = 1$$

$$\left\{ \frac{5}{4} \right\}$$

$$17) \log_4(x^2 - 3) + \log_4 10 = 1$$

$$\left\{ \frac{\sqrt{85}}{5}, -\frac{\sqrt{85}}{5} \right\}$$

$$18) \log_7 2 + \log_7 (x - 5) = 2$$

$$\left\{ \frac{59}{2} \right\}$$

$$19) \log_5 3 - \log_5 5x = 2$$

$$\left\{ \frac{3}{125} \right\}$$

$$20) \log_3(x^2 + 8) - \log_3 4 = 3$$

$$\{10, -10\}$$

$$21) \ln(x + 7) + \ln(x + 3) = \ln 77$$

$$\{4\}$$

$$22) \ln(x + 1) - \ln(x - 1) = 3$$

$$\left\{ \frac{-1 - e^3}{1 - e^3} \right\}$$

$$23) \ln(x + 2) - \ln(x - 1) = 1$$

$$\left\{ \frac{-2 - e}{1 - e} \right\}$$

$$24) \ln(x + 3) - \ln(x + 2) = 5$$

$$\left\{ \frac{-3 + 2e^5}{1 - e^5} \right\}$$

Marion

$$1) \log(n+9) = \log 4n$$

$$n+9 = 4n$$

$$\begin{array}{r} 9 \\ - 3n \\ \hline 3 \end{array}$$

$$3 = n$$

$$2) \log -5x = \log(10-3x)$$

$$-5x = 10-3x$$

$$\begin{array}{r} -2x \\ - 10 \\ \hline -2 \end{array}$$

$$x = -5$$

$$3) \log(-3m-1) = \log(-4m-6)$$

$$4) \log a = \log(4a-9)$$

$$\begin{array}{r} -3m-1 = -4m-6 \\ +4m+1 \\ \hline m = -5 \end{array}$$

$$\begin{array}{r} a = 4a-9 \\ \frac{a}{3} = \frac{3a}{3} \end{array}$$

$$3 = a$$

$$5) -4\log_3 -9m = -4$$

$$\log_3 -9m = 1$$

$$\begin{array}{r} 3^1 = -9m \\ -9 \quad -9 \\ \hline \end{array}$$

$$\begin{array}{r} -1 \\ \hline 3 \\ m \end{array}$$

$$6) 7\log_9(x+8) = 7$$

$$\log_9(x+8) = 1$$

$$9^1 = x+8$$

$$1 = x$$

$$7) -8 + \log_9(m+1) = -8$$

$$\log_9(m+1) = 0$$

$$9^0 = m+1$$

$$1 = m+1$$

$$0 = m$$

$$8) -2\log_8(a+1) = -8$$

$$\log_8(a+1) = 4$$

$$8^4 = a+1$$

$$4096 = a+1$$

$$4095 = a$$

$$9) \log_2(a^2 - 6a) = \log_2(10 + 3a)$$

$$a^2 - 6a = 10 + 3a$$

$$a^2 - 9a - 10 = 0$$

$$(a-10)(a+1) = 0$$

$$\therefore a = 10, -1$$

$$10) \log_{15}(x^2 + 13) = \log_{15}(9x - 1)$$

$$x^2 + 13 = 9x - 1$$

$$x^2 + 9x + 14 = 0$$

$$(x+7)(x+2) = 0$$

$$\therefore x = -7, -2$$

$$11) \log_{19}(x^2 + 17) = \log_{19}(8x + 2)$$

$$x^2 + 17 = 8x + 2$$

$$x^2 - 8x + 15 = 0$$

$$(x-5)(x-3) = 0$$

$$\therefore x = 5, 3$$

$$12) \log_{12}(m^2 + 73) = \log_{12}(17m + 3)$$

$$m^2 + 73 = 17m + 3$$

$$m^2 - 17m + 70 = 0$$

$$(m-7)(m-10) = 0$$

$$\therefore m = 7, 10$$

$$13) \log x - \log 6 = \log 15$$

$$\log\left(\frac{x}{6}\right) = \log(15)$$

$$\frac{x}{6} = 15$$
$$\therefore x = 90$$

$$14) \log 7 + \log x = 2$$

$$\log 7x = 2$$

$$\frac{10^2}{7} = \frac{7x}{7}$$

$$\frac{10^2}{7} = x$$

$$\therefore \frac{100}{7} = x$$

$$15) \log x + \log 2 = \log 2$$

$$\log 2x = \log 2$$

$$\frac{2x}{2} = \frac{2}{2}$$

$$\therefore x = 1$$

$$16) \log x + \log 8 = 1$$

$$\log(8x) = 1$$

$$\frac{10}{8} = \frac{8x}{8}$$

$$\therefore \frac{5}{4} = x$$

$$17) \log_4(x^2 - 3) + \log_4 10 = 1$$

$$\log_4(10(x^2 - 3)) = 1$$

$$4^1 = 10(x^2 - 3)$$

$$0 = 10x^2 - 30 - 4$$

$$0 = 10x^2 - 34$$

$$\therefore x = \frac{\sqrt{85}}{5}, -\frac{\sqrt{85}}{5}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{0 \pm \sqrt{0^2 - 4(10)(-34)}}{2(10)}$$

$$= \frac{\pm \sqrt{1360}}{20}$$

$$= \frac{\pm 4\sqrt{85}}{20}$$

$$= \frac{\pm \sqrt{85}}{5}$$

$$18) \log_7 2 + \log_7 (x-5) = 2$$

$$\log_7(2(x-5)) = 2$$

$$7^2 = 2(x-5)$$

$$\frac{7^2}{2} = x-5$$

$$\frac{49}{2} = x-5$$

$$\frac{49}{2} + 5 = x$$

$$\therefore \frac{59}{2} = x$$

$$19) \log_5 3 - \log_5 5x = 2$$

$$\log_5\left(\frac{3}{5x}\right) = 2$$

$$5^2 = \frac{3}{5x}$$

$$25 = \frac{3}{5x}$$

$$125x = 3$$

$$\therefore x = \frac{3}{125}$$

$$21) \ln(x+7) + \ln(x+3) = \ln 77$$

$$\ln((x+7)(x+3)) = \ln 77$$

$$(x+7)(x+3) = 77$$

$$x^2 + 10x + 21 = 77$$

$$x^2 + 10x - 56 = 0$$

$$(x-4)(x+14) = 0$$

$$x = 4, -14$$

$$\therefore x = 4$$

Not in domain

$$20) \log_3(x^2 + 8) - \log_3 4 = 3$$

$$\log_3\left(\frac{x^2 + 8}{4}\right) = 3$$

$$3^3 = \frac{x^2 + 8}{4}$$

$$4(3^3) = x^2 + 8$$

$$108 = x^2 + 8$$

$$0 = x^2 - 100$$

$$0 = (x-10)(x+10)$$

$$\therefore x = 10, -10$$

$$22) \ln(x+1) - \ln(x-1) = 3$$

$$\ln\left(\frac{x+1}{x-1}\right) = 3$$

$$e^3 = \frac{x+1}{x-1}$$

$$e^3(x-1) = x+1$$

$$e^3x - e^3 = x+1$$

$$e^3x - x = e^3 + 1$$

$$x(e^3 - 1) = e^3 + 1$$

$$\therefore x = \frac{e^3 + 1}{e^3 - 1}$$

$$23) \ln(x+2) - \ln(x-1) = 1$$

$$\ln\left(\frac{x+2}{x-1}\right) = 1$$

$$e^1 = \frac{x+2}{x-1}$$

$$e(x-1) = x+2$$

$$ex - e = x+2$$

$$ex - x = e+2$$

$$x(e-1) = e+2$$

$$\therefore x = \frac{e+2}{e-1}$$

$$24) \ln(x+3) - \ln(x+2) = 5$$

$$\ln\left(\frac{x+3}{x+2}\right) = 5$$

$$e^5 = \frac{x+3}{x+2}$$

$$e^5(x+2) = x+3$$

$$e^5x + 2e^5 = x+3$$

$$e^5x - x = -2e^5 + 3$$

$$x(e^5 - 1) = -2e^5 + 3$$

$$\therefore x = \frac{-2e^5 + 3}{e^5 - 1}$$