MHF4U: Grade 12 Advanced Functions (Catholic)
Unit 1: Introduction to Polynomial Functions
Activity 4: Factor and Remainder Theorem

Formative Assignment - Factoring Polynomials

Prepare full solutions for each of the following. Then check your answers at the end of this document. Talk to your teacher if you need any further assistance with this material.

1. Factor each of the following functions:

   a. $f(x) = x^4 + 3x^3 - 38x^2 + 24x + 64$

   b. $f(x) = 6x^4 - 19x^3 - 2x^2 + 44x - 24$

2. Determine whether $(2x-5)$ is a factor of each polynomial:

   a. $f(x) = 2x^3 - 5x^2 - 2x + 5$
   b. $g(x) = 3x^3 + 2x^2 - 3x - 2$
   c. $h(x) = 2x^4 - 7x^3 - 13x^2 + 63x - 45$
   d. $f(x) = 6x^4 + x^3 - 7x^2 - x + 1$

3. Sketch the graph of $f(x) = 3x^3 + x^2 - 22x - 24$. First, fully factor $f(x)$ and then analyze the graph, as you did in Activity 3. Use your information to sketch the graph. Check your results by graphing $f(x)$ using GeoGebra or GraphCalc.

4. Fully factor:
   a. $h(x) = x^3 - 8$

   b. $f(x) = 8x^3 - 27$

   c. $g(x) = 27x^3 - 64$

5. Look at the results in question 4. Can you see a pattern? Each of the functions given represents a ***'difference of cubes'***. Try to come up with a pattern for factoring polynomials of the form: $f(x) = a^3 - b^3$.

6. What would be different in your pattern, if you were factoring polynomials of the form $f(x) = a^3 + b^3$?

Solutions

1. Factor each of the following functions:

   a. $f(x) = x^4 + 3x^3 - 38x^2 + 24x + 64$
   $f(x) = (x-2)(x-4)(x+8)(x+1)$

   b. $f(x) = 6x^4 - 19x^3 - 2x^2 + 44x - 24$
   $f(x) = (x-2)^2(3x-2)(2x+3)$

2. Determine whether $(2x-5)$ is a factor of each polynomial:

   a. $f(x) = 2x^3 - 5x^2 - 2x + 5$
   $f\left(\frac{5}{2}\right) = 0$ so $(2x-5)$ is a factor of $f(x)$.

   b. $g(x) = 3x^3 + 2x^2 - 3x - 2$
   $g\left(\frac{5}{2}\right) = 49.875$ so, $(2x-5)$ is not a factor of $g(x)$.

   c. $h(x) = 2x^4 - 7x^3 - 13x^2 + 63x - 45$
   $h\left(\frac{5}{2}\right) = 0$ so $(2x-5)$ is a factor of *h(x).*

   d. $f(x) = 6x^4 + x^3 - 7x^2 - x + 1$
   $f\left(\frac{5}{2}\right) = 204.75$ so $(2x-5)$ is not a factor of *f(x).*

3. Sketch the graph of $f(x) = 3x^3 + x^2 - 22x - 24$. First, fully factor *f(x)* and then analyze the graph, as you did in Activity 3. Use your information to sketch the graph. Check your results by graphing *f(x)* using GeoGebra or GraphCalc.

   ANSWER:
   Factor: $f(x) = (x+2)(x-3)(3x+4)$

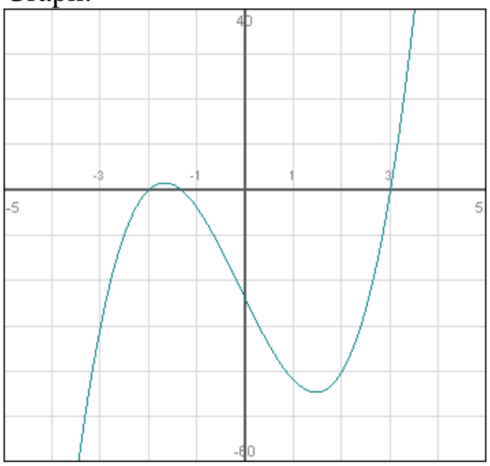
   x-intercepts: *x*=-2, -4/3, 3
   y-intercept: *y*=-24

Odd function, with a positive lead coefficient, so end behaviours are:

$x \to \infty, y \to \infty$

$x \to -\infty, y \to -\infty$

Graph:



4. Fully factor:

   a. $h(x) = x^3 - 8$

      $h(x) = (x-2)(x^2 + 2x + 4)$

   b. $f(x) = 8x^3 - 27$

      $f(x) = (2x-3)(4x^2 + 6x + 9)$

   c. $g(x) = 27x^3 - 64$

      $g(x) = (3x-4)(9x^2 + 12x + 16)$

5. Look at the results in question 4. Can you see a pattern? Each of the functions given represents a ***'difference of cubes'***. Try to come up with a pattern for factoring polynomials of the form: $f(x) = a^3 - b^3$

   $$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

6. What would be different in your pattern, if you were factoring polynomials of the form $f(x) = a^3 + b^3$?

   $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ ← This is called a 'sum of cubes'.