## Unit 4: Trigonometric Functions

# Activity 6: Solving Problems

**Homework/Formative Assessment**

1. In Toronto, Ontario on December 21, 2010, there was 8 hours and 55 minutes of daylight. On June 21, 2011, there was 15 hours and 27 minutes of daylight.

   a) Create a function that models the number of hours of daylight in Toronto as a function of the day of the year.

   b) Use your function to predict the number of hours of daylight on February 14.

   c) Use your function to determine which dates in the year that have 12 hours of daylight.

2. At a local beach, the depth of the water at a pier varies from a minimum of 3.2 metres to a maximum of 7.4 metres, with a period of 12 hours.

   a) If there was a high tide at 10:00 AM, determine the height of the water at 2:30 PM. Consider t=0 at midnight.

   b) Determine the times between 10:00 AM and midnight, when the depth of the water was 5 metres.

**Homework/Formative Assessment SOLUTIONS**

1. Recall the data in the overview of this activity. In Toronto, Ontario on December 21, 2010, there was 8 hours and 55 minutes of daylight. On June 21, 2011, there was 15 hours and 27 minutes of daylight.

   a) Create a function that models the number of hours of daylight in Toronto as a function of the day of the year.

   You first must convert the times to hours: 8 hours, 55 minutes = 8.92 hours (or 535 min)

   And 15 hours 27 minutes = 15.45 hours. (or 927min)

   You also need to determine that December 21 is day 355 of the year and June 21 is day 172.

   $a = \frac{15.45 - 8.92}{2} = 3.27$ $\quad b = \frac{2\pi}{365}$ $\quad c = 172$ $\quad d = \frac{15.45 + 8.92}{2} = 12.19$

   So, the function is $N(d) = 3.27\cos\left[\frac{2\pi}{365}(d - 172)\right] + 12.19$

   b) Use your function to predict the number of hours of daylight on February 14.

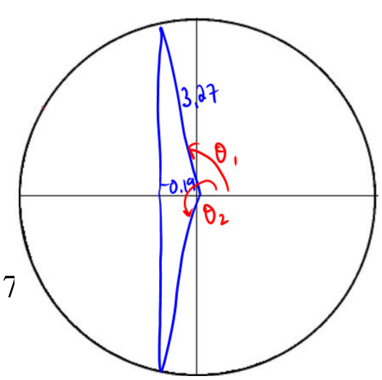
   February 14 = day #45 of the year,

   $$N(45) = 3.27\cos\left[\frac{2\pi}{365}(45 - 172)\right] + 12.19$$
   $$N(45) = 10.30$$

   So you would predict 10.3 hours or 10 hours, 18 minutes of daylight.

   c) Use your function to determine which dates in the year that have 12 hours of daylight.

   $$12 = 3.27\cos\left[\frac{2\pi}{365}(d - 172)\right] + 12.19$$
   $$\cos^{-1}\left(\frac{-0.19}{3.27}\right) = \frac{2\pi}{365}(d - 172)$$
   $$1.62 = \frac{2\pi}{365}(d - 172) \quad \text{or} \quad 4.65 = \frac{2\pi}{365}(d - 17$$
   $$266 = d \quad \text{or} \quad 442 = d$$




   Day 266 corresponds to September 23 and for $d = 442$, we can immediately subtract 365 and determine the date of day 77, which is March 18.

2. At a local beach, the depth of the water at a pier varies from a minimum of 3.2 metres to a maximum of 7.4 metres, with a period of 12 hours.

   a) If there was a high tide at 10:00 AM, determine the height of the water at 2:30 PM. Consider t=0 at midnight

   A function to model this scenario is:

$$h(t) = 2.1\cos\left[\frac{\pi}{6}(t-10)\right] + 5.3$$

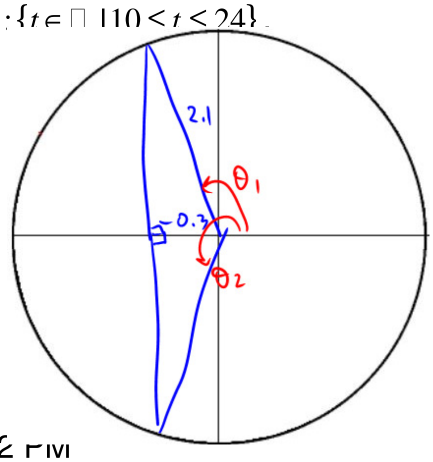
   You would need to switch the time 2:30 PM to 14.5 hours and evaluate the function:

$$h(14.5) = 2.1\cos\left[\frac{\pi}{6}(14.5-10)\right] + 5.3$$
$$h(14.5) = 3.8$$

   The water would be at a depth of 3.8 metres.

   b) Determine the times between 10:00 AM and midnight, when the depth of the water was 5 metres.

   The domain restriction used here will be $D:\{t \in \mathbb{R} \mid 10 < t < 24\}$.




$$5 = 2.1\cos\left[\frac{\pi}{6}(t-10)\right] + 5.3$$
$$\cos^{-1}\left(\frac{-0.3}{2.1}\right) = \frac{\pi}{6}(t-10)$$
$$1.71 = \frac{\pi}{6}(t-10) \quad \text{or} \quad 4.57 = \frac{\pi}{6}(t-10)$$
$$13.3 = t \quad \text{or} \quad 18.7 = \text{t}$$

   13.3 hours = 1:18PM and 18.7 hours = 6:42 PM

   Therefore we would expect a water depth of 5 metres at 1:18 PM and 6:42 PM.