Unit 5: Characteristics of Functions

Activity 2: Comparing Average and Instantaneous Rates of Change

Homework/Formative Assessment

1. Determine the average rate of change of the function $f(x)=2^x$ over the interval $2 \le x \le 4$.

2. Determine the average rate of change of the function $f(x)=2^x$ over the interval $3 \le x \le 4$.

3. Determine the average rate of change of the function $f(x)=2^x$ over the interval $3.9 \le x \le 4$.

4. Determine the average rate of change of the function $f(x)=2^x$ over the interval $3.999 \le x \le 4$.

5. Use your answers from questions 1 to 4 to estimate the instantaneous rate of change of the function $f(x)=2^x$ at $x=4$ to two decimal places.

6. If the $f(x)=2^x$ represents the number of bacteria present at time x in hours. What are the units of the rate of change?

7. Determine the instantaneous rate of change for each function at the indicated point:
   a) $f(x)=\frac{1}{x}; x=5$
   b) $g(x)=2x^4-5x^3; x=1$
   c) $p(x)=3\cos x-2; x=3$

8. Use the algebraic method, difference quotient, $I.R.O.C.$ □ $\frac{f(x+h)-f(x)}{h}$, determine the slope of the tangent line to $f(x)=-x^2+5x-3$ at $x=2$.

9. Use your answer from #8 to determine the equation of the tangent line at $x=2$.

Homework/Formative Assessment SOLUTION

1. Determine the average rate of change of the function $f(x) = 2^x$ over the interval $2 \le x \le 4$.

$$A.R.O.C. = \frac{f(4) - f(2)}{4-2}$$
$$= \frac{16-4}{2}$$
$$= 6$$

2. Determine the average rate of change of the function $f(x) = 2^x$ over the interval $3 \le x \le 4$.

$$A.R.O.C. = \frac{f(4) - f(3)}{4-3}$$
$$= \frac{16-8}{1}$$
$$= 8$$

3. Determine the average rate of change of the function $f(x) = 2^x$ over the interval $3.9 \le x \le 4$.

$$A.R.O.C. = \frac{f(4) - f(3.9)}{4-3.9}$$
$$= \frac{16-14.928528}{0.1}$$
$$= 10.71472$$

4. Determine the average rate of change of the function $f(x) = 2^x$ over the interval $3.999 \le x \le 4$.

$$A.R.O.C. = \frac{f(4) - f(3.999)}{4-3.999}$$
$$= \frac{16-15.988913}{0.001}$$
$$= 11.087$$

5. Use your answers from questions 1 to 4 to estimate the instantaneous rate of change of the function $f(x) = 2^x$ at $x = 4$ to two decimal places.
   The instantaneous rate of change is approximately 11.09

6. If the $f(x) = 2^x$ represents the number of bacteria present at time x in hours. What are the units of the rate of change?

   Units are always

$$\frac{dependent}{independent}$$

   So here the units are

$$\frac{bacteria}{hour}$$

7. Determine the instantaneous rate of change for each function at the indicated point:

a) $f(x) = \frac{1}{x}; x = 5$

$$IROC \approx \frac{f(5) - f(4.999)}{5-4.999}$$
$$= \frac{0.2-0.20004}{0.001}$$
$$= -0.04$$

b) $g(x) = 2x^4 - 5x^3; x = 1$

$$IROC \approx \frac{g(1) - g(0.999)}{1-0.999}$$
$$= \frac{-3-(-2.993003)}{0.001}$$
$$= -6.997$$

c) $p(x)=3\cos x-2; x=3$

$$IROC \approx \frac{p(3)-p(2.999)}{3-2.999}$$
$$=\frac{-4.969977-(-4.969553)}{0.001}$$
$$=-0.424$$

8. Use the algebraic method, difference quotient,
$$I.R.O.C. \approx \frac{f(x+h)-f(x)}{h},$$
determine the slope of the tangent line to $f(x)=-x^2+5x-3$ at $x=2$.

$$I.R.O.C. \approx \frac{f(x+h)-f(x)}{h}$$
$$=\frac{\left[-(2+h)^2+5(2+h)-3\right]-\left(-(2)^2+5(2)-3\right)}{h}$$
$$=\frac{\left[-4-4h-h^2+10+5h-3\right]+4-10+3}{h}$$
$$=\frac{h-h^2}{h}$$
$$=\frac{h(1-h)}{h}$$
$$=1-h$$

And, as $h\rightarrow 0$, the instantaneous rate of change goes to 1
$\therefore slope=1$

9. Use your answer from #7 to determine the equation of the tangent line at $x=2$.
You know that the slope is 1 and the value of $f(2)=3$. You just need to determine the y-intercept. Sub in slope and the point (2, 3)

$$y=mx+b$$
$$3=(1)(2)+b$$
$$3-2=b$$
$$1=b$$

So, the equation of the tangent is $y=x+1$.