This Worksheet May be done with technology:

Free Graphing technology If you do not currently have graphing software installed on your computer, please take the time to install some:
GeoGebra: http://www.geogebra.org/cms/en/download
GraphCalc: http://www.graphcalc.com/download.shtml

Part 1: Odd or Even Symmetry - general functions

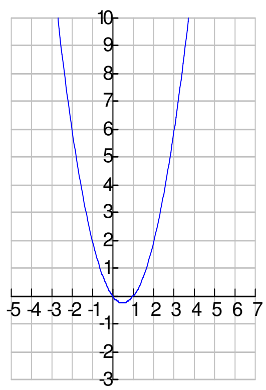
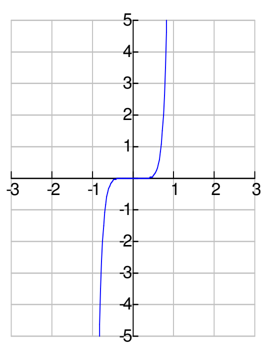
Once you have completed this worksheet, at the end there are solutions to check your work.

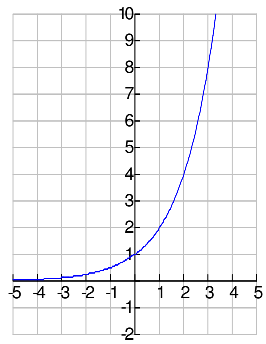
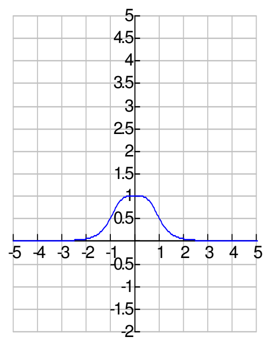
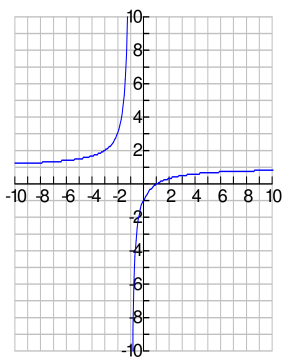
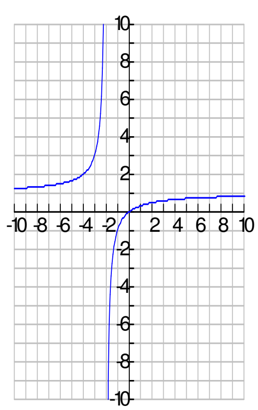
a) If a function is **even** then
- *f(-x)* =________________
- The function is symmetrical about______________________________

b) If a function is **odd** then
- *F(-x)* = ________________
- The function is symmetrical about ______________________________

c) If a function is **neither odd nor even** then
- *f(-x)* _______________________________
- The function is ____

d) Can a function be both even and odd at the same time?

Function	Graph	Odd or even or neither?	*Algebraic Proof*
$f(x)=x^2-x$			
$f(x)=(x+x^3)^5$			

$f(x) = 2^x$			
$f(x)=\frac{1}{x^4+1}$			
$F(x) = \frac{x-1}{x+1}$			
$f(x)=\frac{x}{x+2}$			

Part 2: Odd or Even Symmetry - polynomial functions

For each of the following polynomial functions, fill in the required information, then look for patterns. Use technology GeoGebra or GraphCalc to help you get the sketches.

Equation and sketch	Odd/Even Degree	Odd/Even/Neither symmetry	Number of zeros
$y = x^4 - 2x^2 + 1$			
$y = x^6 + 5x^4 - x^2 + 1$			

What do you notice about all the powers on each term of these polynomials?

____

What do you notice about the number of zeros?

____

Equation and sketch	Odd/Even Degree	Odd/Even/Neither symmetry	Number of zeros
$y = x^5 - 2x^3 - x$			
$y = x^7 + 5x^5 - x^3 + x$			

What do you notice about all the powers on each term of these polynomials?

____

What do you notice about the number of zeros?

____

Equation and sketch	Odd/Even Degree	Odd/Even/Neither symmetry	Number of zeros
$y = x^4 - 2x^3 - x$			
$y = x^6 - 2x^5 - 4x^4 + 6x^3 + 7x^2 - 4x - 4$			
$y = x^7 + 5x^5 - x^3 + 5$			
$y = x^5 - x^4 - 5x^3 + x^2 + 8x + 4$			

What do you notice about all the powers on each term of these polynomials?

____

What do you notice about the number of zeros?

____

Part 1: Odd or Even functions SOLUTIONS

a) If a function is **even** then

- $f(-x) = f(x)$

- The function is symmetrical about the y-axis.

b) If a function is **odd** then

- $f(-x) = -f(x)$

- The function is symmetrical about the origin.

c) If a function is **neither odd nor even** then

- $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$

- The function is not symmetrical about the origin or the y-axis.

d) Can a function be both even and odd at the same time?

Yes – but it is true only for the function that is the horizontal line at the x-axis, f(x)=0.

Proof:

f(-x)=-f(x) must also equal f(x)

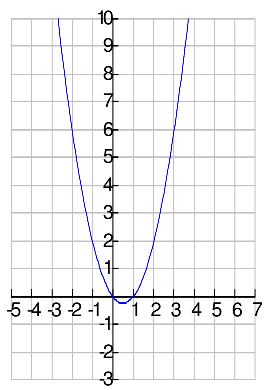
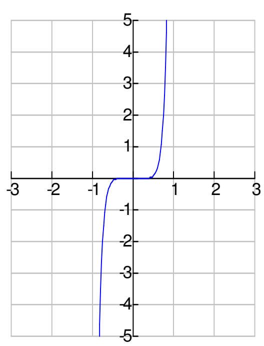
so –f(x)=f(x)

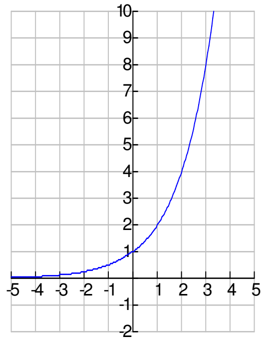
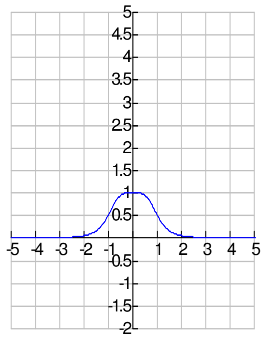
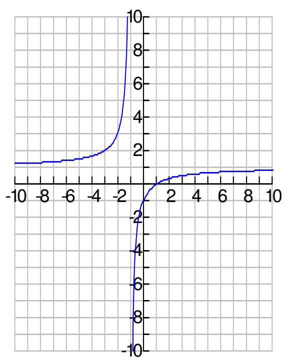
or –y=y

0=2y

0=y

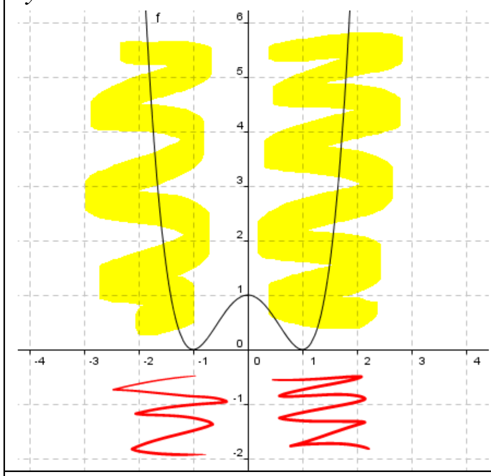
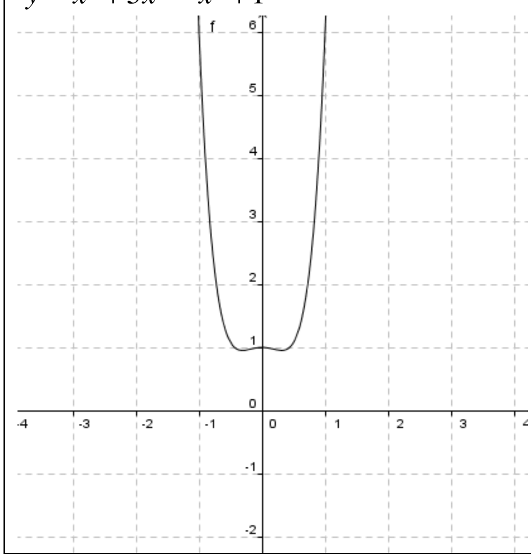
or 0=f(x)

Function	Graph	Odd or even or neither?	*Compare f(-x) with f(x)*
$f(x) = x^2 - x$		We can see that the function is not symmetrical about the y-axis or the origin. Therefore this function is neither odd nor even.	$f(-x) = (-x)^2 - (-x)$ $= x^2 + x$ $\neq f(x)$
$f(x) = (x + x^3)^5$		This function is symmetrical about the origin. This function is odd.	$f(-x) = ((-x) + (-x)^3)^5$ $= (-x - x^3)^5$ $= (-1)^5(x + x^3)^5$ $= -1(x + x^3)^5$ $= -f(x)$

$f(x) = 2^x$		This function is not symmetrical about the y – axis or the origin. This function is neither odd nor even.	$f(-x) = 2^{-x}$ $f(-x) \neq f(x)$
$f(x)=\dfrac{1}{x^4+1}$		This function is even since it is symmetrical about the y-axis.	$f(-x)=\dfrac{1}{(-x)^4+1}$ $=\dfrac{1}{x^4+1}$ $= f(x)$
$f(x) = \dfrac{x-1}{x+1}$		This function is neither odd nor even since it is not symmetrical about the origin or the y-axis.	$f(x) = \dfrac{-x-1}{-x+1}$ $= \dfrac{-(x+1)}{-(x-1)}$ $= \dfrac{x+1}{x-1}$ $f(-x) \neq f(x)$

Part 2: Odd or Even polynomial functions SOLUTIONS

For each of the following polynomial functions, fill in the required information, then look for patterns. Use technology GeoGebra or GraphCalc to help you get the sketches.

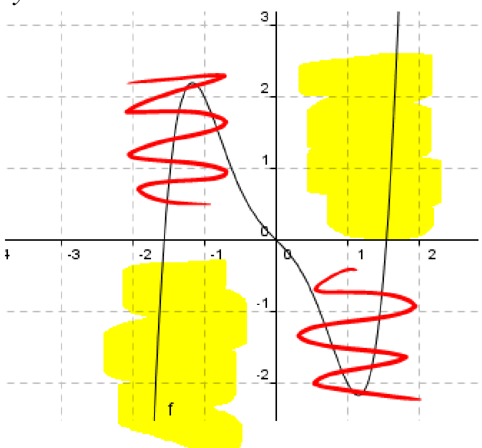
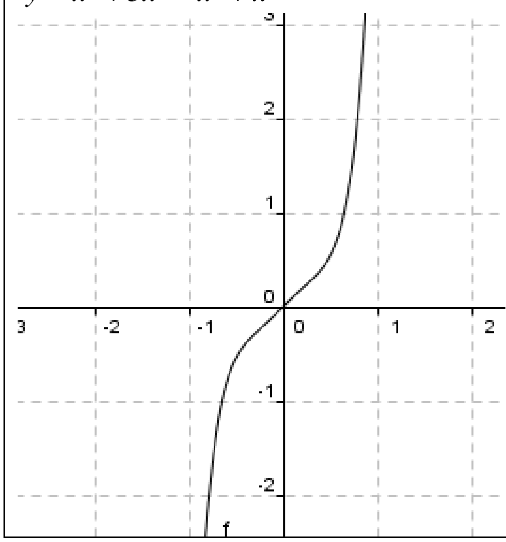
Equation and sketch	Odd/Even Degree	Odd/Even/Neither symmetry	Number of zeros
$y = x^4 - 2x^2 + 1$ 	Even Degree=4	Even symmetry Since quadrant 1 looks like 2 (highlighted yellow), and quadrant 4 looks like 3 (red)	2
$y = x^6 + 5x^4 - x^2 + 1$ 	Even Degree=6	Even symmetry	none

What do you notice about all the powers on each term of these polynomials?

All the powers are even and the even symmetry is preserved.

What do you notice about the number of zeros?

The number of zeros are even in number. (No zeros, means zero, which can be considered even.)

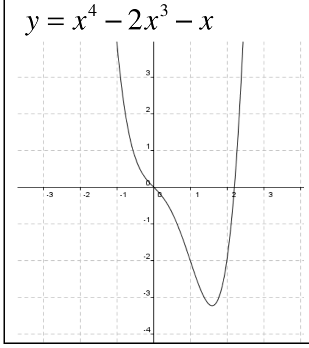
Equation and sketch	Odd/Even Degree	Odd/Even/Neither symmetry	Number of zeros
$y = x^5 - 2x^3 - x$ 	Odd Degree=5	Odd symmetry Since quadrant 1 looks like 3 (highlighted yellow), and quadrant 4 looks like 2 (red)	3
$y = x^7 + 5x^5 - x^3 + x$ 	Odd Degree=7	Odd symmetry	1

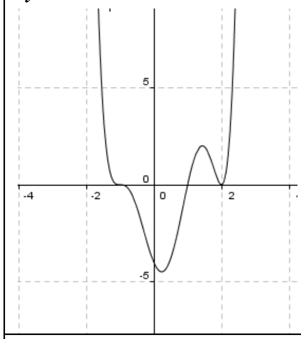
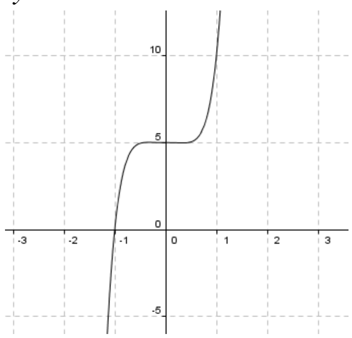
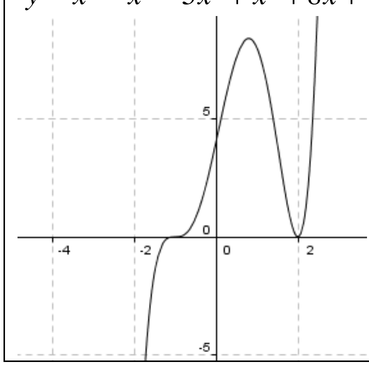
What do you notice about all the powers on each term of these polynomials?

All the powers are odd and the odd symmetry is preserved.

What do you notice about the number of zeros?

The number of zeros are odd in number.

Equation and sketch	Odd/Even Degree	Odd/Even/Neither symmetry	Number of zeros
$y = x^4 - 2x^3 - x$ 	Even Degree=4	Neither symmetry Since none of the quadrants present as the same. Notice that the powers on the terms are a mix of even and odd powers.	2

$y = x^6 - 2x^5 - 4x^4 + 6x^3 + 7x^2 - 4x - 4$  	Even Degree=6	Neither	3
$y = x^7 + 5x^5 - x^3 + 5$  	Odd Degree=7	neither symmetry Notice that the powers on the terms look like all are odd powers. However the symmetry is neither. Why? Consider the term 5 as $5x^o$, which can be considered as an even number.	1
$y = x^5 - x^4 - 5x^3 + x^2 + 8x + 4$  	Odd Degree=5	Neither symmetry	2

What do you notice about all the powers on each term of these polynomials?

All the powers are a mix of odd and even powers and so symmetry is NOT preserved.

What do you notice about the number of zeros?

There is no pattern for the number of zeros when the powers on the terms of the polynomial are not consistent.