# Remainder Theorem and Factor Theorem

**Remainder Theorem:**
When a polynomial $f(x)$ is divided by $x-a$, the remainder is $f(a)$

1. Find the remainder when $2x^3+3x^2-17x-30$ is divided by each of the following:

   (a) $x-1$ (b) $x-2$ (c) $x-3$

   (d) $x+1$ (e) $x+2$ (f) $x+3$

**Factor Theorem**:
If $x=a$ is substituted into a polynomial for $x$, and the remainder is 0, then $x-a$ is a factor of the polynomial.

2. Using the above Theorem and your results from question 1 which of the given binomials are factors of $2x^3+3x^2-17x-30$?

3. Using the binomials you determined were factors of $2x^3+3x^2-17x-30$, complete the division (i.e. divide $2x^3+3x^2-17x-30$ by your chosen ( $x-a$ ) and remember to fully factor your result in each case.

# 1.10.1 Remainder Theorem and Factor Theorem (Answers)

1. Find the remainder when $2x^3+3x^2-17x-30$ is divided by each of the following:

(a) $x-1$

$\therefore a=1$

$f(1)=2(1)^3+3(1)^2-17(1)-30$

$f(1)=2+3-17-30$

$f(1)=-42$

(b) $x-2$

$a=2$

$f(a)=-36$

(c) $x-3$

$a=3$

$f(a)=0$

(d) $x+1$

$a=-1$

$f(a)=-12$

(e) $x+2$

$a=-2$

$f(a)=0$

(f) $x+3$

$a=-3$

$f(a)=-6$

2. Using the above Theorem and your results from question 1 which of the given binomials are factors of $2x^3+3x^2-17x-30$ ?

From results → (c) $x-3$ and (e) $x+2$ are factors

3. Using the binomials you determined were factors of $2x^3+3x^2-17x-30$ complete the division (i.e. divide $2x^3+3x^2-17x-30$ by your chosen $x-a$ ) and remember to fully factor your result in each case.

(c) $x-3$

$$
\begin{array}{r|l}
 & 2x^2+9x+10 \\
\hline
x-3 & 2x^3+3x^2-17x-30 \\
 & \underline{2x^3-6x^2} \quad \downarrow \quad \downarrow \\
 & 9x^2-17x \quad \downarrow \\
 & \underline{9x^2-27x} \quad \downarrow \\
 & 10x-30 \\
 & \underline{10x-30} \\
 & 0
\end{array}
$$

**Result**: $(x-3)(2x^2+9x+10)$

$(x-3)(2x+5)(x+2)$

(e) $x+2$

$$
\begin{array}{r|l}
 & 2x^2-x-15 \\
\hline
x+2 & 2x^3+3x^2-17x-30 \\
 & \underline{2x^3+4x^2} \quad \downarrow \quad \downarrow \\
 & -x^2-17x \quad \downarrow \\
 & \underline{-x^2-2x} \quad \downarrow \\
 & -15x-30 \\
 & \underline{-15x-30} \\
 & 0
\end{array}
$$

**Result**: $(x+2)(2x^2-x-15)$

$(x+2)(2x+5)(x-3)$

***(Note:*** The results are the same just rearranged.)

# Dividing Polynomials Practice

Complete the polynomial divisions below:

1. Without using long division, find each remainder:

   (a) $(2x^2+6x+8)\div(x+1)$

   (b) $(x^2+4x+12)\div(x-4)$

   (c) $(x^3+6x^2-4x+3)\div(x+2)$

   (d) $(3x^3+7x^2-2x-11)\div(x-2)$

2. Find each remainder:

   (a) $(2x^2+x-6)\div(x+2)$

   (b) $(x^3+6x^2-4x+2)\div(x+1)$

   (c) $(x^3+x^2-12x-13)\div(x-2)$

   (d) $(x^4-x^3-3x^2+4x+2)\div(x+2)$

3. When $x^3+kx^2-4x+2$ is divided by $x+2$ the remainder is 26, find *k*.

4. When $2x^3-3x^2+kx-1$ is divided by $x-1$ the remainder is 2, find *k*.

**ANSWERS**:

1. (a) 4 (b) 44 (c) 27 (d) 37
2. (a) 0 (b) 11 (c) -25 (d) 6
3. 6
4. 4