

Unit 4: Trigonometric Functions

Activity 9: Solving trigonometric Equations

Homework/Formative Assessment

Solve the following on the interval $0 \leq x \leq 2\pi$:

1. $2\cos^2x = 1 + \sin x$
2. $2\cos x + \sqrt{3} = 0$
3. $\sin^2x + 2\cos x - 2 = 0$
4. $\cos x(2\sin x + 1)(-\cos x + 1) = 0$
5. $2\sin x \tan x + \tan x - 2\sin x - 1 = 0$
6. $6\tan^2x - 4\sin^2x = 1$

Homework/Formative Assessment SOLUTIONS

Solve the following on the interval $0 \leq x \leq 2\pi$:

1.

$$2\cos^2 x = 1 + \sin x$$

$$2(1 - \sin^2 x) - 1 - \sin x = 0$$

$$2 - 2\sin^2 x - 1 - \sin x = 0$$

$$-2\sin^2 x - \sin x + 1 = 0$$

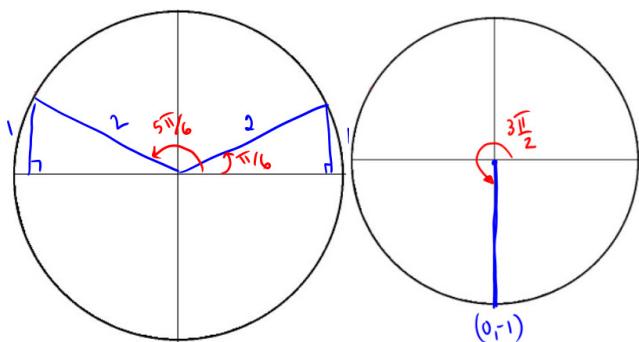
$$2\sin^2 x + \sin x - 1 = 0$$

$$(2\sin x - 1)(\sin x + 1) = 0$$

$$2\sin x - 1 = 0 \text{ or } \sin x + 1 = 0$$

$$\sin x = \frac{1}{2} \text{ or } \sin x = -1$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6} \text{ or } x = \frac{3\pi}{2}$$



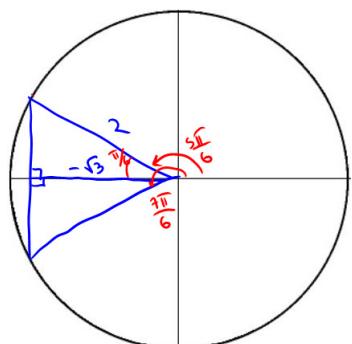
2.

$$2\cos x + \sqrt{3} = 0$$

$$2\cos x = -\sqrt{3}$$

$$\cos x = -\frac{\sqrt{3}}{2}$$

$$x = \frac{5\pi}{6}, \frac{7\pi}{6}$$



3.

$$\sin^2 x + 2 \cos x - 2 = 0$$

$$(1 - \cos^2 x) + 2 \cos x - 2 = 0$$

$$-\cos^2 x + 2 \cos x - 1 = 0$$

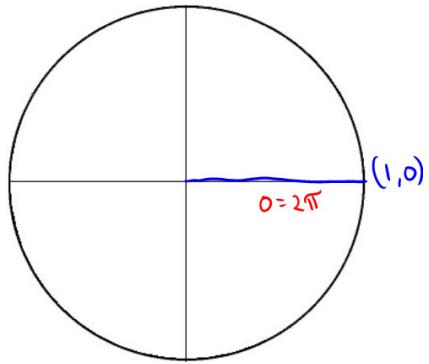
$$\cos^2 x - 2 \cos x + 1 = 0$$

$$(\cos x - 1)(\cos x - 1) = 0$$

$$\cos x - 1 = 0$$

$$\cos x = 1$$

$$x = 0, 2\pi$$



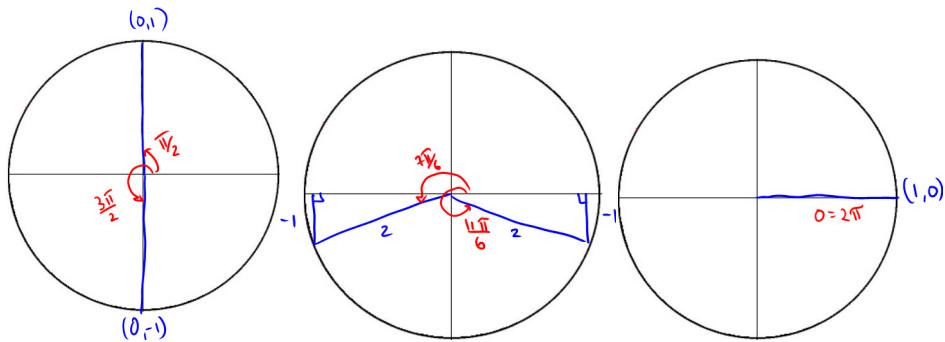
4.

$$\cos x(2 \sin x + 1)(-\cos x + 1) = 0$$

$$\cos x = 0 \text{ or } 2 \sin x + 1 = 0 \text{ or } -\cos x + 1 = 0$$

$$\sin x = -\frac{1}{2} \quad \cos x = 1$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2} \quad x = \frac{7\pi}{6}, \frac{11\pi}{6} \quad x = 0, 2\pi$$



5.

$$2 \sin x \tan x + \tan x - 2 \sin x - 1 = 0$$

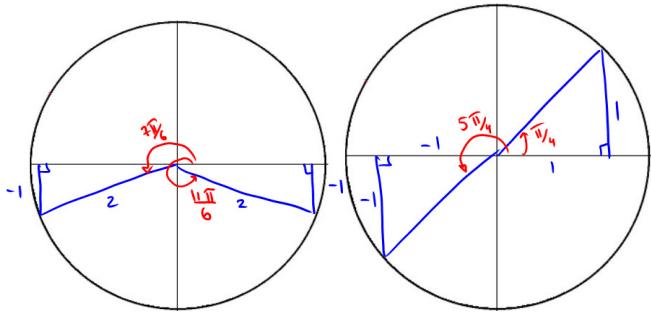
$$\tan x(2 \sin x + 1) - 1(2 \sin x + 1) = 0$$

$$(2 \sin x + 1)(\tan x - 1) = 0$$

$$2 \sin x + 1 = 0 \text{ or } \tan x - 1 = 0$$

$$\sin x = -\frac{1}{2} \text{ or } \tan x = 1$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6} \text{ or } x = \frac{\pi}{4}, \frac{5\pi}{4}$$



6.

$$6 \tan^2 x - 4 \sin^2 x = 1$$

$$6 \tan^2 x - 4 \sin^2 x - 1 = 0$$

$$6 \left(\frac{\sin^2 x}{\cos^2 x} \right) - 4 \sin^2 x - 1 = 0$$

lowest common denominator

$$\frac{6 \sin^2 x - 4 \sin^2 x \cos^2 x - \cos^2 x}{\cos^2 x} = 0$$

$$\frac{6 \sin^2 x - 4 \sin^2 x (1 - \sin^2 x) - (1 - \sin^2 x)}{\cos^2 x} = 0$$

$$\frac{6 \sin^2 x - 4 \sin^2 x + 4 \sin^4 x - 1 + \sin^2 x}{\cos^2 x} = 0$$

$$4 \sin^4 x + 3 \sin^2 x - 1 = 0$$

$$(4 \sin^2 x - 1)(\sin^2 x + 1) = 0$$

$$4 \sin^2 x - 1 = 0 \text{ or } \sin^2 x + 1 = 0$$

$$\sin^2 x = \frac{1}{4} \text{ or } \sin^2 x = -1$$

$$\sin x = \pm \frac{1}{2} \text{ or N/A}$$

$$x = \sin^{-1} \left(\pm \frac{1}{2} \right)$$

$$= \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

