Homework: Applications of polynomial functions
Provide complete solutions to the following problems:

Homework/Formative Assignment: Where Do We Use It?

Note to Students: This is a formative assignment. It is not to be submitted. Mark it yourself, using the answers provided, and contact your instructor for assistance if needed.

1. For which values of $k$ does the function $f(x) = x^3 + 6x^2 + kx - 4$ give the same remainder when divided by $(x - 1)$ and $(x + 2)$?

2. A rectangular shipping container that the Food Bank uses to store tinned food has a volume of 2500 cm$^3$. The container is 4 times as wide as it is deep, and 5 cm taller than it is wide. What are the dimensions of the container?

3. An open-top rectangular shipping container has a volume of 1008 cm$^3$. If the piece of rectangular cardboard that the container is made from has dimensions 30 cm by 20 cm, what are the dimensions of the container? Remember, you will be cutting squares out of the corners of the cardboard material to create the container.

4. A school is hosting a charity dance to raise funds for a community centre. School dances are usually attended by 600 students and the regular price is $10 per student. Student council knows that for every $1 increase in price, the number of guests attending will be reduced by 30. Determine the price the student council should charge for the dance if they want to raise $6480 for the community centre.
1. For which values of \( k \) does the function \( f(x) = x^3 + 6x^2 + kx - 4 \) give the same remainder when divided by \((x-1)\) and \((x+2)\)?

By remainder theorem the remainders are given by \( f(1) \) and \( f(-2) \), set them to be equal and solve for \( k \).

\[
\begin{align*}
(1)^3 + 6(1)^2 + k(1) - 4 &= (-2)^3 + 6(-2)^2 + k(-2) - 4 \\
1 + 6 + k - 4 &= -8 + 24 - 2k - 4 \\
k + 2k &= -8 + 24 - 4 - 1 - 6 + 4 \\
3k &= 9 \\
k &= 3
\end{align*}
\]

2. A rectangular shipping container that the Food Bank uses to store tinned food has a volume of 2500 cm\(^3\). The container is 4 times as wide as it is deep, and 5cm taller than it is wide. What are the dimensions of the container?

Let \( x = \)length (depth)

So, \( w = 4x \)

And, \( h = w + 5 = 4x + 5 \)

In \( V = lwh \), you have

\[
2500 = (x)(4x)(4x + 5)
\]

\[
0 = 16x^3 + 20x^2 - 2500
\]

\[
0 = 4(x - 5)(4x^2 + 25x + 125)
\]

\( x = 5 \) (no real roots for \( 4x^2 + 25x + 125 \) since the discriminant is negative)

Which gives you length = 5cm, width = 20cm and height = 25cm.

3. An open-top rectangular shipping container has a volume of 1008 cm\(^3\). If the piece of rectangular cardboard that the container is made from has dimensions 30 cm by 20 cm, what are the dimensions of the container? Remember, you will be cutting squares out of the corners of the cardboard material to create the container.

Let \( x = \)height (piece cut out from each corner)

\[
v = lwh
\]

\[
v = (30 - 2x)(20 - 2x)(x)
\]

\[
v = 4x^3 - 100x^2 + 600x
\]

Substituting 1008 in for the volume you have:

\[
1008 = 4x^3 - 100x^2 + 600x
\]

\[
0 = 4x^3 - 100x^2 + 600x - 1008
\]

\[
0 = 4(x - 3)(x^2 - 22x + 84)
\]

\( x = 3 \) or using the quadratic formula, \( x^2 - 22x + 84 = 0 \)

\[
x = 17.1 \text{ or } x = 4.9
\]
The feasible answers here are values of $x$ between zero and ten so the dimensions could be:

$x = 3$  \hspace{1cm} x = 4.9$

$l = 30 - 2(3) = 24$  \hspace{1cm}  $l = 30 - 2(4.9) = 20.2$

$w = 20 - 2(3) = 14$  \hspace{1cm}  $w = 20 - 2(4.9) = 10.2$

$h = 3$  \hspace{1cm}  $h = 4.9$

4. A school is hosting a charity dance to raise funds for a community centre. School dances are usually attended by 600 students and the regular price is $10 per student. Student council knows that for every $1 increase in price, the number of guests attending will be reduced by 30. Determine the price the student council should charge for the dance if they want to raise $6480 for the community centre.

Currently, the school has 600 students attending at $10 each.

If you let $x$ be the number of price increases, you have,

Revenue=(price)(quantity)

$R(x) = (10 + x)(600 - 30x)$

$R(x) = -30x^2 + 300x + 6000$

Substituting a revenue of $6480 in for revenue you can solve for $x$:

$6480 = -30x^2 + 300x + 6000$

$0 = -30x^2 + 300x - 480$

$0 = -30(x^2 - 10x + 16)$

$0 = -30(x - 2)(x - 8)$

$x = 2$  \hspace{1cm}  $x = 8$

So your solutions are,

Raise the price twice to $12, which will draw 540 guests (revenue = $6480) or, raise the price to $18, which will draw 360 guests (again, revenue = $6480).