Trigonometric Identities Extra Practice

MHF4U1

The following identities involve the reciprocal, quotient, and Pythagorean relationships. Prove each one.

1. $\sin x \tan x = \sec x - \cos x$

2. $\cos^4 x - \sin^4 x = 1 - 2\sin^2 x$

3. $\csc^2 x + \sec^2 x = \csc^2 x \sec^2 x$

4. $\cos^2 x \cos^2 y + \sin^2 x \sin^2 y + \sin^2 x \cos^2 y + \sin^2 y \cos^2 x = 1$

5. $\sec^2 x - \sec^2 y = \tan^2 x - \tan^2 y$

6. $\dfrac{\tan x + \tan y}{\cot x + \cot y} = \tan x \tan y$

7. $(\sec x - \cos x)(\csc x - \sin x) = \dfrac{\tan x}{1 + \tan^2 x}$

8. $\cos^6 x + \sin^6 x = 1 - 3\sin^2 x + 3\sin^4 x$

9. $\sec^6 x - \tan^6 x = 1 + 3\tan^2 x \sec^2 x$

The following involve the addition and subtraction formulas.

10. $1 + \cot x \tan y = \dfrac{\sin(x+y)}{\sin x \cos y}$

11. $\cos(x+y)\cos y + \sin(x+y)\sin y = \cos x$

12. $\sin x - \tan y \cos x = \dfrac{\sin(x-y)}{\cos y}$

13. $\cos\left(\dfrac{3\pi}{4} + x\right) + \sin\left(\dfrac{3\pi}{4} - x\right) = 0$

14. $\dfrac{\tan\left(\dfrac{\pi}{4} + x\right) - \tan\left(\dfrac{\pi}{4} - x\right)}{\tan\left(\dfrac{\pi}{4} + x\right) + \tan\left(\dfrac{\pi}{4} - x\right)} = 2\sin x \cos x$

15. $\sin(x+y)\sin(x-y)=\cos^2 y-\cos^2 x$

16. $\tan(x+y)\tan(x-y)=\dfrac{\sin^2 x-\sin^2 y}{\cos^2 x-\sin^2 y}$

17. $\dfrac{\tan(x-y)+\tan y}{1-\tan(x-y)\tan y}=\tan x$

18. $\sin 5x=\sin x\left(\cos^2 2x-\sin^2 2x\right)+2\cos x\cos 2x\sin 2x$

The following involve related and co-related angles.

19. $\sin\left(\dfrac{\pi}{2}-x\right)\cot\left(\dfrac{\pi}{2}+x\right)=-\sin x$

20. $\cos(-x)+\cos(\pi-x)=\cos(\pi+x)+\cos x$

21. $\dfrac{\sin(\pi-x)\cot\left(\dfrac{\pi}{2}-x\right)\cos(2\pi-x)}{\tan(\pi+x)\tan\left(\dfrac{\pi}{2}+x\right)\sin(-x)}=\sin x$

22. $\dfrac{\sin(-x)}{\sin(\pi+x)}-\dfrac{\tan\left(\dfrac{\pi}{2}+x\right)}{\cot x}+\dfrac{\cos x}{\sin\left(\dfrac{\pi}{2}+x\right)}=3$

23. $\dfrac{\csc(\pi-x)\cos(-x)}{\sec(\pi+x)\cos\left(\dfrac{\pi}{2}+x\right)}=\cot^2 x$

24. $\dfrac{\cos\left(\dfrac{\pi}{2}+x\right)\sec(-x)\tan(\pi-x)}{\sec(2\pi+x)\sin(\pi+x)\cot\left(\dfrac{\pi}{2}-x\right)}=-1$

25. $\dfrac{\sin(\pi-x)\cos(\pi+x)\tan(2\pi-x)}{\sec\left(\dfrac{\pi}{2}+x\right)\csc\left(\dfrac{3\pi}{2}-x\right)\cot\left(\dfrac{3\pi}{2}+x\right)}=\sin^4 x-\sin^2 x$

The following involve the double angle formulas.

26. $\dfrac{\sin 2x}{1+\cos 2x} = \tan x$

27. $\dfrac{1+\cos x}{\sin x} = \cot \dfrac{x}{2}$

28. $2\csc 2x = \sec x \csc x$

29. $2\cot 2x = \cot x - \tan x$

30. $\dfrac{\cos 2x}{1+\sin 2x} = \tan\left(\dfrac{\pi}{4} - x\right)$

31. $\dfrac{\cos x - \sin x}{\cos x + \sin x} = \sec 2x - \tan 2x$

32. $\dfrac{1-\cos 2x+\sin 2x}{1+\cos 2x+\sin 2x} = \tan x$

33. $\cos^6 x - \sin^6 x = \cos 2x\left(1 - \dfrac{1}{4}\sin^2 2x\right)$

34. $4\left(\cos^6 x + \sin^6 x\right) = 1 + 3\cos^2 2x$

35. $\sec x - \tan x = \tan\left(\dfrac{\pi}{4} - \dfrac{x}{2}\right)$

36. $\dfrac{\sin 2x \cos x}{(1+\cos 2x)(1+\cos x)} = \tan \dfrac{x}{2}$

The following involve a variety of formulas and identities.

37. $\sin^2 x + \cos^4 x = \cos^2 x + \sin^4 x$

38. $\tan x - \cot x = (\tan x - 1)(\cot x + 1)$

39. $\cos x = \sin x \tan^2 x \cot^3 x$

40. $(\sin x + \cos x)(\tan x + \cot x) = \sec x + \csc x$

41. $\sin^4 x + \cos^4 x = \sin^2 x\left(\csc^2 x - 2\cos^2 x\right)$

42. $\sin^3 x+\cos^3 x=(1-\sin x\cos x)(\sin x+\cos x)$

43. $\cos\left(\frac{\pi}{12}-x\right)\sec\frac{\pi}{12}-\sin\left(\frac{\pi}{12}-x\right)\csc\frac{\pi}{12}=4\sin x$

44. $\tan(x-y)+\tan(x-z)=\frac{\sec^2 y(\tan x-\tan z)}{(1+\tan x\tan y)(1+\tan y\tan z)}$ typo?

45. $\sin 8x=8\sin x\cos x\cos 2x\cos 4x$

46. $\sin x=1-2\sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)$

47. $\sin(x+y)+\sin(x-y)=2\sin x\cos y$

48. $\frac{\sin(x-y)}{\sin x\sin y}+\frac{\sin(y-z)}{\sin y\sin z}+\frac{\sin(z-x)}{\sin z\sin x}=0$

49. $\tan x+\tan(\pi-x)+\cot\left(\frac{\pi}{2}+x\right)=\tan(2\pi-x)$

50. $\sin\left(\frac{\pi}{2}+x\right)\cos(\pi-x)\cot\left(\frac{3\pi}{2}+x\right)=\sin\left(\frac{\pi}{2}-x\right)\sin\left(\frac{3\pi}{2}-x\right)\cot\left(\frac{\pi}{2}+x\right)$

51. $\tan\left(\frac{\pi}{2}-x\right)-\cot\left(\frac{3\pi}{2}-x\right)+\tan(2\pi-x)-\cot(\pi-x)=\frac{4-2\sec^2 x}{\tan x}$

52. $\tan(x+y+z)=\frac{\tan x+\tan y+\tan z-\tan x\tan y\tan z}{1-\tan x\tan y-\tan x\tan z-\tan y\tan z}$

53. $\csc^2\left(\frac{\pi}{2}-x\right)=1+\sin^2 x\csc^2\left(\frac{\pi}{2}-x\right)$

54. $\tan\left(\frac{\pi}{4}+x\right)+\tan\left(\frac{\pi}{4}-x\right)=2\sec 2x$

55. $\frac{1-\sin 2x}{\cos 2x}=\frac{\cos 2x}{1+\sin 2x}$

56. $\frac{\sin 4x}{1-\cos 4x}\bullet\frac{1-\cos 2x}{\cos 2x}=\tan x$