3.13 – Rational Equations and Inequalities

Rational equations can be solve using the same technique as when solving polynomial equations (i.e. graphically or algebraically) with the added caveat of having to watch for restrictions.
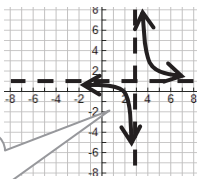
Rational inequalities, like polynomial inequalities, are best solved graphically.

Example 1: Solve the following rational equations

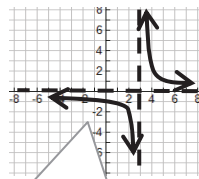a) $\dfrac{x-2}{x-3} = 0$ b) $\dfrac{1}{x-3} = 0$ c) $1 = \dfrac{x-2}{x^2+x-6}$ d) $1 = \dfrac{x-2}{x^2+x-6}$

$x - 2 = 0$

$x = 2$

$1 = 0$?

∴ no solution

$x^2 + x - 6 = x - 2$

$x^2 - 4 = 0$

$(x-2)(x+2) = 0$

so x = 2 or x = -2 ?

∴ x = -2

$1 = \dfrac{x-2}{(x-2)(x+3)}$

$1 = \dfrac{1}{x+3}$

$x + 3 = 1$

$x = -2$

Graphically x=2 represents the x-intercept

1=0 makes no sense. Graph show that no x-intercept

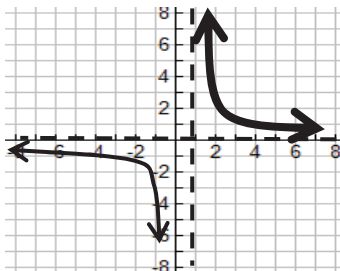But if check restrictions would see that x=2 is inadmisssible

This is only solution

Factoring 1st exposes any restrictions so can see possible inadmisssible solutions.

Example 2: Solve the following rational inequalities

a) $\dfrac{1}{x-1} \geq 0$ b) $\dfrac{1}{x^2-1} \leq 0$ c) $\dfrac{1}{x+1} \leq 5$

graph linear reciprocal and highligh areas ≥ 0

graph using reciprocal technique on $y = x^2 - 1$

graph linear reciprocal and highlight areas ≤ 5

∴ x > 1
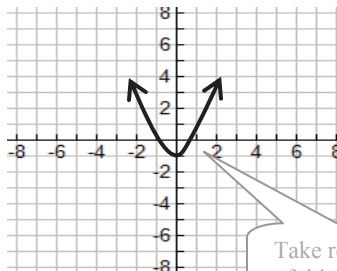
Take reciprocal of this graph
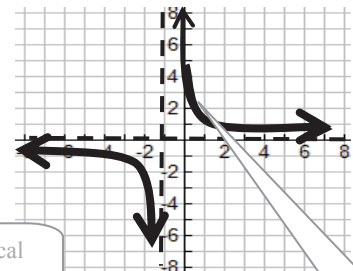
$1 = 5(x+1)$

$1 = 5x + 5$

$-4 = 5x$

$\dfrac{-4}{5} = x$

Will need to use algebra (with equal sign) to help get the exact answer for this section

Although the question uses the $\geq$ symbol the value of x=1 is restricticted so need to use > symbol

Highlight region and describe interval

∴ -1 < x < 1

∴ x <-1 or x ≥ -4/5

Need to be carefull with signs for these

3.13 – Rational Equations and Inequatities Practice Questions

1. Solve for unknown

a) $\dfrac{x}{2} + \dfrac{x}{3} = 5$

b) $-\dfrac{2}{3}x - 4 = -\dfrac{4}{5}$

c) $\dfrac{x+1}{2} = 3$

d) $\dfrac{2x+1}{3} = -5$

e) $\dfrac{3}{2} = \dfrac{7}{x}$

f) $0 = \dfrac{2}{x-1} - 1$

g) $\dfrac{x-5}{x+2} = 2$

h) $\dfrac{x+1}{(x+1)(x-2)} = -1$

i) $\dfrac{2}{x-1} = \dfrac{3}{x+2}$

j) $\dfrac{x-1}{x^2+2x-3} - 2 = 0$

k) $\dfrac{3}{4x^2} = \dfrac{5}{3x^3}$

l) $\dfrac{x}{x^2-6} = 1$

m) $\dfrac{1}{x-1} - \dfrac{2}{x+3} = 2$

2. Solve the following inequalities.

a) $\dfrac{1}{x+2} > 1$

b) $\dfrac{1}{x^2-4} \le 0$

c) $\dfrac{x}{x^2-6} \ge 1$

d) $\dfrac{2}{x-1} + 3 \ge 3$

e) $\dfrac{2}{x-1} + 3 < 0$

f) $\dfrac{1}{(x+2)(x-3)^2} > 0$

g) $\dfrac{2}{(x-1)^2} + 1 < 0$

h) $\dfrac{5}{x-2} + \dfrac{3}{2-x} \ge 1$

i) $\dfrac{x}{2x-4} - \dfrac{3}{x-6} + 1 \le \dfrac{3}{2}$

Answers 1. a) 6 **b)** -4.8 **c)** 5 **d)** -8 **e)** 14/3 **f)** 3 **g)** -9 **h)** 1 **i)** 7 **j)** -5/2 (x=1 is inadmissible) **k)** 20/9 (x=0 is inadmissible)
l) x=-2,3 **m)** x=-3.9, 1.4 (use quadratic formula) **2. a)** -2<x<-1 **b)** -2<x<2 **c)** -√6 < x ≤ -2 or +√6 < x ≤ 3
d) x>1 **e)** 1/3 < x < 1 **f)** -2 < x < 3 or x>3 **g)** no real solution (graph is above x-axis so is never less than zero)
h) 2 < x ≤ 4 **i)** 0 ≤ x < 2 or x > 6