MHF4U_2011: Advanced Functions, Grade 12, University Preparation

Unit 1: Introduction to Polynomial Functions
Activity 8: Solving Inequalities Part 1

HW Assignment – Solving Polynomial Inequalities

Solve for x:

1. $x^3 - x^2 - 20x > 0$

2. $x^3 + 10x^2 + 21x < 0$

3. $2x^3 - x^2 - 15x + 18 \geq 0$

4. $x^3 + x^2 + x < 14$

5. $x^4 + 9x + 18 \leq x^3 + 11x^2$

Check your solution set by graphing each of the corresponding polynomial functions.

HW Assignment – Solving Polynomial Inequalities SOLUTIONS

Solve for x:

1. $x^3 - x^2 - 20x > 0$

$-4 < x < 0$, and $x > 5$

2. $x^3 + 10x^2 + 21x < 0$

$x < -7$, and $-3 < x < 0$

3. $2x^3 - x^2 - 15x + 18 \geq 0$

$-3 \leq x \leq 1.5$, and $x \geq 2$

4. $x^3 + x^2 + x < 14$

$x < 2$

5. $x^4 + 9x + 18 \leq x^3 + 11x^2$

$-3 \leq x \leq -1$, and $2 \leq x \leq 3$

(Day 10) Link 2

Solving Polynomial Inequalities

Justin Fus

1. $x^3 - x^2 - 20x > 0$

Related Eqtn: $x^3 - x^2 - 20x = 0$ by RRT: root $\frac{p}{q} = \pm \frac{20, 10, 5, 4, 2, 1}{1}$

Factor Theorem: $\begin{cases} x = 5 \text{ is a root} \\ \Rightarrow (x-5) \text{ is a factor} \end{cases}$

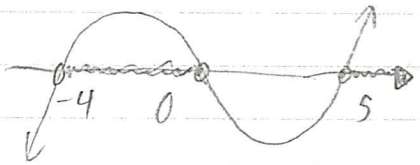
5	1	−1	−20	0
	↓	5	20	0
	1	4	0	0

$(x^2 + 4x)(x-5) > 0$

↓ factor x

$x(x+4)(x-5) > 0$

Plot:



Zeros: 0 cut
−4 cut
5 cut

Shade values w/ output > 0 $\therefore x \in (-4, 0) \cup (5, \infty)$

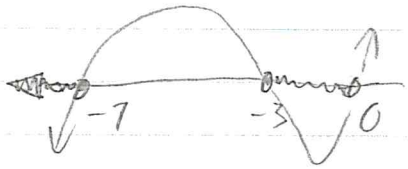
2. $x^3 + 10x^2 + 21x < 0$

$x(x^2 + 10x + 21) < 0$

$x(x+7)(x+3) < 0$

Zeros: 0 cut
−7 cut
−3 cut

Plot:



Shade values w/ output < 0

$\therefore x \in (-\infty, -7) \cup (-3, 0)$

3. $2x^3 - x^2 - 15x + 18 \geq 0$

Rel. Eqtn: $2x^3 - x^2 - 15x + 18 \geq 0$ by RRT: root $\frac{p}{q} = \pm \frac{18, 9, 6, 3, 2, 1}{2, 1}$

Factor Theorem: $\begin{cases} x = -3 \text{ is a root} \\ \Rightarrow (x+3) \text{ is a factor} \end{cases}$

$$\begin{array}{r|rrrr} -3 & 2 & -1 & -15 & 18 \\ & \downarrow & -6 & 21 & -12 \\ \hline & 2 & -7 & 6 & \boxed{0} \end{array}$$

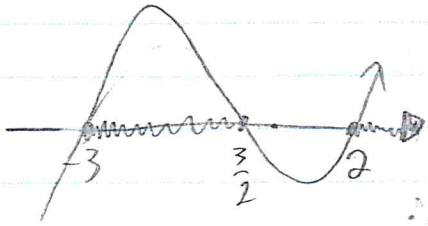
$(x+3)(2x^2-7x+6)$

$2 \cdot 1 \quad -3 \cdot -2$

$(x+3)(2x-3)(x-2)$

Zeros: -3 rat
$\frac{3}{2}$ rat
2 rat

Plot



shade values w/ output ≥ 0

$\therefore x \in [-3, \frac{3}{2}] \cup [2, \infty)$

4. $x^3 + x^2 + x < 14$

$x^3 + x^2 + x - 14 < 0$ by RRT: root $\frac{p}{q} = \pm \frac{14, 7, 2, 1}{1}$

Rel. Eqtn: $x^3 + x^2 + x - 14$

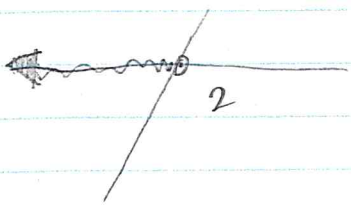
Factor Theorem: $\begin{cases} x = 2 \text{ is a root} \\ \Rightarrow (x-2) \text{ is a factor} \end{cases}$

$$\begin{array}{r|rrrr} 2 & 1 & 1 & 1 & -14 \\ & \downarrow & 2 & 6 & 14 \\ \hline & 1 & 3 & 7 & \boxed{0} \end{array}$$

$(x-2)(x^2+3x+7) < 0$

↑ has no zeros since $b^2 - 4ac < 0$ (-19)

Zeros: 2 rat



← can't plot rest without calculus (all we know is that it will go up after $x = 2$ since L.C. > 0)

Shade values with output < 0

$\therefore x < 2$

(Day 10) Link 2

Solving Inequalities cont... Justify Eqs

5. $x^4 + 9x + 18 \le x^3 + 11x^2$

$x^4 - x^3 - 11x^2 + 9x + 18 \le 0$

Rel. Eqtn: $x^4 - x^3 - 11x^2 + 9x + 18 = 0$

by RRT: roots $\frac{p}{q} = \pm \frac{18, 9, 6, 3, 2, 1}{1}$

Factor Theorem: $\begin{cases} x = 3 \text{ is a root} \\ \Rightarrow (x-3) \text{ is a factor} \end{cases}$

3	1	-1	-11	9	18
	↓	3	6	-15	-18
	1	2	-5	-6	0

$(x-3)(x^3 + 2x^2 - 5x - 6) \le 0$

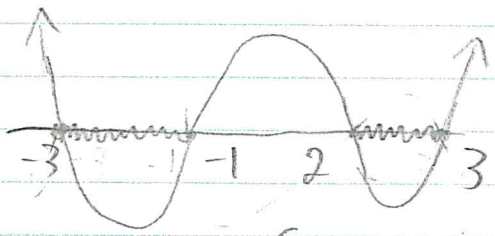
Factor Theorem: $\begin{cases} x = 2 \text{ is a zero} \\ \Rightarrow (x-2) \text{ is a factor} \end{cases}$

2	1	2	-5	-6	
	↓	2	8	6	
	1	4	3	6	

$(x-3)(x-2)(x^2 + 4x + 3) \le 0$

$(x-3)(x-2)(x+3)(x+1) \le 0$

Plot:



zeros: 3, 2, -3, -1 (all cont)

Shade values w/ outputs ≤ 0

$\therefore x \in [-3, -1] \cup [2, 3]$