

Unit 4: Trigonometric Functions

Activity 8: Trigonometric Identities

Homework/Formative Assessment

1. Simplify the following:

a) $\frac{1}{\sin^2 x} - \frac{1}{\tan^2 x}$

b) $\frac{\sin x \cos x}{1 - \sin^2 x}$

2. Prove the following identities. Explain steps:

a) $\tan x + \frac{1}{\tan x} = \frac{1}{\sin x \cos x}$

b) $\sec^2 x - 2 \sec x \cos x + \cos^2 x = \tan^2 x - \sin^2 x$

c) $1 + \cot x \tan y = \frac{\sin(x + y)}{\sin x \cos y}$

d) $\frac{1 - \sin 2x}{\cos 2x} = \frac{\cos 2x}{1 + \sin 2x}$

Homework/Formative Assessment SOLUTIONS

1. Simplify the following:

a)

$$\begin{aligned} & \frac{1}{\sin^2 x} - \frac{1}{\tan^2 x} \\ &= \frac{1}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x} \\ &= \frac{1 - \cos^2 x}{\sin^2 x} \\ &= \frac{\sin^2 x}{\sin^2 x} \\ &= 1 \end{aligned}$$

b)

$$\begin{aligned} & \frac{\sin x \cos x}{1 - \sin^2 x} \\ &= \frac{\sin x \cos x}{\cos^2 x} \\ &= \frac{\sin x}{\cos x} \\ &= \tan x \end{aligned}$$

2. Prove the following identities. Explain steps:

a) **KEEP IN MIND THAT THESE STEPS ARE NOT THE ONLY WAY TO PROVE THESE.**

$$L.S. = R.S.$$

$$\tan x + \frac{1}{\tan x} = \frac{1}{\sin x \cos x}$$

change to sine and cosine

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{1}{\sin x \cos x}$$

common denominator

$$\frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \frac{1}{\sin x \cos x}$$

pythagorean identity

$$\frac{1}{\sin x \cos x} = \frac{1}{\sin x \cos x}$$

$$\therefore L.S. = R.S.$$

b)

$$L.S. = R.S.$$

$$\sec^2 x - 2 \sec x \cos x + \cos^2 = \tan^2 x - \sin^2 x$$

change everything to sine and cosine

$$\frac{1}{\cos^2 x} - 2 \left(\frac{1}{\cos x} \right) \cos x + \cos^2 x = \frac{\sin^2 x}{\cos^2 x} - \sin^2 x$$

LS simplify

RS common denominator

$$\frac{1}{\cos^2 x} - 2 + \cos^2 x = \frac{\sin^2 x - \sin^2 x \cos^2 x}{\cos^2 x}$$

LS common denominator

RS factor

$$\frac{1 - 2 \cos^2 x + \cos^4 x}{\cos^2 x} = \frac{\sin^2 x (1 - \cos^2 x)}{\cos^2 x}$$

pythagorean identity

$$\frac{1 - 2 \cos^2 x + \cos^4 x}{\cos^2 x} = \frac{(1 - \cos^2 x)(1 - \cos^2 x)}{\cos^2 x}$$

expand

$$\frac{1 - 2 \cos^2 x + \cos^4 x}{\cos^2 x} = \frac{1 - 2 \cos^2 x + \cos^4 x}{\cos^2 x}$$

$$\therefore L.S. = R.S.$$

c)

$$L.S. = R.S.$$

$$1 + \cot x \tan y = \frac{\sin(x+y)}{\sin x \cos y}$$

change to sine and cosine

use the compound identity

$$1 + \left(\frac{\cos x}{\sin x} \right) \left(\frac{\sin y}{\cos y} \right) = \frac{\sin x \cos y + \sin y \cos x}{\sin x \cos y}$$

RS distribute denominator

$$1 + \frac{\cos x \sin y}{\sin x \cos y} = \frac{\sin x \cos y}{\sin x \cos y} + \frac{\sin y \cos x}{\sin x \cos y}$$

simplify

$$1 + \frac{\cos x \sin y}{\sin x \cos y} = 1 + \frac{\cos x \sin y}{\sin x \cos y}$$

$$\therefore L.S. = R.S.$$

d)

$$L.S. = R.S.$$

$$\frac{1 - \sin 2x}{\cos 2x} = \frac{\cos 2x}{1 + \sin 2x}$$

use double angle identities

$$\frac{1 - 2 \sin x \cos x}{\cos^2 x - \sin^2 x} = \frac{\cos^2 x - \sin^2 x}{1 + 2 \sin x \cos x}$$

change 1 into sine and cosine using pythagorean

$$\frac{\cos^2 x + \sin^2 x - 2 \sin x \cos x}{\cos^2 x - \sin^2 x} = \frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x + 2 \sin x \cos x}$$

rearrange for factoring

$$\frac{\cos^2 x - 2 \sin x \cos x + \sin^2 x}{\cos^2 x - \sin^2 x} = \frac{\cos^2 x - \sin^2 x}{\cos^2 x + 2 \sin x \cos x + \sin^2 x}$$

factor

$$\frac{(\cos x - \sin x)(\cos x - \sin x)}{(\cos x + \sin x)(\cos x - \sin x)} = \frac{(\cos x + \sin x)(\cos x - \sin x)}{(\cos x + \sin x)(\cos x + \sin x)}$$

simplify

$$\frac{(\cos x - \sin x)}{(\cos x + \sin x)} = \frac{(\cos x - \sin x)}{(\cos x + \sin x)}$$

$$\therefore L.S. = R.S.$$