Unit 2: Advanced Polynomial and Rational Functions

Activity 6: Graphing rational functions part 1

<u>Formative</u> Assignment

1. For each of the following rational functions, sketch the graph by first finding the following features. Show all of your work and reasoning.

- Domain.
- x-intercept(s).
- y-intercept(s).
- Vertical Asymptote(s).
- Horizontal Asymptote(s).
- Symmetry (Odd, Even, Neither).
- Key Points.

a. $f(x)=\dfrac{-3}{2x-1}$

b. $f(x)=\dfrac{2}{x^2+3}$

c. $f(x)=\dfrac{4}{x^2-2x-3}$ *(Hint: Factor the denominator first to see vertical asymptotes.)*

d. $f(x)=\dfrac{4}{x^2+4x+7}$ *(Hint: Complete the square in the denominator first since this one has no vertical asymptotes. This will enable you to see where the turning point will occur.)*

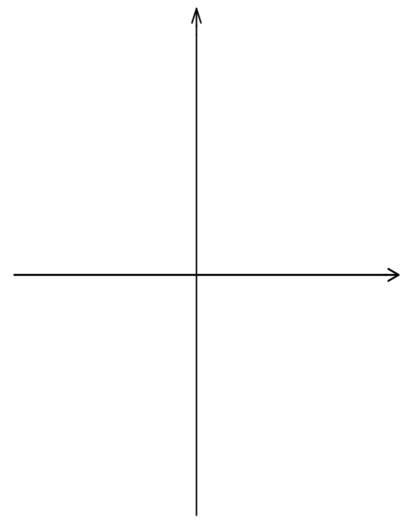
2. Complete the following chart for **EACH** of the given Rational Functions:

a) $f(x)=\dfrac{x}{x-1}$

b) $g(x)=\dfrac{-x}{x-1}$

c) $h(x)=\dfrac{x+1}{x-1}$

d) $k(x)=\dfrac{2x-1}{x-1}$

Characteristics	**Rational function:** y =
Graph(use graphing technology)	
Domain (put this in set notation)	
Range (put this in set notation)	
Vertical asymptote(s) (give the equation)	
Horizontal asymptote (give the equation)	
x-intercept	
y-intercept	
Positive interval	
Negative interval	
Interval(s) of increase	
Interval(s) of decrease	

Formative Assignment SOLUTIONS

a) $f(x)=\dfrac{-3}{2x-1}$

- Domain

$\{x \in R \mid x \neq \frac{1}{2}\}$

- x-intercept(s)

None

- y-intercept(s)

3

- Vertical Asymptote(s)

$x=\frac{1}{2}$

- Horizontal Asymptote(s)

$y=0$

- Symmetry (Odd, Even, Neither)

Neither

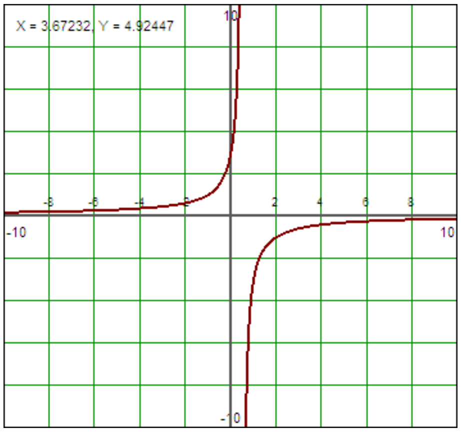
- Key Points – take some x values on all sides of the vertical asymptotes.

These can vary – any points that provide the 'bend' of the two branches will be fine. eg.

x	f(x)
0	3
-0.5	1.5
-1	1
-1.5	0.75
1	-3
1.5	-1.5

Use a table of values to examine the behaviour of the function on either side of a vertical asymptote. To conclude that as $x \to \frac{1}{2}$ ***on the left side*** $f(x) \to \infty$ ***and as*** $x \to \frac{1}{2}$ ***on the right side*** $f(x) \to -\infty$

Use technology to verify:



b) $f(x)=\dfrac{2}{x^2+3}$

- Domain

$\{x \in R\}$

- x-intercept(s)

None

- y-intercept(s)

$\dfrac{2}{3}$

- Vertical Asymptote(s)

None

- Horizontal Asymptote(s)

$y=0$

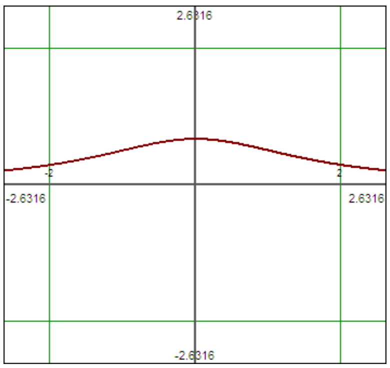
- Symmetry (Odd, Even, Neither)

Even since f(x)=f(-x)

- Key Points – vertex of the parabola on the bottom is $(0,3)$ so taking point on either side of this vertex will be enough to show the proper bend in the graph.

x	f(x)
-1	0.5
0	0.666667
1	0.5

Use technology to verify:



c) $f(x)=\dfrac{4}{x^2-2x-3}$

First, factor:

$$f(x)=\frac{4}{(x-3)(x+1)}$$

- Domain

$$\{x \in R \mid x \neq 3,-1\}$$

- x-intercept(s)

None

- y-intercept(s)

y=-4/3

- Vertical Asymptote(s)

$$x=-1$$
$$x=3$$

- Horizontal Asymptote(s)

$y = 0$

- Symmetry (Odd, Even, Neither)

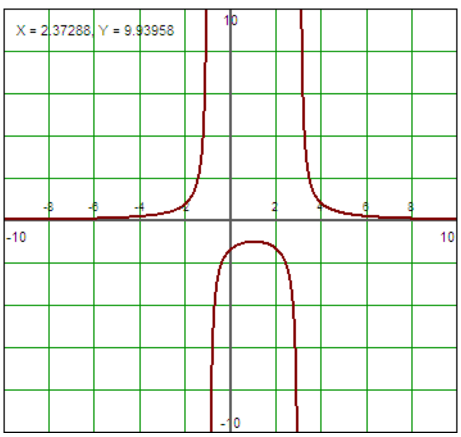
Neither

- Key Points – take some x values on all sides of the vertical asymptotes.

Enough to get the proper bend in the graph.

x	f(x)
-2	0.8
-1.5	1.777778
1	-1
2	-1.33333
3.5	1.777778
4.5	0.484848

Use technology to verify:



d. $f(x) = \dfrac{4}{x^2 + 4x + 7}$

- Domain

$\{x \in R\}$

- x-intercept(s)

None

- y-intercept(s)

y= 4/7

- Vertical Asymptote(s)

None

- Horizontal Asymptote(s)

$$y = 0$$

- Symmetry (Odd, Even, Neither)

Neither

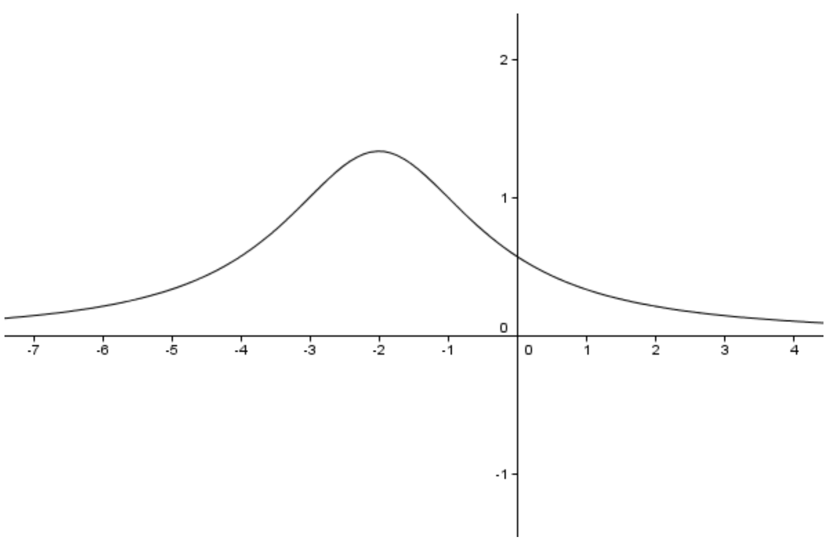
- Key Points

Complete the square:

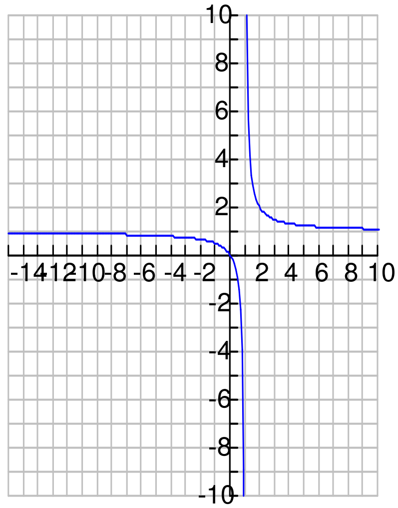
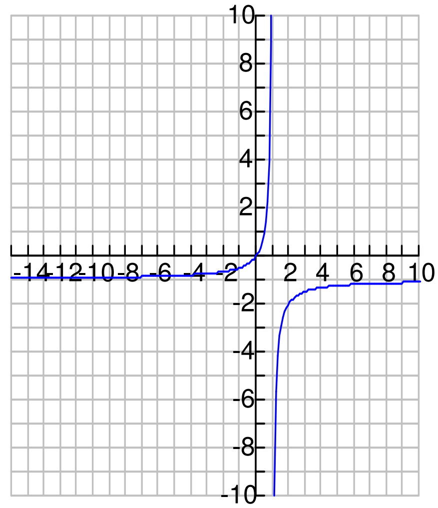
$f(x) = \dfrac{4}{(x+2)^2 + 3}$ vertex of the parabola on the bottom is $(-2, 3)$, so take key points around that to get the proper bend in the graph.

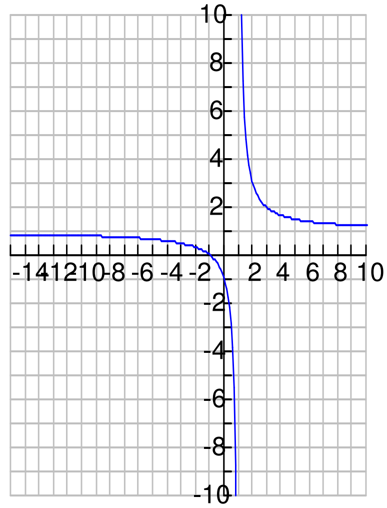
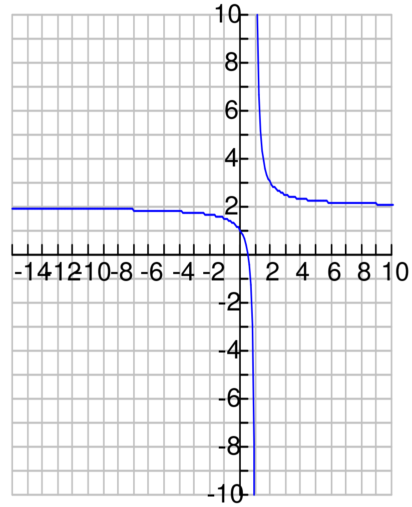
x	f(x)
-3	1
-2	1.3333
-1	1

Use technology to verify:



2.

Characteristics	Rational function a): $f(x)=\dfrac{x}{x-1}$	Rational function b): $g(x)=\dfrac{-x}{x-1}$
Graph(use graphing technology)		
Domain (put this in set notation)	$\{x\in R \mid x\neq 1\}$	$\{x\in R \mid x\neq 1\}$
Range (put this in set notation)	$\{y\in R \mid y\neq 1\}$	$\{y\in R \mid y\neq -1\}$
Vertical asymptote(s) (give the equation)	$x=1$	$x=1$
Horizontal asymptote (give the equation)	$y=1$	$y=-1$
x-intercept	$x=0$	$x=0$
y-intercept	$y=0$	$y=0$
Positive interval	$\{x\in R \mid x<0; x>1\}$	$\{x\in R \mid 0<x<1\}$
Negative interval	$\{x\in R \mid 0<x<1\}$	$\{x\in R \mid x<0; x>1\}$
Interval(s) of increase	***none***	***Function is always increasing***
Interval(s) of decrease	***Function is always decreasing***	***none***

Characteristics	**Rational function c):** $h(x)=\dfrac{x+1}{x-1}$	**Rational function d):** $k(x)=\dfrac{2x-1}{x-1}$
Graph(use graphing technology)		
Domain (put this in set notation)	$\{x\in R \mid x\neq 1\}$	$\{x\in R \mid x\neq 1\}$
Range (put this in set notation)	$\{y\in R \mid y\neq 1\}$	$\{y\in R \mid y\neq 2\}$
Vertical asymptote(s) (give the equation)	$x=1$	$x=1$
Horizontal asymptote (give the equation)	$y=1$	$y=2$
x-intercept	$x=-1$	$x=\frac{1}{2}$
y-intercept	$y=-1$	$y=1$
Positive interval	$\{x\in R \mid x<-1; x>1\}$	$\{x\in R \mid x<\frac{1}{2}; x>1\}$
Negative interval	$\{x\in R \mid -1<x<1\}$	$\{x\in R \mid \frac{1}{2}<x<1\}$
Interval(s) of increase	***None***	***None***
Interval(s) of decrease	***Function is always decreasing***	***Function is always decreasing***