MHF4U_2011: Advanced Functions, Grade 12, University Preparation

Unit 5: Characteristics of Functions

Activity 5: Investigating Compound Functions

Homework/Formative Assessment

1. Using $f(x)=x$ (odd), $g(x)=x^2$ (even), $h(x)=x^3$ (odd) and $j(x)=x^4$ (even), find if the compound functions are odd or even on the table below. Write a statement about odd/even compound functions and the operation that makes them. Use GraphCalc or Geogebra to help you.

Addition of Compound Functions

Compound Function	Resulting Function (Odd/Even)
k(x) = $x^2 + x^3$ (even + odd)	
l(x) = $x^2 + x^4$ (even + even)	
m(x) = $x + x^3$ (odd + odd)	

Subtraction of Compound Functions

Compound Function	Resulting Function (Odd/Even)
k(x) = $x^2 - x^3$ (even - odd)	
l(x) = $x^2 - x^4$ (even - even)	
m(x) = $x - x^3$ (odd - odd)	

Multiplication of Compound Functions

Compound Function	Resulting Function (Odd/Even)
k(x) = $(x^2)(x^3)$ (even × odd)	
l(x) = $(x^2)(x^4)$ (even × even)	
m(x) = $(x)(x^3)$ (odd × odd)	

Division of Compound Functions

Compound Function	Resulting Function (Odd/Even)
k(x) = $(x^2) / (x^3)$ (even / odd)	
l(x) = $(x^2) / (x^4)$ (even / even)	
m(x) = $(x) / (x^3)$ (odd / odd)	

2. A pendulum is pulled back and let go. The motion of the pendulum can be approximated by the function $h(x) = f(x)g(x)$, where $h(x)$ is the distance the pendulum swings away from its resting point in centimetres and x is the time in seconds. In this scenario, $f(x) = 8\cos\frac{2\pi}{5}x$ represents the pendulum swinging in a vacuum without friction and $g(x) = 0.97^x$ is the function that models the dampening of the motion due to air resistance, and friction between the arm of the pendulum and the point of attachment.

a) Use graphing technology to graph $h(x) = \left[8\cos\left(\frac{2\pi}{5}x\right)\right](0.97)^x$.

b) Describe, in your own words, the function you have graphed and discuss why using compound functions would be appropriate in this situation.

c) Use your graph to determine how far the pendulum gets from the resting point at 15 seconds. Verifying using the given function,

$$\begin{aligned} h(15) &= \left[8\cos\left(\frac{2\pi}{5}(15)\right)\right](0.97)^{15} \\ &= 5.07 \end{aligned}$$

Homework/Formative Assessment SOLUTIONS

1. Using $f(x) = x$ (odd), $g(x) = x^2$ (even), $h(x) = x^3$ (odd) and $j(x) = x^4$ (even), find if the compound functions are odd or even on the table below. Write a statement about odd/even compound functions and the operation that makes them. Use GraphCalc or Geometer's Sketchpad.

Additiontion of Compound Functions

Compound Function	**Resulting Function (Odd/Even)**
$k(x) = x^2 + x^3$ (even + odd)	Neither
$l(x) = x^2 + x^4$ (even + even)	Even
$m(x) = x + x^3$ (odd + odd)	Odd

Subtraction of Compound Functions

Compound Function	**Resulting Function (Odd/Even)**
$k(x) = x^2 - x^3$ (even - odd)	Neither
$l(x) = x^2 - x^4$ (even - even)	Even
$m(x) = x - x^3$ (odd - odd)	Odd

Multiplication of Compound Functions

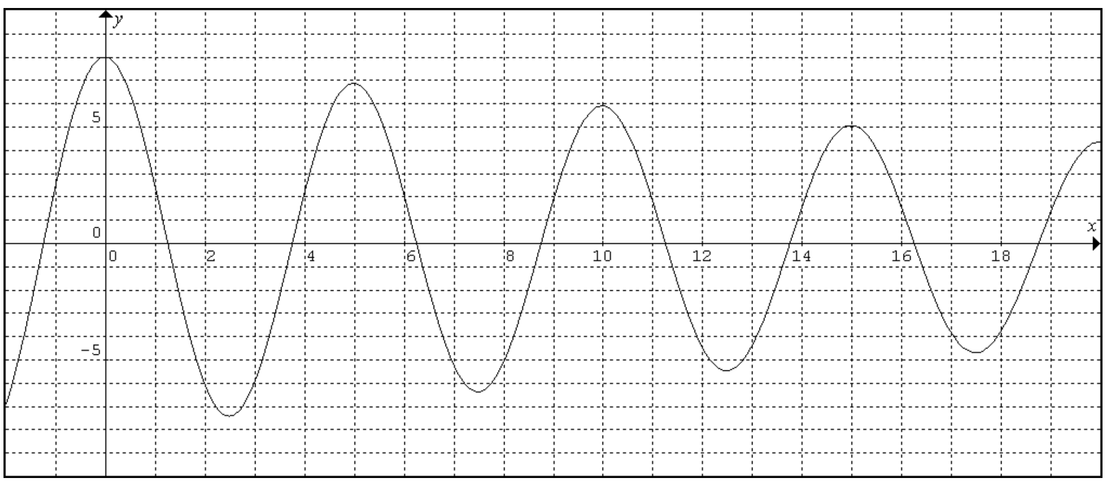
Compound Function	**Resulting Function (Odd/Even)**
$k(x) = (x^2)(x^3)$ (even × odd)	Odd
$l(x) = (x^2)(x^4)$ (even × even)	Even
$m(x) = (x)(x^3)$ (odd × odd)	Even

Division of Compound Functions

Compound Function	**Resulting Function (Odd/Even)**
$k(x) = (x^2) / (x^3)$ (even / odd)	Odd
$l(x) = (x^2) / (x^4)$ (even / even)	Even
$m(x) = (x) / (x^3)$ (odd / odd)	Even

2. A pendulum is pulled back and let go. The motion of the pendulum can be approximated by the function $h(x) = f(x)g(x)$, where $h(x)$ is the distance the pendulum swings away from its resting point in centimetres and x is the time in seconds. In this scenario, $f(x) = 8\cos\frac{2\pi}{5}x$ represents the pendulum swinging in a vacuum without friction and $g(x) = 0.97^x$ is the function that models the dampening of the motion due to air resistance, and friction between the arm of the pendulum and the point of attachment.

a) Use graphing technology to graph $h(x) = \left[8\cos\left(\frac{2\pi}{5}x\right)\right](0.97)^x$.



b) Describe, in your own words, the function you have graphed and discuss why using compound functions would be appropriate in this situation.

As the pendulum swings, its motion is constantly slowing down. The function shows a periodic sinusoidal function that begins with an amplitude of 8 but is always decreasing. This motion makes sense because eventually the pendulum would slow to a stop. The use of a compound function is appropriate here because the motion is periodic but is also always slowing down.

c) Use your graph to determine how far the pendulum gets from the resting point at 15 seconds.

At 15 seconds, the pendulum gets from its resting position is approximately 5cm

Verifying using the given function,

$$\begin{aligned} h(15) &= \left[8\cos\left(\frac{2\pi}{5}(15)\right)\right](0.97)^{15} \\ &= 5.07 \end{aligned}$$