MHF4U_2011: Advanced Functions, Grade 12, University Preparation

Unit 2: Advanced Polynomial and Rational Functions

Activity 7: Graphing rational functions part 2

Formative Assignment

Do **NOT** submit this to your instructor. Check your solutions when you are done. If you need help with these concepts, email your instructor for assistance.

1. Graph each of the following Rational Functions, by analyzing the function expression to first determine:

- Domain
- x-intercept(s)
- y-intercept(s)
- Vertical Asymptote(s)
- Horizontal Asymptote(s)
- Oblique Asymptote(s)
- Symmetry (Odd, Even, Neither)
- Key Points
- Graph the above without technology, then use technology to check your work

a. $$f(x)=\frac{5x-11}{2x+3}$$

b. $$f(x)=\frac{2x+1}{x^2-4x-5}$$

c. $$f(x)=\frac{2x^2-5x+2}{x+1}$$

2. For each of the following, create a function that has a graph with the given features:

a. A vertical asymptote at x=2; a horizontal asymptote at y=0; no x-intercept; y-intercept is 3

b. A vertical asymptote at x=1; an oblique asymptote at y=2x-1

3. The profit function for a company selling widgets is given by:

$p(x) = -2x^2 + 5x - 2$ where x is the number of widgets sold.

The average profit per widget is given by: $AP(x) = \frac{p(x)}{x}$.

a. Write the defining statement for AP(x).

b. What is the domain of the function, considering that AP(x) is a model for a real-life situation?

c. What are the break-even quantities of x (ie – when profit is zero)?

4. The value of a new car, in dollars, x years after it is purchased is modelled by:

$$f(x) = \frac{42000 + 5x}{2 + 0.5x}$$

a. What was the value of the car when it was new?

b. What was the value of the car after 5 years?

c. What is happening to the value of the car as time passes?

d. What is the scrap value of the car (the least value the car can have)?

Formative Assignment: SOLUTIONS

1. Graph each of the following Rational Functions, by analyzing the function expression to first determine:

- Domain
- x-intercept(s)
- y-intercept(s)
- Vertical Asymptote(s)
- Horizontal Asymptote(s)
- Oblique Asymptote(s)
- Symmetry (Odd, Even, Neither)
- Key Points
- Graph the above without technology, then use technology to check your work

a. $f(x)=\dfrac{5x-11}{2x+3}$

Domain: $\{x \in R \mid x \neq -\frac{3}{2}\}$

x-intercept: $\frac{11}{5}$

y-intercept: $-\frac{11}{3}$

Vertical Asymptote: $x=-\frac{3}{2}$

Horizontal Asymptote: $y=\frac{5}{2}$ (since the degrees are the same – look at the leading coefficients of both numerator and denominator.)

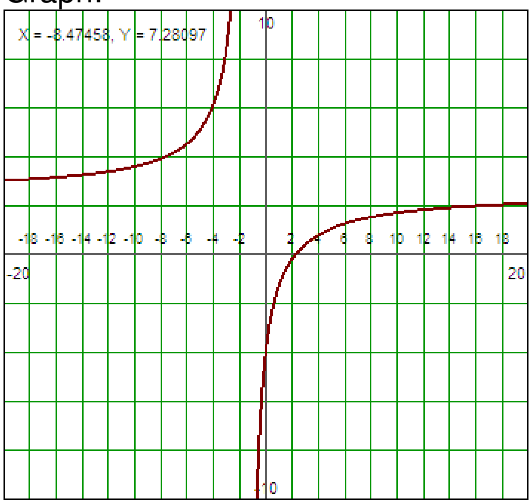
Oblique Asymptote: None since there is a horizontal asymptote.

Symmetry: Neither. Check that $\begin{array}{l} f(-x) \neq f(x) \text{ not even} \\ f(-x) \neq -f(x) \text{ not odd} \end{array}$

Key Points: Take some x values on either side of the vertical asymptote. These points can vary, but here are a few possibilities:

x	f(x)
-2.5	11.75
-3	8.6667
-0.5	-6.75
0	-3.6667

Graph:



b. $$f(x)=\frac{2x+1}{x^2-4x-5}=\frac{2x+1}{(x-5)(x+1)}$$

Domain: $\{x \in R \mid x \neq 5,-1\}$ (Factor denominator first)

x-intercept: $-\frac{1}{2}$

y-intercept: $-\frac{1}{5}$

Vertical Asymptotes: $x=5; x=-1$

Horizontal Asymptote: $\boldsymbol{y = 0}$ (since if you were to rewrite the function so that both numerator and denominator had equal degrees it would look like $f(x) = y = \dfrac{0x^2 + 2x + 1}{x^2 - 4x - 5}$ and then you just look at the leading coefficients. It will be $y = \dfrac{0}{1} = 0$)

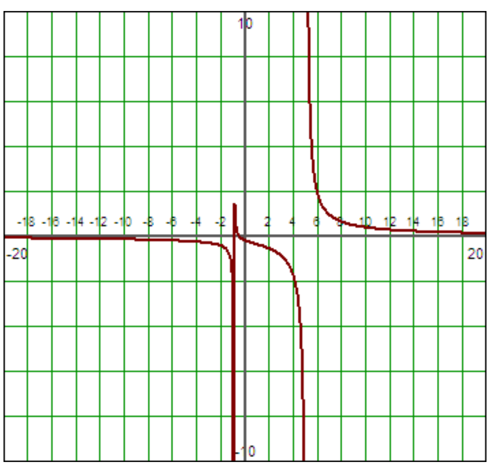
Oblique Asymptote: None, can't have both Horizontal and Oblique.

Symmetry: Neither. Check that $\begin{array}{l} f(-x) \neq f(x) \text{ not even} \\ f(-x) \neq -f(x) \text{ not odd} \end{array}$

Key Points: Take values on both sides of vertical asymptotes. The points may vary, but here are some possibilities:

x	**f(x)**
-3	-0.3125
-2	-0.428
0	-0.2
4	-1.8
6	1.857
7	0.937

Graph:



c. $f(x)=\dfrac{2x^2-5x+2}{x+1}=\dfrac{(2x-1)(x-2)}{x+1}$

Domain: $\{x \in R \mid x \neq -1\}$

x-intercepts: $\dfrac{1}{2}, 2$ Factor the numerator to see this.

y-intercept: 2

Vertical Asymptote: $x=-1$

Horizontal Asymptote: None. (since if you were to rewrite the function so that both numerator and denominator had equal degrees it would look like $f(x)=y=\dfrac{2x^2-5x+2}{0x^2+x+1}$ and then you just look at the leading coefficients. It will be $y=\dfrac{2}{0}=undefined$)

Oblique Asymptote: There will be one since the degree of the numerator is one higher than degree of denominator. To find it use synthetic division.

$$\begin{array}{r|rrr} -1 & 2 & -5 & 2 \\ & & -2 & 14 \\ \hline & 2 & -7 & 16 \end{array}$$

$$f(x)=(quotient)+\frac{remainder}{divisor}$$

$$f(x)=(2x-7)+\frac{16}{x+1}$$

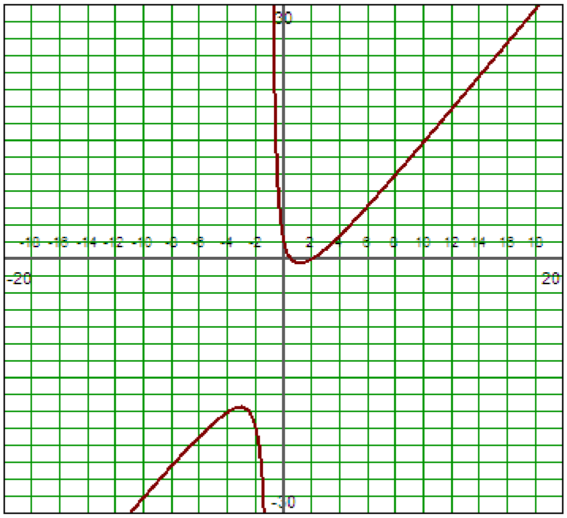
So as x goes very big the last term $\dfrac{16}{x+1}$ goes to zero and the oblique asymptote is $y=2x-7$ *(to help you sketch the graph, graph this line, and make the branches approach it)*

Symmetry: Neither. Check that $\begin{array}{l} f(-x)\neq f(x) \text{ not even} \\ f(-x)\neq -f(x) \text{ not odd} \end{array}$

Key Points: Take values on both sides of vertical asymptotes. The points may vary, but here are some possibilities:

x	f(x)
-3	-17.5
-2	-20
0	2
1	-0.5

Graph:



2. For each of the following, create a function that has a graph with the given features:

a. A vertical asymptote at x=2; a horizontal asymptote at y=0; no x-intercept; y-intercept is 3

Your answers may vary, but here is one example:

$$f(x)=\frac{-6}{x-2}$$

Thought process:
VA at x=2 implies there must be a factor of x-2 in the denominator.
HA at y=0 implies that the degree in denominator must be bigger than degree of numerator
No x-int implies that the numerator cannot be solved for zeros
Create the numerator that is never zero and is of smaller degree than denominator:

$$f(x)=\frac{a}{x-2}$$

y-int =3 implies that when you sub in x=0 the result should be 3

$$3=\frac{a}{0-2}$$

$$-6=a$$

b. A vertical asymptote at x=1; an oblique asymptote at y=2x-1

One solution is

$$f(x)=\frac{2x^2-3x+5}{x-1}$$

(there could be other answers – if unsure of yours, check with your instructor.)

Thought process:
VA at x=1 implies x-1 is in the denominator
OA at y=2x-1 implies that once you do long/synthetic division your quotient is 2x-1. You can use the following formula.

$$f(x)=(quotient)+\frac{remainder}{divisor}$$

$$f(x)=(2x-1)+\frac{a}{x-1}$$

Combine the two term by doing LCD, lowest common denominator and pick any value for *a* since there are no other conditions to satisfy.

$$f(x)=\frac{(2x-1)(x-1)+a}{x-1}$$

$$f(x)=\frac{2x^2-3x+1+a}{x-1}$$

For the solution above, *a=4* is chosen randomly.

3. The profit function for a company selling widgets is given by:

$p(x)=-2x^2+5x-2$ where x is the number of widgets sold.

The average profit per widget is given by: $AP(x)=\dfrac{p(x)}{x}$.

a. Write the defining statement for AP(x).

Solution is

$$AP(x)=\frac{-2x^2+5x-2}{x}=\frac{-(2x-1)(x-2)}{x}$$

b. What is the domain of the function, considering that AP(x) is a model for a real-life situation?

Normally, the only restriction on the domain for such a function as this would be

$$\{x \in R \mid x \neq 0\}$$

But because 'x' in this function represents a number of units of widget sold, you must further restrict the domain to:

$$\{x \in Integers \mid x > 0\}$$

c. What are the break-even quantities of x (ie. – when profit is zero)?

We need the y-intercept for the function:

Factor the numerator to get:

$$-(2x-1)(x-2)=0$$

$$x=\frac{1}{2},2$$

The best answer, due to the nature of the quantity 'x', which cannot be a fraction, is x=2 so when 2 units are sold, the profit is zero – costs equal revenues.

4. The value of a new car, in dollars, x years after it is purchased is modelled by:

$$f(x)=\frac{42000+5x}{2+0.5x}$$

a. What was the value of the car when it was new?

Solution: new means time is zero.

$$f(0)=\frac{42000+5(0)}{2+0.5(0)}$$

$$f(0)=\frac{42000}{2}$$

$$f(0)=21000$$

So the car was worth $21,000 when new.

b. What was the value of the car after 5 years?

Solution:

$$f(5) = \frac{42000 + 5(5)}{2 + 0.5(5)}$$

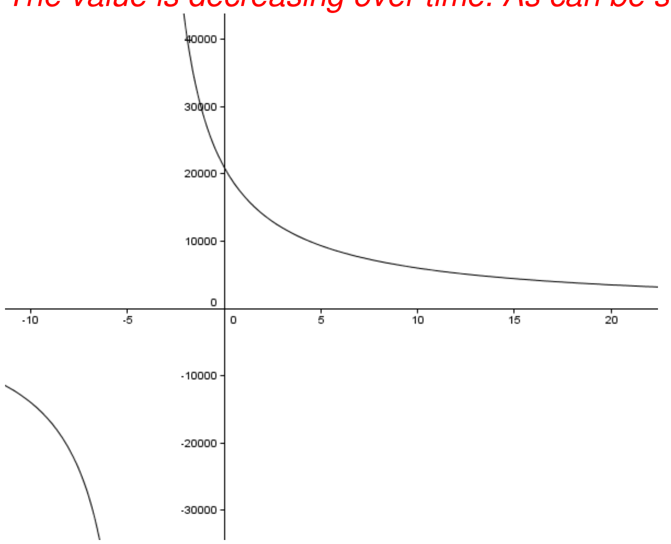
$$f(5) = \frac{42025}{4.5}$$

$$f(5) = 9338.89$$

So after five years, the car is worth $9,338.89

c. What is happening to the value of the car as time passes?

The value is decreasing over time. As can be seen from the graph:



d. What is the scrap value of the car (the least value the car can have)?

Consider the horizontal asymptote of this function's graph:

$y = 10$ (since if you were to rewrite the function it would look like $f(x) = y = \frac{5x + 42000}{0.5x + 2}$ and then you just look at the leading coefficients. It will be $y = \frac{5}{0.5} = 10$)

So, theoretically, the car value would tend toward $10 over time. Realistically, it won't make it! It will break down before it nears the value of $10.