MHF4U_2011: Advanced Functions, Grade 12, University Preparation

Unit 5: Characteristics of Functions

Activity 6: Composition of Functions

Homework/Composite Functions

1. Given $f(x)=x+1$ and $g(x)=3x$, determine $f(g(x))$ and state the domain of the composite function.

2. Given $f(x)=x^2+1$ and $g(x)=\sqrt{2x}$, determine $f(g(x))$ and state the domain of the composite function.

3. If $f(x)=x^2$ and $g(x)=\dfrac{x-2}{x+2}$, determine $f(g(2))$ and $g(f(2))$.

4. Suppose that $h(x)=(x+1)^2-9(x+1)$. Determine $f(x)$ and $g(x)$ if $h(x)=f(g(x))$.

5. Sally is trying to determine how many calories she burns on her Sunday morning jog. She can jog at a constant speed of 6 km/h and she has read that she can burn 600 calories per 1800m.
   a) Create a composite function that relates calories burned to time in hours.
   b) Use your new function to determine how many calories she burns if she runs for 1.5 hours.

Homework/Composite Functions SOLUTIONS

1. Given $f(x)=x+1$ and $g(x)=3x$, determine $f(g(x))$ and state the domain of the composite function.

$$\begin{aligned} f(g(x)) &= (3x)+1 \\ &= 3x+1 \end{aligned}$$

Since the range of $g(x)$ is $y\in\mathbb{R}$, the domain of $f(g(x))$ is $x\in\mathbb{R}$.

2. Given $f(x)=x^2+1$ and $g(x)=\sqrt{2x}$, determine $f(g(x))$ and state the domain of the composite function.

$$\begin{aligned} f(g(x)) &= \left(\sqrt{2x}\right)^2+1 \\ &= 2x+1 \end{aligned}$$

Since the range of $g(x)$ is $y\in\mathbb{R}$, the domain of $f(g(x))$ is $x\in\mathbb{R}$.

3. If $f(x)=x^2$ and $g(x)=\dfrac{x-2}{x+2}$, determine $f(g(2))$ and $g(f(2))$.

$$\begin{aligned} g(2) &= \frac{2-2}{2+2} \\ &= \frac{0}{4} \\ &= 0 \\ f(0) &= \left(0^2\right) \\ &= 0 \end{aligned}$$

and

$$\begin{aligned} f(2) &= (2)^2 \\ &= 4 \\ g(4) &= \frac{4-2}{4+2} \\ &= \frac{2}{6} \\ &= \frac{1}{3} \end{aligned}$$

4. Suppose that $h(x)=(x+1)^2-9(x+1)$. Determine $f(x)$ and $g(x)$ if $h(x)=f(g(x))$.

You can see that $h(x)$ is a combination of $x+1$ substituted into x^2-9, so:
$g(x)=x+1$ and $f(x)=x^2-9$x

5. Sally is trying to determine how many calories she burns on her Sunday morning jog. She can jog at a constant speed of 6 km/h and she has read that she can burn 270 calories per 1800m

   a) Create a composite function that relates calories burned to time in hours.

   Relating distance and time, $d(t) = 6(t)$

   $d(t) = 6t$

   (Convert 1800m to 1.8km so units match.)

   Relating calories burned and distance, $c(d) = \frac{270}{1.8}(d)$

   $c(d) = 150d$

   Finally, relating calories burned to time you have:

   $c(d(t)) = 150(6t)$

   $c(d(t)) = 900t$

   b) Use your new function to determine how many calories she burns if she runs for 1.5 hours.

   For $t = 1.5$,

   $c(d(1.5)) = 900(1.5)$

   $= 1350$

   $\therefore$ Sally would burn 1350 calories on her 1.5 hour run.