

Double- and Half-Angle Identities

Use a double-angle identity to find the exact value of each expression.

1) $\sin 120^\circ$

2) $\tan 60^\circ$

3) $\cos \frac{4\pi}{3}$

4) $\sin \frac{5\pi}{3}$

Use a half-angle identity to find the exact value of each expression.

5) $\tan 45^\circ$

6) $\sin 165^\circ$

7) $\sin \frac{5\pi}{6}$

8) $\cos 30^\circ$

Use a double-angle or half-angle identity to find the exact value of each expression.

9) $\cot \frac{\pi}{3}$

10) $\cot \frac{2\pi}{3}$

11) $\sec \frac{5\pi}{12}$

12) $\cot 60^\circ$

13) $\cot 240^\circ$

14) $\cot \frac{5\pi}{3}$

$$15) \sin \theta = -\frac{7}{25} \text{ and } 270^\circ < \theta < 360^\circ$$

Find $\cos \frac{\theta}{2}$

$$16) \cos \theta = \frac{1}{3} \text{ and } 0^\circ < \theta < 90^\circ$$

Find $\sin 2\theta$

$$17) \cos \theta = \frac{4}{5} \text{ and } 270^\circ < \theta < 360^\circ$$

Find $\sin 2\theta$

$$18) \cos \theta = \frac{2\sqrt{5}}{5} \text{ and } 0^\circ < \theta < 90^\circ$$

Find $\sin \frac{\theta}{2}$

$$19) \cos \theta = -\frac{4}{5} \text{ and } 90^\circ < \theta < 180^\circ$$

Find $\sin \frac{\theta}{2}$

$$20) \cos \theta = -\frac{15}{17} \text{ and } 180^\circ < \theta < 270^\circ$$

Find $\tan \frac{\theta}{2}$

$$21) \tan \theta = -\frac{7}{24} \text{ and } \frac{3\pi}{2} < \theta < 2\pi$$

Find $\cot \frac{\theta}{2}$

$$22) \cot \theta = \frac{4}{3} \text{ and } \pi < \theta < \frac{3\pi}{2}$$

Find $\sin 2\theta$

$$23) \cot \theta = \frac{4}{3} \text{ and } \pi < \theta < \frac{3\pi}{2}$$

Find $\cot 2\theta$

$$24) \tan \theta = 2 \text{ and } 0 < \theta < \frac{\pi}{2}$$

Find $\sin \frac{\theta}{2}$

$$25) \sin \theta = -\frac{3}{5} \text{ and } \frac{3\pi}{2} < \theta < 2\pi$$

Find $\tan \frac{\theta}{2}$

$$26) \cot \theta = -\frac{3\sqrt{91}}{91} \text{ and } \frac{3\pi}{2} < \theta < 2\pi$$

Find $\sin \frac{\theta}{2}$

Double- and Half-Angle Identities

Use a double-angle identity to find the exact value of each expression.

1) $\sin 120^\circ$

$$\frac{\sqrt{3}}{2}$$

2) $\tan 60^\circ$

$$\sqrt{3}$$

3) $\cos \frac{4\pi}{3}$

$$-\frac{1}{2}$$

4) $\sin \frac{5\pi}{3}$

$$-\frac{\sqrt{3}}{2}$$

Use a half-angle identity to find the exact value of each expression.

5) $\tan 45^\circ$

$$1$$

6) $\sin 165^\circ$

$$\frac{\sqrt{6} - \sqrt{2}}{4}$$

7) $\sin \frac{5\pi}{6}$

$$\frac{1}{2}$$

8) $\cos 30^\circ$

$$\frac{\sqrt{3}}{2}$$

Use a double-angle or half-angle identity to find the exact value of each expression.

9) $\cot \frac{\pi}{3}$

$$\frac{\sqrt{3}}{3}$$

10) $\cot \frac{2\pi}{3}$

$$-\frac{\sqrt{3}}{3}$$

11) $\sec \frac{5\pi}{12}$

$$\sqrt{6} + \sqrt{2}$$

12) $\cot 60^\circ$

$$\frac{\sqrt{3}}{3}$$

13) $\cot 240^\circ$

$$\frac{\sqrt{3}}{3}$$

14) $\cot \frac{5\pi}{3}$

$$-\frac{\sqrt{3}}{3}$$

15) $\sin \theta = -\frac{7}{25}$ and $270^\circ < \theta < 360^\circ$

Find $\cos \frac{\theta}{2}$

$$-\frac{7\sqrt{2}}{10}$$

16) $\cos \theta = \frac{1}{3}$ and $0^\circ < \theta < 90^\circ$

Find $\sin 2\theta$

$$\frac{4\sqrt{2}}{9}$$

17) $\cos \theta = \frac{4}{5}$ and $270^\circ < \theta < 360^\circ$

Find $\sin 2\theta$

$$-\frac{24}{25}$$

18) $\cos \theta = \frac{2\sqrt{5}}{5}$ and $0^\circ < \theta < 90^\circ$

Find $\sin \frac{\theta}{2}$

$$\frac{\sqrt{50 - 20\sqrt{5}}}{10}$$

19) $\cos \theta = -\frac{4}{5}$ and $90^\circ < \theta < 180^\circ$

Find $\sin \frac{\theta}{2}$

$$\frac{3\sqrt{10}}{10}$$

20) $\cos \theta = -\frac{15}{17}$ and $180^\circ < \theta < 270^\circ$

Find $\tan \frac{\theta}{2}$

$$-4$$

21) $\tan \theta = -\frac{7}{24}$ and $\frac{3\pi}{2} < \theta < 2\pi$

Find $\cot \frac{\theta}{2}$

$$-7$$

22) $\cot \theta = \frac{4}{3}$ and $\pi < \theta < \frac{3\pi}{2}$

Find $\sin 2\theta$

$$\frac{24}{25}$$

23) $\cot \theta = \frac{4}{3}$ and $\pi < \theta < \frac{3\pi}{2}$

Find $\cot 2\theta$

$$\frac{7}{24}$$

24) $\tan \theta = 2$ and $0 < \theta < \frac{\pi}{2}$

Find $\sin \frac{\theta}{2}$

$$\frac{\sqrt{50 - 10\sqrt{5}}}{10}$$

25) $\sin \theta = -\frac{3}{5}$ and $\frac{3\pi}{2} < \theta < 2\pi$

Find $\tan \frac{\theta}{2}$

$$-\frac{1}{3}$$

26) $\cot \theta = -\frac{3\sqrt{91}}{91}$ and $\frac{3\pi}{2} < \theta < 2\pi$

Find $\sin \frac{\theta}{2}$

$$\frac{\sqrt{35}}{10}$$

Double and Half Angle Identities

$$1) \sin 120^\circ = 2 \sin 60^\circ \cos 60^\circ \quad 2) \tan 60^\circ = \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}$$

$$\begin{aligned} &= 2 \left(\frac{\sqrt{3}}{2} \right) \left(\frac{1}{2} \right) \\ &= \frac{\sqrt{3}}{2} \end{aligned}$$

$$\begin{aligned} &= \frac{2 \left(\frac{1}{\sqrt{3}} \right)}{1 - \frac{1}{3}} \\ &= \frac{2}{\frac{2\sqrt{3}}{3}} \end{aligned}$$

$$3) \cos \frac{4\pi}{3} = 2 \cos^2 \frac{4\pi}{6} - 1$$

$$\begin{aligned} &= 2 \left(\frac{1}{4} \right) - 1 \\ &= \frac{1}{2} - 1 \\ &= -\frac{1}{2} \end{aligned}$$

$$\begin{aligned} &= \frac{2}{3} \\ &= \frac{3}{\sqrt{3}} \cdot \sqrt{3} \end{aligned}$$

$$4) \sin \frac{5\pi}{3} = 2 \sin \frac{5\pi}{6} \cos \frac{5\pi}{6}$$

$$\begin{aligned} &= 2 \left(\frac{1}{2} \right) \left(-\frac{\sqrt{3}}{2} \right) \\ &= -\frac{\sqrt{3}}{2} \end{aligned}$$

$$\begin{aligned} &= \frac{3\sqrt{3}}{3} = \sqrt{3} \end{aligned}$$

$$5) \tan 45^\circ = \frac{\sqrt{1 - \cos 2(45^\circ)}}{2}$$

$$\begin{aligned} &= \frac{\sqrt{1 + \cos 2(45^\circ)}}{2} \\ &= \frac{\sqrt{\frac{1}{2}}}{\frac{\sqrt{2}}{2}} \\ &= 1 \end{aligned}$$

$$6) \sin 165^\circ = \sqrt{\frac{1 - \cos 330^\circ}{2}}$$

$$\begin{aligned} &= \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} \\ &= \sqrt{\frac{2 - \sqrt{3}}{2}} \\ &= \sqrt{\frac{2 - \sqrt{3}}{4}} \end{aligned}$$

$$7) \sin \frac{5\pi}{6} = \sqrt{\frac{1 - \cos \frac{5\pi}{3}}{2}}$$

$$\begin{aligned} &= \sqrt{\frac{1 - 0.5}{2}} \\ &= \sqrt{\frac{1}{4}} \\ &= \frac{1}{2} \end{aligned}$$

$$8) \cos 30^\circ = \sqrt{\frac{1 + \cos 60^\circ}{2}}$$

$$\begin{aligned} &= \sqrt{\frac{1 + 0.5}{2}} \\ &= \sqrt{\frac{3}{4}} \\ &= \frac{\sqrt{3}}{2} \end{aligned}$$

$$9) \cot \frac{\pi}{3} = \frac{1 - \tan^2 \frac{\pi}{8}}{2 \tan \frac{\pi}{8}}$$

$$\begin{aligned} &= \frac{1 - \frac{1}{3}}{\frac{2}{\sqrt{3}}} \\ &= \frac{\frac{2\sqrt{3}}{3}}{\frac{2}{\sqrt{3}}} \\ &= \frac{\sqrt{3}}{3} \end{aligned}$$

$$10) \cot \frac{2\pi}{3} = \frac{1 - \tan^2 \frac{\pi}{3}}{2 \tan \frac{\pi}{3}}$$

$$\begin{aligned} &= \frac{1 - 3}{2\sqrt{3}} \\ &= \frac{-2}{2\sqrt{3}} \\ &= \frac{-1}{\sqrt{3}} \\ &= -\frac{\sqrt{3}}{3} \end{aligned}$$

$$11) \sec \frac{5\pi}{12} = \frac{1}{\sqrt{\frac{1 + \cos \frac{5\pi}{6}}{2}}}$$

$$\begin{aligned} &\sqrt{\frac{1}{2 + \frac{\sqrt{3}}{2}}} \\ &= \sqrt{\frac{1}{2 + \sqrt{3}}} \cdot \sqrt{\frac{2 - \sqrt{3}}{4}} \\ &= \frac{\sqrt{2 - \sqrt{3}}}{2 - \sqrt{3}} \cdot \frac{2 - \sqrt{3}}{4} \\ &= \frac{\sqrt{2 - \sqrt{3}}}{4} \end{aligned}$$

$$12) \cot 60^\circ = \frac{1 - \tan^2 30^\circ}{2 \tan 30^\circ}$$

$$\begin{aligned} &= \frac{1 - \frac{1}{3}}{\frac{2}{\sqrt{3}}} \\ &= \frac{\frac{2}{3}}{\frac{2}{\sqrt{3}}} \\ &= \frac{\sqrt{3}}{3} \end{aligned}$$

$$= \sqrt{\frac{2 - \sqrt{3}}{4}} \cdot \frac{4}{2 - \sqrt{3}}$$

$$= \frac{4\sqrt{2 - \sqrt{3}}}{8 - 4\sqrt{3}}$$

$$= \frac{\sqrt{2 - \sqrt{3}}}{2 - \sqrt{3}}$$

$$13) \cot 240^\circ = \frac{1 - \tan^2 120^\circ}{2 \tan 120^\circ}$$

$$= \frac{1 - 3}{-2\sqrt{3}} \\ = \frac{-2}{-2\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}} \text{ and } \cot 120^\circ \\ = \frac{\sqrt{3}}{3}$$

$$14) \cot \frac{5\pi}{3} = \frac{1 - \tan^2 150^\circ}{2 \tan 150^\circ}$$

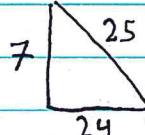
$$= \frac{1 - \frac{1}{3}}{-2} \\ = \frac{2}{3}$$

$$= -\frac{\sqrt{3}}{3}$$

$$15) \sin^2 \theta = \frac{49}{625}$$

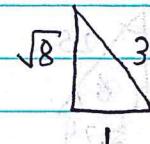
~~$\frac{49}{625} =$~~

$$\cos \theta = \frac{24}{25}$$



$$16) \sin^2 x$$

$$\sin 2x = \sqrt{1 - \cos^2 x}$$



$$\cos^2 x = \frac{1 + \frac{24}{25}}{2}$$

$$\cos x = \sqrt{\frac{49}{50}}$$

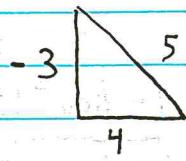
$$= \frac{7}{\sqrt{50}} \\ = \frac{7}{5\sqrt{2}} \cdot \sqrt{2}$$

$$= \frac{7\sqrt{2}}{10}$$

$$\sin 2\theta = 2 \sin \theta \cos \theta \\ = 2 \left(\frac{\sqrt{8}}{5}\right) \left(\frac{1}{\sqrt{3}}\right)$$

$$= \frac{4\sqrt{2}}{9}$$

17)



$$\begin{aligned}\sin 2\theta &= 2 \sin \theta \cos \theta \\ &\approx 2 \left(\frac{3}{5}\right) \left(\frac{4}{5}\right) \\ &= \frac{-24}{25}\end{aligned}$$

$$18) \quad \sin x = \sqrt{\frac{1 - \cos 2x}{2}}$$

$$= \sqrt{\frac{1 - 2\sqrt{5}}{2}}$$

$$= \sqrt{\frac{5 - 2\sqrt{5}}{10} \cdot \frac{10}{10}} \\ = \frac{\sqrt{50 - 20\sqrt{5}}}{10}$$

19)

$$\begin{aligned}\sin x &= \sqrt{\frac{1 + \frac{4}{5}}{2}} \\ &= \sqrt{\frac{9}{10}} \\ &= \frac{3}{\sqrt{10}} \cdot \sqrt{10} \\ &= \frac{3\sqrt{10}}{10}\end{aligned}$$

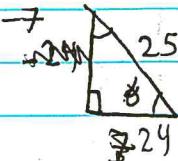
$$20) \quad \tan \theta = \frac{\sqrt{\frac{1 - \cos 2x}{2}}}{\sqrt{\frac{1 + \cos 2x}{2}}}$$

$$= \sqrt{\frac{1 + \frac{17}{25}}{2}}$$

$$\sqrt{\frac{1 - \frac{17}{25}}{2}}$$

$$= \sqrt{\frac{32}{25}} \\ = \sqrt{\frac{2}{\frac{25}{17}}} \\ = \sqrt{\frac{17}{25}}$$

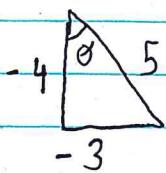
21)



$$\begin{aligned}\cot \theta &= \frac{\sqrt{\frac{1 + \frac{24}{25}}{2}}}{\sqrt{\frac{1 - \frac{24}{25}}{2}}} \\ &= \sqrt{\frac{\frac{49}{25}}{\frac{1}{2}}} \\ &= \frac{7}{\sqrt{\frac{50}{2}}} \\ &= \frac{7}{\sqrt{50}} \\ &= \frac{7}{\sqrt{50}} \\ &= -\frac{7}{\sqrt{50}} \\ &= -\frac{1}{\sqrt{50}} \\ &= -\frac{1}{5\sqrt{2}}\end{aligned}$$

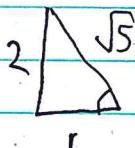
$$= -4 \quad \text{neg since } \tan \theta \text{ is neg in quad II}$$

22)



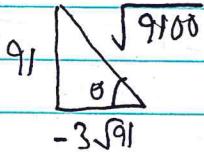
$$\begin{aligned}\sin 2\theta &= 2 \sin \theta \cos \theta \\ &= 2 \left(\frac{-3}{5}\right) \left(\frac{-4}{5}\right) \\ &= \frac{24}{25}\end{aligned}$$

24)



$$\begin{aligned}\sin \theta &= \sqrt{\frac{1-\frac{1}{5}}{2}} \\ &= \sqrt{\frac{5-1}{2}} \\ &= \frac{\sqrt{5-1}}{2} \\ &= \frac{\sqrt{4}}{2} \\ &= \frac{2}{2}\end{aligned}$$

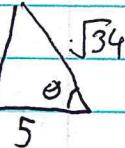
26)



$$\begin{aligned}\sin \theta &= \sqrt{1 + \frac{3\sqrt{91}}{\sqrt{9100}}} \\ &= \sqrt{1 + \frac{3(91)}{9100}} \\ &= \sqrt{\frac{5915}{9100}} \\ &= \sqrt{\frac{13 \cdot 10}{20 \cdot 10}} = \frac{\sqrt{130}}{10} = \frac{\sqrt{35}}{10}\end{aligned}$$

$$\begin{aligned}23) \cot 2\theta &= \frac{2 \left(\frac{3}{4}\right)}{1 - \frac{9}{16}} \\ &= \frac{\frac{3}{2}}{\frac{7}{16}} \\ &= \frac{6}{4} \\ &= \frac{7(4)}{16(6)} \\ &= \frac{28}{96}\end{aligned}$$

$$25) \tan \theta = \frac{\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}}$$



$$\begin{aligned}&= \sqrt{1 - \frac{5}{\sqrt{34}}} \\ &= \sqrt{\frac{1+5}{2}}\end{aligned}$$

$$= \sqrt{\frac{\sqrt{34}-5}{2\sqrt{34}}}$$

$$= \sqrt{\frac{\sqrt{34}+5}{2\sqrt{34}}}$$

$$= \sqrt{\frac{34-5\sqrt{34}}{68}}$$

$$= \sqrt{\frac{34+5\sqrt{34}}{68}}$$

$$= -\frac{1}{3}$$

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