Activity 2: Families of polynomial functions

Formative Assignment

PART A

For each of the following functions:

i) State the value and the degree of each of the x-intercepts.
ii) State the value of the y-intercept.
iii) State the degree of the function and the end behaviour it will exhibit.
iv) Sketch the function, showing all intercepts.

1. $f(x) = -x(x-4)(x+3)^2$
2. $f(x) = -1(x-5)^3$
3. $f(x) = 0.5(x+2)(x-5)^3(x+4)$
4. $f(x) = -(x+3)^2(x-2)^2(x-5)^2$
5. $f(x) = 0.3(x-1)^3(x+3)^6$

PART B

1. Given the following data sets, use Finite Differences and algebra to find a unique function equation to define the function containing the data set:

a)

x	$f(x)$
1	-9
2	-10
3	-7
4	0
5	11
6	26

b)

x	$f(x)$
1	-34
2	-42
3	-38
4	-16
5	30
6	106

2. a) Write the equation of the family of quartic functions that have zeroes of: -2 (double root) and 3 (double root).

b) Write the specific equation for the function from a) that passes through the point (1, 108).

3. a) Write the equation of the family of polynomial functions, of degree 4, that have zeroes at 2 and 5.

b) Write a possible unique equation for a function that is a member of the family in a) and graph this function.

4. Given the following data set, find a unique defining equation of polynomial format that contains this data set. You may need to use regression software (either Excel or CurveExpert or the TI83+) to arrive at a possible equation after deciding the best degree to use for the representation.

x	$f(x)$
-20	-20544
-10	-2643
0	-2
10	2379
20	19500
30	66361
40	157962
50	309303

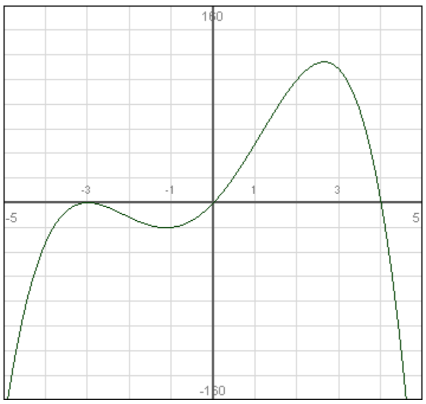
Solutions:
PART A

1. $f(x) = -x(x-4)(x+3)^2$

i) x-intercepts: x=0 (cut), x=4 (cut), and x=-3 (bounce).

ii) y-intercept: y=0.

iii) This is a negative 4th degree function so the end behaviour is as $x \to \infty$, $f(x) \to -\infty$, and as $x \to -\infty$, $f(x) \to -\infty$.

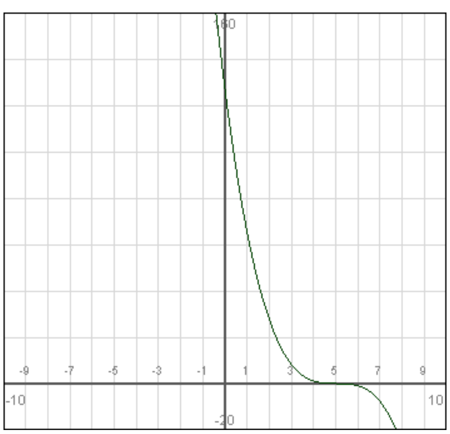
iv) 

2. $f(x) = -1(x-5)^3$

i) x-intercepts: x=5 (saddle).

ii) y-intercept: y=125.

iii) This is a negative 3rd degree functions so as $x \to \infty$, $f(x) \to -\infty$ and as $x \to -\infty$, $f(x) \to \infty$.

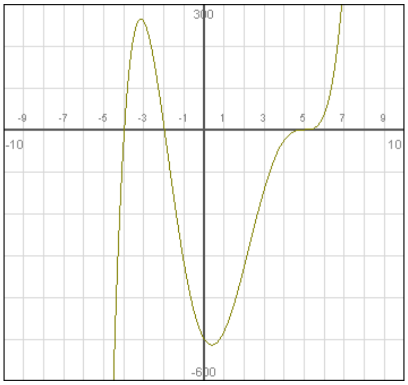
iv) 

3. $f(x)=0.5(x+2)(x-5)^3(x+4)$

i) The x-intercepts are x=-2 (cut), x=5 (saddle) and x=-4 (cut).

ii) The y-intercept is y=-500.

iii) This is a positive 5th degree polynomial function so as $x \to \infty$, $f(x) \to \infty$ and as $x \to -\infty$, $f(x) \to -\infty$.

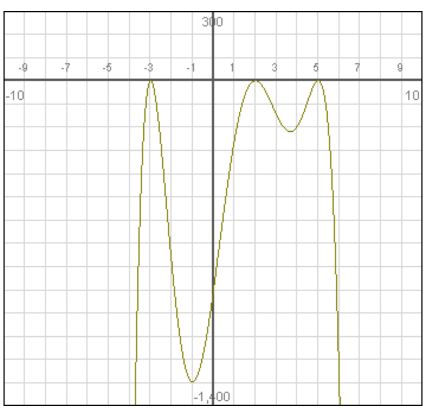
iv) 

4. $f(x)=-(x+3)^2(x-2)^2(x-5)^2$

i) The x-intercepts are x=-3 (bounce), x=2 (bounce) and x=5 (bounce).

ii) The y-intercept is at y=-900.

iii) This is a negative 6th degree polynomial function so, as $x \to \infty$, $f(x) \to -\infty$ and as $x \to -\infty$, $f(x) \to -\infty$.

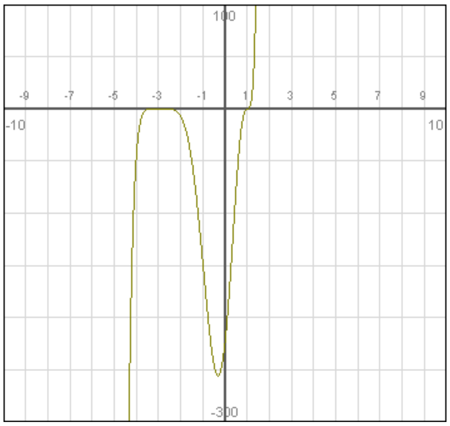
iv) 

5. $f(x) = 0.3(x-1)^3(x+3)^6$

i) The x-intercepts are at x=1 (saddle), and x=-3 (bounce – acting like a 6th degree function).

ii) The y-intercept is y=-218.7.

iii) This is a positive 9th degree polynomial function so as $x \to \infty$, $f(x) \to \infty$ and as $x \to -\infty$, $f(x) \to -\infty$.

iv) 

PART B

1.a)

x	$f(x)$	**First Level Difference** $\Delta\ f(x)$	**Second Level Difference** $\Delta^2\ f(x)$
1	-9		
2	-10	-1	
3	-7	3	4
4	0	7	4
5	11	11	4
6	26	15	4

Quadratic function

$a = 2$ **(since 4/2! and data elements involve sequential domain elements)**

So the function is:

$$f(x) = 2x^2 + bx + c$$

Substitute two points from given data set:

1. $$f(1) = 2 + b + c = -9$$
$$b + c = -11$$

2. $$f(2) = 8 + 2b + c = -10$$
$$2b + c = -18$$

Subtract: 2) – 1)

3) $b = -7$

Substitute in 1) to get:
$-7 + c = -11$

$c = -4$
So final function statement is:

$$f(x) = 2x^2 - 7x - 4$$

b)

x	$f(x)$	**First Level Difference** $\Delta\ f(x)$	**Second Level Difference** $\Delta^2\ f(x)$	**Third Level Difference** $\Delta^3\ f(x)$
1	-34			
2	-42	-8		
3	-38	4	12	
4	-16	22	18	6
5	30	46	24	6
6	106	76	30	6

Cubic function
$a = 1$ **(since 6/3! and data elements involve sequential domain elements)**
So function definition is:

$$f(x) = x^3 + bx^2 + cx + d$$

Substitute three points from the given data set to get:

1.) $$f(1) = 1 + b + c + d = -34$$
$$b + c + d = -35$$

2.) $$f(2) = 8 + 4b + 2c + d = -42$$
$$4b + 2c + d = -50$$

3.) $$f(3) = 27 + 9b + 3c + d = -38$$
$$9b + 3c + d = -65$$

2.) – 1.) produces:

4.) $3b + c = -15$

3.) - 2.) produces:

5.) $5b + c = -15$

Then: 5.) – 4.) produces:

6.) $2b = 0$
$b = 0$

Substitute b=0 in 1.) and 2.) and subtract to get:

$c = -15$

Substitute $b = 0$ and $c = -15$ in 1.) to get:
$d = -20$

So function statement is:

$$f(x) = x^3 - 15x - 20$$

2. a) Write the equation of the family of quartic functions that have zeroes of:
-2 (double root) and 3 (double root).

$$f(x) = k(x+2)^2(x-3)^2$$

b) Write the specific equation for the function from a) that passes through the point (1, 108)

$$f(1) = k(1+2)^2(1-3)^2 = 108$$

$$k = 3$$

So specific function definition is:

$$f(x) = 3(x+2)^2(x-3)^2$$

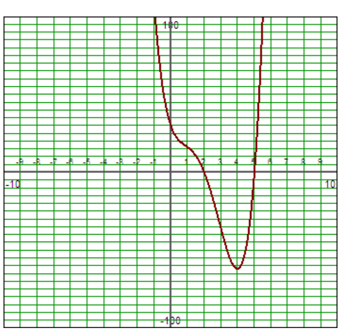
3. a) Write the equation of the family of polynomial functions, of degree 4, that have zeroes at 2 and 5.

$$f(x) = (x-2)(x-5)(ax^2 + bx + c)$$

b) Write a possible unique equation for a function that is a member of the family in a) and graph this function.

One possibility:

$$f(x) = (x-2)(x-5)(2x^2 - x + 3)$$



4. Given the following data set, find a unique defining equation of polynomial format that contains this data set. You may need to use regression software (either **Excel** or **CurveExpert** or the **TI83+**) to arrive at a possible equation after deciding the best degree to use for the representation.

x	$f(x)$			
-20	-20544			
-10	-2643	17901		
0	-2	2641	-15260	
10	2379	2381	-260	15000
20	19500	17121	14740	15000
30	66361	46861	29740	15000
40	157962	91601	44740	15000
50	309303	151341	59740	15000

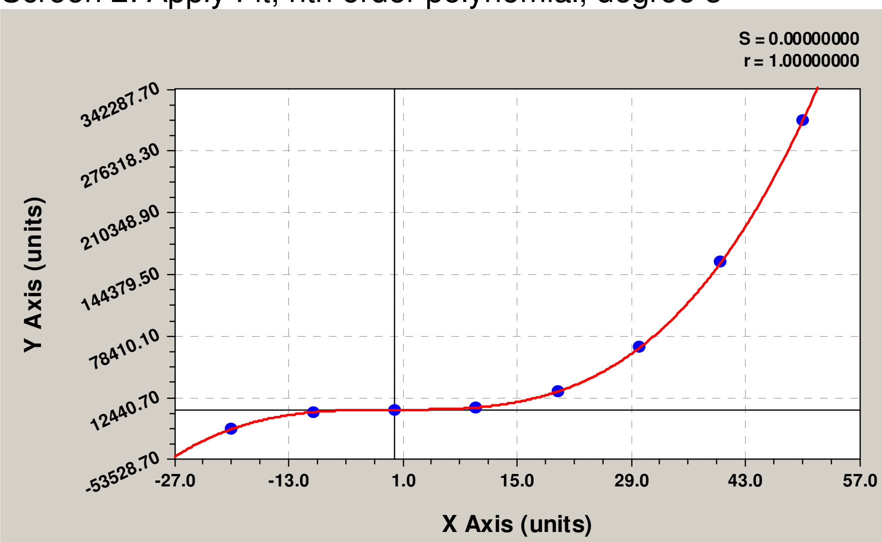
Cubic Function

You cannot, however, predict the value of the lead coefficient in this case, as the data elements do not involve sequential domain elements ie. skip counting for the x values.

Using Curve Experts:

Screen 1: Enter data

Screen 2: Apply Fit, nth order polynomial, degree 3



Screen 3: Click Info

3rd degree Polynomial Fit:

3rd degree Polynomial Fit: y=a+bx+cx^2+dx^3...

Coefficient Data:

a = -2

b = 1.1

c = -1.3

d = 2.5

So the function definition is:

$$f(x) = 2.5x^3 - 1.3x^2 + 1.1x - 2$$