

## Solving Polynomials

1. Solve the following then sketch

(a)  $f(x) = 2x^3 - 3x^2 - 8x - 3 = 0$

(b)  $f(x) = 2x^3 + 3x^2 - 17x - 30 = 0$

2. Solve the following then sketch

(a)  $x^3 + 5x^2 + 2x - 8 = 0$

(b)  $x^3 - 7x - 6 = 0$

(c)  $x^3 - 28x - 48 = 0$

(d)  $2x^3 - x^2 - 13x - 6 = 0$

3. Using the “equating coefficients” method solve the following. (first one is done for you)

(a)  $x^3 - 6x^2 + 3x + 10 = 0$

(b)  $x^3 + 2x^2 - 5x - 6 = 0$

$f(-1)=0$  so  $x+1$  is a factor

$$x^3 - 6x^2 + 3x + 10 = (x+1)\underbrace{(ax^2 + bx + c)}_c$$

$$\therefore a = 1, c = 10$$

$$x^3 - 6x^2 + 3x + 10 = (x+1)\underbrace{(x^2 + bx + 10)}_{+bx}$$

$$\therefore 10x + bx = 3x$$

$$(10+b)x = 3x$$

$$(10+b) = 3$$

$$b = -7$$

$$x^3 - 6x^2 + 3x + 10 = (x+1)(x^2 - 7x + 10)$$

$$(x+1)(x-2)(x-5)$$

therefore  $x = -1, 2, 5$

(c)  $x^3 + x^2 - 9x - 9 = 0$

(d)  $x^3 + 4x^2 + 5x + 2 = 0$

(e)  $2x^3 - x^2 - 13x - 6 = 0$

(f)  $2x^3 + x^2 - 27x - 36 = 0$

## Solving More Polynomials

Complete the exercises below:

1. Which of the following polynomials have  $x + 2$  as a factor?

(a)  $x^3 + 5x^2 + 2x - 8$

(b)  $x^3 - 9x^2 + 17x - 6$

(c)  $x^3 - 28x - 48$

(d)  $x^3 + 4x^2 - 7x - 10$

2. Which of the following polynomials have  $x - 3$  as a factor?

(a)  $x^3 - 7x - 6$

(b)  $x^3 + 9x^2 + 23x + 15$

(c)  $x^3 + 8x^2 + 17x + 10$

(d)  $2x^3 - x^2 - 13x - 6$

3. Which polynomials from exercise #1 have  $x - 2$  as a factor?

4. Which polynomials from exercise #2 have  $x + 5$  as a factor?

You now have one factor for each of the polynomials from exercise #1 and exercise #2 – using this information and the “equating coefficients” method complete the following:

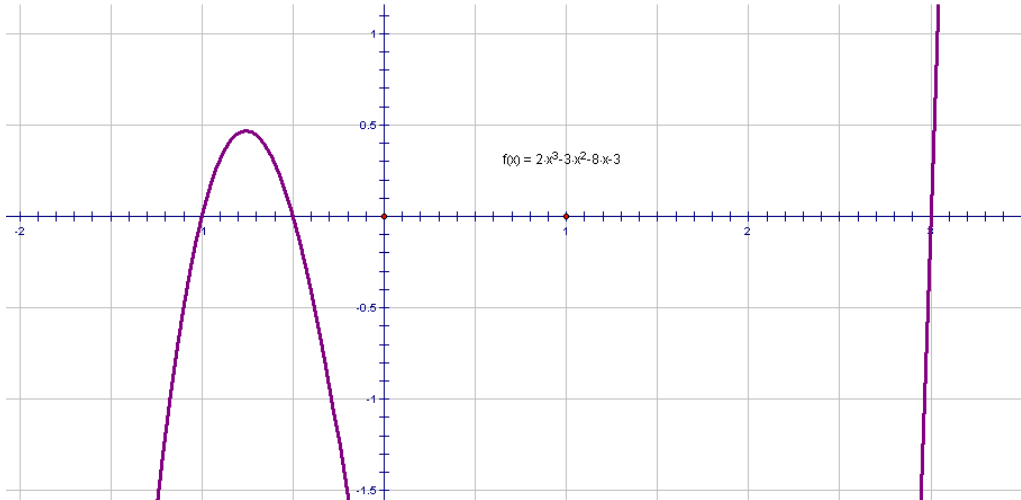
5. Factor completely the polynomials from exercise #1

6. Factor completely the polynomials from exercise #2

## Solving Polynomials (Answers)

1. Solve the following by graphing:

(a)  $f(x) = 2x^3 - 3x^2 - 8x - 3 = 0$



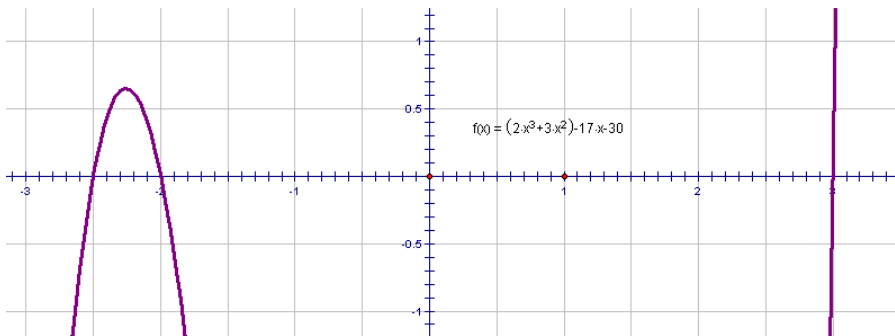
From the graph we can see:

$$x = -1 \quad x = -\frac{1}{2}$$
$$(x+1) \quad (2x+1)$$

$$x = 3$$
$$(x-3)$$

$$\therefore f(x) = (x+3)(2x+1)(x-3)$$

(b)  $f(x) = 2x^3 + 3x^2 - 17x - 30 = 0$



From the graph we can see:

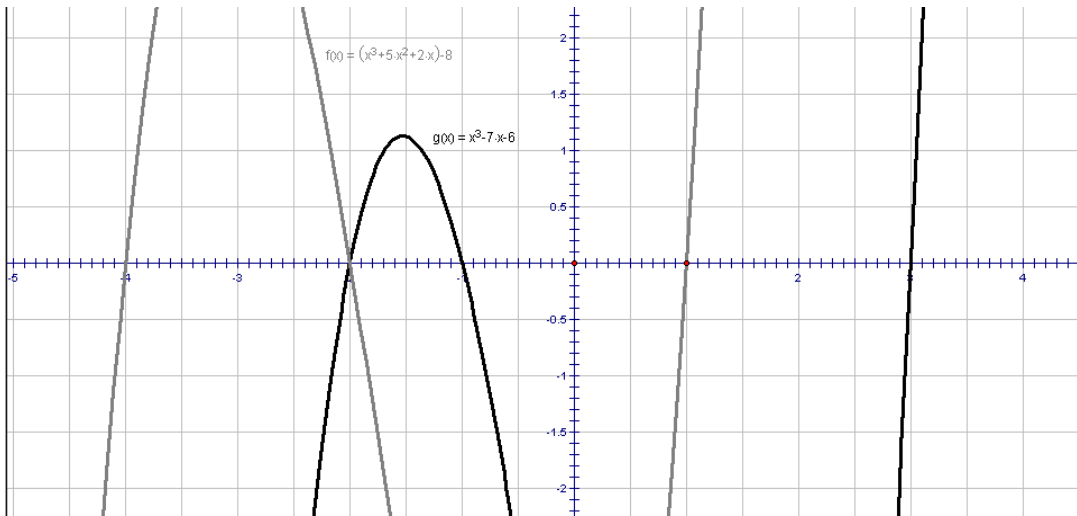
$$x = -2.5 \quad x = -2$$
$$(2x+5) \quad (x+2)$$

$$x = 3$$
$$(x-3)$$

$$\therefore f(x) = (2x+5)(x+2)(x-3) = 0$$

2. Solve the following by graphing:

(a)  $x^3 + 5x^2 + 2x - 8 = 0$  ( $f(x) \rightarrow$  grey line)    (b)  $x^3 - 7x - 6 = 0$  ( $g(x) \rightarrow$  black line)

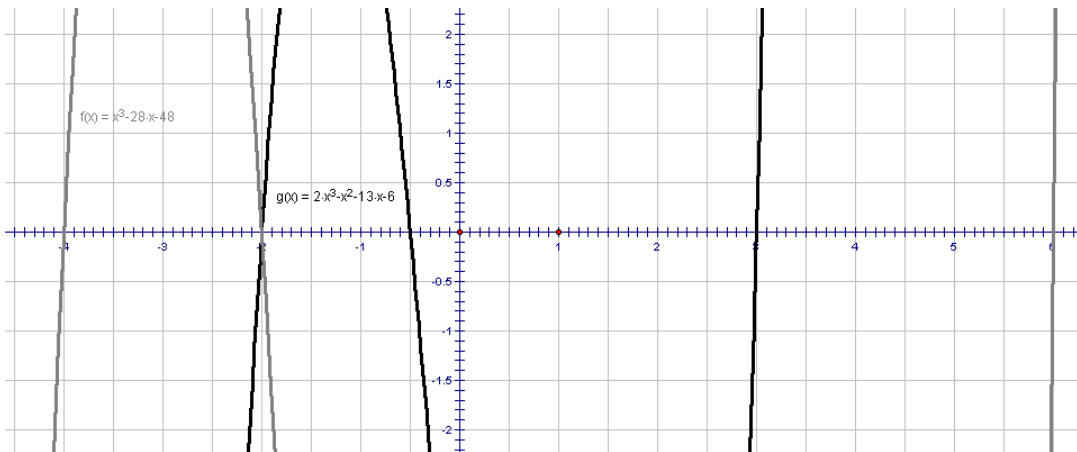


(a) Solution:  $(x + 4)(x + 2)(x - 1)$

(b) Solution:  $(x + 2)(x + 2)(x - 3)$

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(c)  $x^3 - 28x - 48 = 0$  ( $f(x) \rightarrow$  grey line)    (d)  $2x^3 - x^2 - 13x - 6 = 0$  ( $g(x) \rightarrow$  black line)



(c) Solution:  $(x + 4)(x + 2)(x - 6)$

(d) Solution:  $(x + 2)(2x + 1)(x - 3)$

3. Using the "equating coefficients" method solve the following. (first one is done for you)

(a)  $x^3 - 6x^2 + 3x + 10 = 0$   
 $f(-1) = 0$  so  $x+1$  is a factor  
 $x^3 - 6x^2 + 3x + 10 = (x+1)(ax^2 + bx + c)$

$\therefore a = 1, c = 10$

$x^3 - 6x^2 + 3x + 10 = (x+1)(x^2 + bx + 10)$

$\therefore 10x + bx = 3x$

$(10+b)x = 3x$

$(10+b) = 3$

$b = -7$

$x^3 - 6x^2 + 3x + 10 = (x+1)(x^2 - 7x + 10)$   
 $(x+1)(x-2)(x-5)$

therefore  $x = -1, 2, 5$

(c)  $x^3 + x^2 - 9x - 9 = 0$

$f(3) = 0$   $\therefore (x-3)$  is a factor

$f(x) = (x-3)(ax^2 + bx + c)$

Compare  $x^3$  term:  $x^3 = ax^3 \therefore a = 1$

Compare  $x^2$  term:  $x^2 = -3ax^2 + bx^2$

compare last:

$-9 = -3c$   
 $+3 = c$

$1 = -3a + b$   
 $1 = -3 + b$

$\therefore f(x) = (x-3)(x^2 + 4x + 3)$   
 $= (x-3)(x+3)(x+1)$

(b)  $x^3 + 2x^2 - 5x - 6 = 0$   
 $f(2) = 0 \therefore (x-2)$  is a factor

$f(x) = (x-2)(ax^2 + bx + c)$

compare  $x^3$  term:  $x^3 = ax^3 \therefore a = 1$

compare  $x^2$  term:  $2x^2 = -2ax^2 + bx^2$

$2 = -2a + b$

$2 = -2 + b$

$4 = b$

compare constants:  $-6 = -2c$

$3 = c$

$\therefore f(x) = (x-2)(x^2 + 4x + 3)$   
 $= (x-2)(x+3)(x+1)$

(d)  $x^3 + 4x^2 + 5x + 2 = 0$

$f(-2) = 0 \therefore (x+2)$  is a factor

$f(x) = (x+2)(x^2 + bx + c)$

$a = 1$  as before ...

compare  $x^2$  term:  $4x^2 = 2x^2 + bx^2$

$4 = 2 + b$

$2 = b$

compare last:

$2 = 2c$

$1 = c$

$\therefore f(x) = (x+2)(x^2 + 2x + 1)$   
 $= (x+2)(x+1)(x+1)$

(e)  $2x^3 - x^2 - 13x - 6 = 0$

$f(-2) = 0 \therefore (x+2)$  is a factor

$f(x) = (x+2)(2x^2 + bx + c)$  *(a=2 by looking at 1st terms!)*

compare  $x^2$  term:  $-x^2 = 4x^2 + bx^2$

$-1 = 4 + b$

$-5 = b$

compare last:

$-6 = 2c$

$-3 = c$

$\therefore f(x) = (x+2)(2x^2 - 5x - 3)$   
 $= (x+2)(2x+1)(x-3)$

(f)  $2x^3 + x^2 - 27x - 36 = 0$

$f(4) = 0 \therefore (x-4)$  is a factor

$f(x) = (x-4)(2x^2 + bx + c)$

again  $a = 2$

compare  $x^2$  term:  $x^2 = -8x^2 + bx^2$

$1 = -8 + b$

$9 = b$

compare last:

$-36 = -4c$

$9 = c$

$\therefore f(x) = (x-4)(2x^2 + 9x + 9)$   
 $= (x-4)(2x+3)(x+3)$

**Solving More Polynomials ANSWERS:**

- |                          |                        |
|--------------------------|------------------------|
| 1. (a) Yes               | (b) No                 |
| (c) Yes                  | (d) No                 |
| 2. (a) Yes               | (b) No                 |
| (c) No                   | (d) Yes                |
| 3. (a) No                | (b) Yes                |
| (c) No                   | (d) Yes                |
| 4. (a) No                | (b) Yes                |
| (c) Yes                  | (d) No                 |
| 5. (a) $(x+2)(x-1)(x+4)$ | (b) $(x-2)(x^2-7x+3)$  |
| (c) $(x+2)(x+4)(x-6)$    | (d) $(x-2)(x+1)(x+5)$  |
| 6. (a) $(x-3)(x+1)(x+2)$ | (b) $(x+5)(x+1)(x+3)$  |
| (c) $(x+5)(x+1)(x+2)$    | (d) $(x-3)(2x+1)(x+2)$ |