Date: ______________________ Name: ___________________________

Review for FINALS

FINAL EXAM date_______________ Rm: _________

Success Criteria

- ☐ Ensure your Journals are complete and corrected for studying purposes
- ☐ Complete the given Review booklet. Check your answers online.

FORMULAS GIVEN ON EXAM:

FORMULAS:

Addition and Subtraction Formulas

$$\sin(x+y)=\sin x\cos y+\cos x\sin y$$

$$\sin(x-y)=\sin x\cos y-\cos x\sin y$$

$$\cos(x+y)=\cos x\cos y-\sin x\sin y$$

$$\cos(x-y)=\cos x\cos y+\sin x\sin y$$

$$\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x\tan y}$$

$$\tan(x-y)=\frac{\tan x-\tan y}{1+\tan x\tan y}$$

Key Identities

$$1+\tan^2\theta=\sec^2\theta \qquad 1+\cot^2\theta=\csc^2\theta$$

$$\sin(-\theta)=-\sin\theta \qquad \cos(-\theta)=\cos\theta$$

$$\tan(-\theta)=-\tan\theta \qquad \sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$$

$$\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta \qquad \tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta$$

Double Angle Formulas

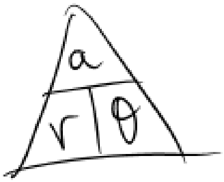
$$\sin 2x=2\sin x\cos x \qquad \cos 2x=\cos^2 x-\sin^2 x$$

$$\tan 2x=\frac{2\tan x}{1-\tan^2 x} \qquad \cos 2x=2\cos^2 x-1$$

$$\cos 2x=1-2\sin^2 x$$

$$M_2-M_1=\log\left(\frac{I_2}{I_1}\right) \qquad L_2-L_1=10\log\left(\frac{I_2}{I_1}\right)$$

$$pH=-\log\left(H^+\right) \qquad L=10\log\left(\frac{I}{I_0}\right) \qquad M=\log\left(\frac{I}{I_0}\right)$$



On Exam:

CAN NOT use Journals

But formula page is provided

Total 14 pages & 19 questions (some with a,b,c…) no multiple choice

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MHF 4U1 Mrs.K's Review For Exam

1. Sketch each of the following
   - a. $y = 0.5^x - 2$
   - b. $y = 3(1.5)^{-x}$
   - c. $y = -2(5)^x + 6$
   - d. $y = -2\sin[\pi x - 3\pi] + 4$
   - e. $y = \tan\left(\frac{\pi}{2}x + \pi\right)$
   - f. $y = \csc(2x) - 3$
   - g. $y = \frac{1}{x^2 - x - 6}$ find the eqtns of all asymptotes
   - h. $y = \frac{1}{x^2 + 3x + 4}$ find the eqtns of all asymptotes
   - i. $y = \frac{x+2}{x^2 - 4x + 4}$ find the eqtns of all asymptotes
   - j. $y = \frac{2x-3}{x+6}$ find the eqtns of all asymptotes
   - k. $y = \frac{5-x}{2x-8}$ find the eqtns of all asymptotes
   - l. $y = \frac{x^2 - 9}{x-2}$ find the eqtns of all asymptotes
   - m. $y = \frac{2x^2 - 5x - 3}{x+4}$ find the eqtns of all asymptotes
   - n. $y = (x+2)^2(3-2x)^3(x)$
     Sketch, state end behaviour, # of possible zeros, # of possible turning points
   - o. $y = -2x^3 - 3x^2 + 11x + 6$
     Sketch, state end behaviour, # of possible zeros, # of possible turning points
2. Determine the inverse for 1a, b, c, j, k
3. Sketch the inverses in 2. a,b,c,j,k
4. Prove
   - a. $(\cos x - \sin x)^2 = 1 - \sin 2x$
   - b. $2\cos x \cos y = \cos(x+y) + \cos(x-y)$
   - c. $(\csc x - \cot x)^2 = 1 - \frac{2\cos x}{1 + \cos x}$
5. Solve
   - a. $2\sec^2 x + \sec x - 1 = 0$
   - b. $\tan x \cos^2 x - \tan x = 0$
   - c. $2\cos^2 x + \sin x - 1 = 0$
6. Find exact values for
   - a. $\sin\left(\frac{7\pi}{12}\right)$
   - b. $\sin\left(\frac{7\pi}{6}\right)\cos\left(\frac{4\pi}{3}\right) - \cos\left(\frac{5\pi}{4}\right)\tan\left(\frac{5\pi}{6}\right)$
   - c. $\cos\left(\frac{3\pi}{8}\right)$
7. Solve
   - a. $\frac{500}{x} + x < 60$
   - b. $x^3 + 5x^2 + 2x - 8 \geq 16x - 8$
   - c. $\frac{2x+1}{2x-3} \geq \frac{x+1}{x-5}$
   - d. $5^{2x+2} - 3126(5^x) = -125$
   - e. $\log_5(x+1) + \log_5(x-3) = 1$
   - f. $2\tan x = \sqrt{3}\tan^2 x - \sqrt{3}$
   - g. $\sqrt{2x-1} + \sqrt{x+11} = 7$ **extra NOT on exam!**
   - h. $\log_9 \frac{9}{5}x = \log_9 \frac{63}{10} + \log_9 2$
   - i. $\log_5 x - 2\log_{\frac{1}{2}} 2^{-1} = 2$
   - j. $\left(\frac{1}{3}\right)^x = \sqrt[16]{81}$
   - k. $2^{x-1} = 6^x$
   - l. $|2x+3| \geq 5$
   - m. $\frac{600}{x-5} = \frac{600}{x} + 20$
8. You board a Ferris wheel of 49m radius at a height of 1m off the ground. The wheel rotates at 2 rotations per minute.
   - a. Write an equation of height in meters versus time in seconds. Do both cosine and sine.
   - b. At what two times in one cycle are you 90m off the ground?
   - c. How high are you after 5 min 20 sec?
   - d. What is the average rate of change on the time interval of [0, 10] seconds?

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9. At high tide the water is at 9 feet. At low tide it is at 3 feet. One day the low tide occurred at 3am and high tide at 9am
   - a. Find the equation that models this if t=3 is 3am. Do both sine and cosine.
   - b. What is the water level at 2pm?

10. Find a real number that exceeds its cube by $\frac{15}{8}$

11. A mass of 1200 grams decays at a half life of 4 weeks.
   - a. Write the equations for the mass in terms of weeks, months and years.
   - b. At what time is there only 0.5 grams left?
   - c. After 2 days what is the mass?
   - d. What is the average rate of change of mass during the 1st two weeks?

12. A certain bacteria triples every 5 seconds. What are the equations that model this in seconds, in minutes and in hours?

13. You kick a ball 2m away from the net so it lands 7m away from the net. As it flew through the air it reached a maximum height of 3m.
   - a. What is the equation to model this?
   - b. Find the rate of change of height versus horizontal distance when the ball is 5 m away from the net.

14. For $f(x)=3x-2$ on [0,6]

$g(x)=x^2-6x$ on [-1,4]

$h(x)=\{(3,2),(5,1),(7,4),(9,3),(11,5)\}$

$i(x)=\{(1,3),(2,5),(3,7),(4,9),(5,11)\}$

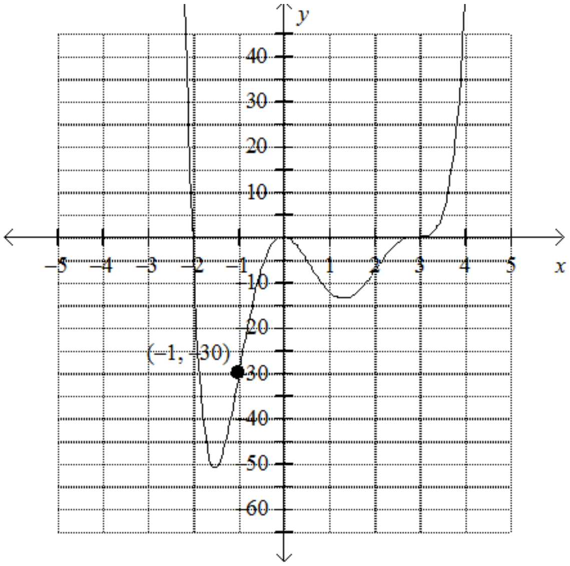
Find

- a. $(f-g)(x)$
- b. $(f\div g)(x)$
- c. $(f\circ g)(x)$
- d. $(h+i)(x)$
- e. $(h\circ i)(x)$

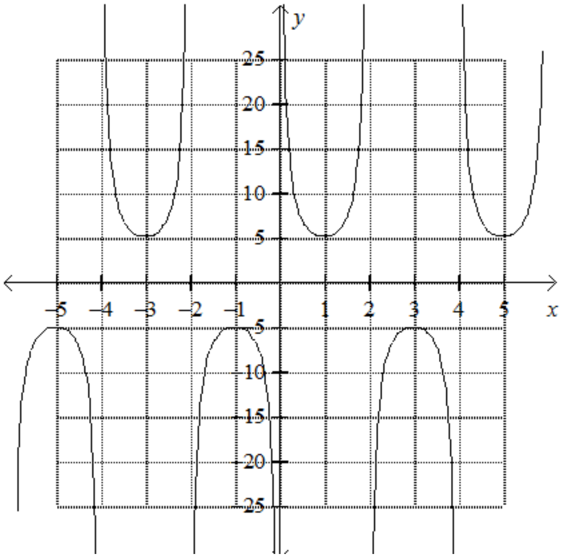
(be able to match graphs for exam too)

15. State the equations for

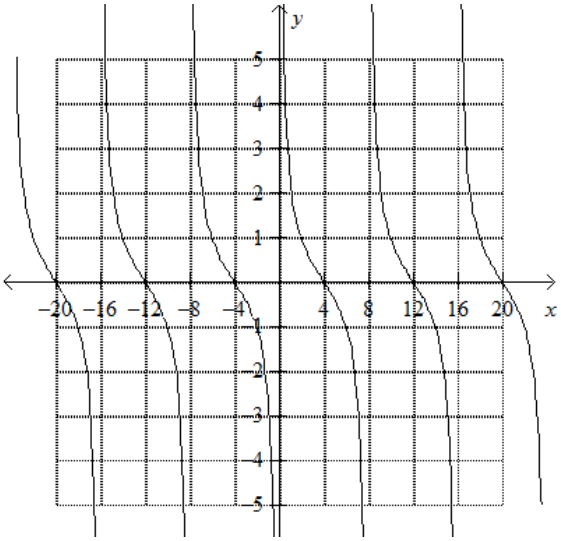
a.



b.



c.



Date: ______________________

16. For 15a.
    a. State when $f(x) < 0$
    b. State when $f(x) \geq -10$
    c. State an interval where average rate of change would be negative.
    d. State a point where instantaneous rate of change would be zero.

17. A grade school is taking a trip to the zoo. A parent group of 6 people is responsible for putting together 225 box lunches for the trip. The group hopes to recruit extra people for the task.
    a. Write an equation that gives the average number "n" of box lunches made per person as a function of the number "p" of parents that can come in and help complete the task.
    b. Sketch the equation.
    c. State the valid domain for this situation.
    d. How many people need to come in so that the average number of box lunches made per person is 15 box lunches?

18. A motorboat travels 30 miles up a river and returned a distance of 27 miles. The entire trip takes 5 hours. If the rate of the motorboat in still water is 12 mph, find the rate of the current of the river.

19. Ero and Jamal set off at the same time on a 30km walk for charity. Ero, who has trained all year for this event, walks 1.4km/h faster than Jamal, but sees a friend on the route and stops to talk for 20 min. Even with this delay, Ero still waits for Jamal to finish for 2 hrs more. How fast was each person walking?

20. If constructing a box with a square base with a volume of 1000 cubic inches. The material for the top and bottom are $3 per 100 square inch and the material for the sides costs $1.25 per 100 sq. in.
    a. If x is the length of the side of the base, what is the cost in terms of x?
    b. For what dimensions will the cost be $39.50?

21. For each of the following find the end behaviour, the possible number of turning points and possible number of zeros.
    a. $f(x) = -x^3 + 2x^4 - 6$
    b. $g(x) = -2x^5 + 2x^4 + x - 8$

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22. For each of the following tables
    a. Determine what type of relationship is given
    b. Determine an equation to model the relationship

i) x	y
-6	0.3
-1	0.6
4	1.2
9	2.4
14	4.8
19	9.6
24	19.2

ii) x	y
0.2	2
0.6	5
1.	8
1.4	11
1.8	14
2.2	17
2.6	20

iii) x	y
-2	29
-1	20
0	13
1	8
2	5
3	4
4	5

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23. A wheel is rotating at an angular velocity of 80π radians/minute, while a point on the circumference of the wheel travels 10π meters in 15 seconds.
    a. Find the radius of the wheel
    b. Find the period in seconds.
    c. State the equations in both sine and cosine to model the height versus time in seconds if the axle of the wheel is located 3 meters off the ground and the position at time zero is at minimum.
    d. Within first 4 seconds find all the different times that a point on the circumference is 2.7 meters off the ground.

24. A ferris wheel has a diameter of 140 meters and completes one revolution in 30 minutes.
    a. Find angular velocity in radians/second.
    b. Find linear velocity in kilometers/hour
    c. Find how far (the curved length) a person has traveled in kilometers over 160 minutes.
    d. What is the period in minutes of this situation?
    e. Write an equation using cosine that would model this situation if you assume that the bottom of the wheel is 1 meter off the ground and the person gets on at 71 meters off the ground at time 0 minutes and then goes down.
    f. Write an equation using sine that would model this situation if you assume that the bottom of the wheel is 3 meters under ground and the person gets on at ground level at time 0 minutes and then goes up.

25. $f(x) = x^4 + 1$
    a. Find a.r.o.c on $x \in [-2, 4]$
    b. Find i.r.o.c at x=3

26. Find domain algebraically, then sketch and find range graphically
    a. $a(x) = \sqrt{2x-5} + 3$
    b. $b(x) = 3\log_5(14-7x) - 4$
    c. $c(x) = \dfrac{x-11}{2x^2 - 15x - 8}$
    d. $d(x) = 2x^2 - 8x - 10$
    e. $e(x) = -2(3.5)^x - 6$
    f. $f(x) = 4 - \sqrt{5-10x}$
    g. $g(x) = -2|x+5|$
    h. $h(x) = \begin{Bmatrix} x+2, x \le -3 \\ 4^x, 0 < x \le 5 \end{Bmatrix}$
    i. $i(x) = 4\log_{0.5}(2x-1)$
    j. $j(x) = \dfrac{3x^2}{2x^3 + 3x^2 - 18x + 8}$
    k. $k(x) = \dfrac{2}{5}x + 6$
    l. $l(x) = \sqrt[3]{2x+6} - 8$
    m. $m(x) = \dfrac{4}{x^2+1}$
    n. $n(x) = -2\sin(\pi(x+4)) + 9$
    o. $\boldsymbol{o(x) = \tan 0.25x + 1}$
    p. $p(x) = x^3(x-5)^2(x+1)$

27. All customers pay a monthly customer fee of \$8.91, plus a fee of 10.49 ¢ per kilowatt hour (kWhr) for the first 400 kWhr supplied in the month, plus a fee of 7.91¢ per kWhr for all usage over 400 kWhr. Write the piecewise equation for this relation.

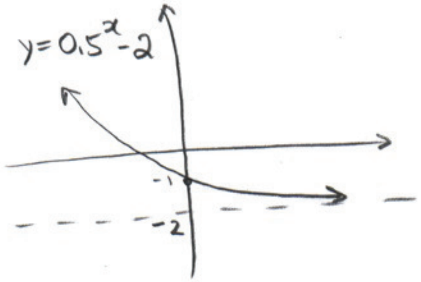
28. A restaurant patron has decided to leave a 15% tip for meals costing up to \$40, an 18% tip for meals costing at least \$40 but less than \$100, and a 20% tip for meals costing \$100 or more. Write a piecewise function to describe the total amount, T, the patron will pay in terms of the meal cost c

29. The pH of water ¡n a small lake in northern Quebec has dropped from 5.4 to 4.8 in the last three years. How many times as acidic as it was three years ago, is the lake now?

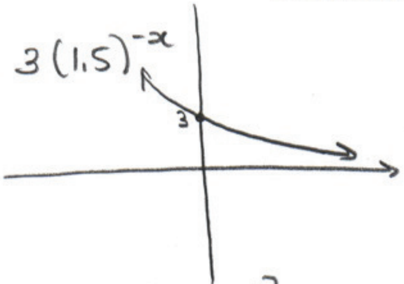
30. Anna can scream at 56 db and Billy can yell at 48 db. How many more times intense is Anna's scream than Billy's yell?

Review Advanced Functions - answers (hopefully without careless errors :))

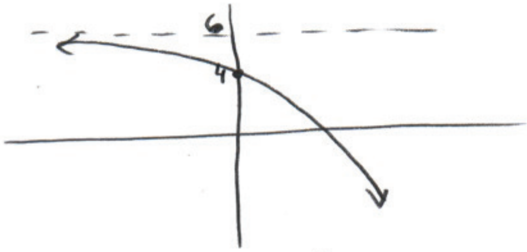
1ⓐ $y=0.5^x-2$



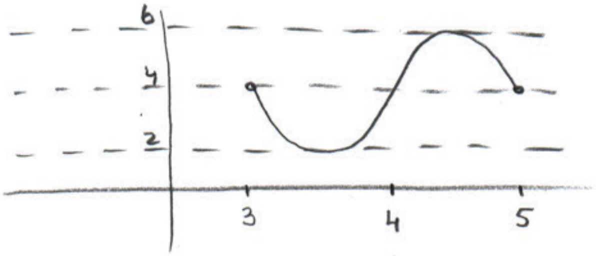
ⓑ $y=3(1.5)^{-x}$



ⓒ $y=-2(5)^x+6$

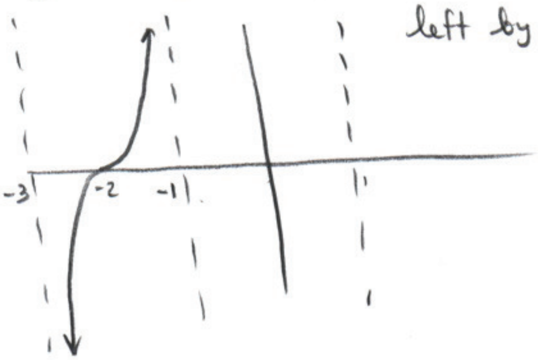


ⓓ $y=-2\sin[\pi(x-3)]+4$

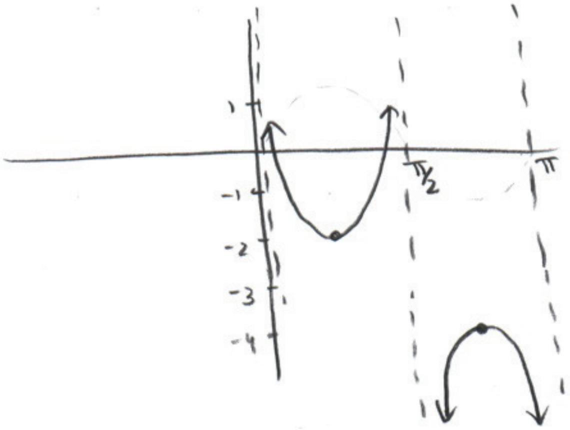


ⓔ $y=\tan\left[\frac{\pi}{2}(x+2)\right]$

period $= \pi \div \frac{\pi}{2} = 2$
left by 2



ⓕ $y=\csc(2x)-3$



ⓖ $y=\dfrac{1}{x^2-x-6}$

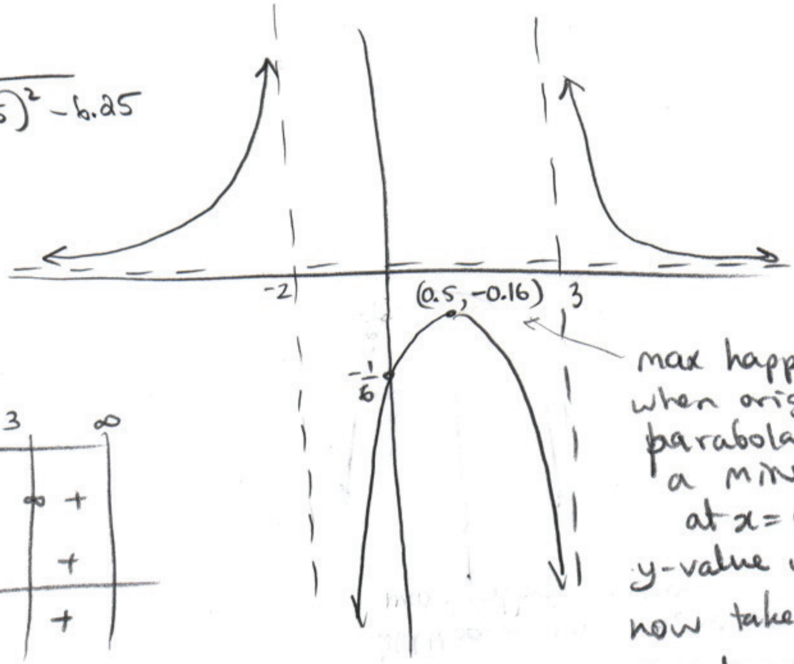
Factor

$$y=\frac{1}{(x-3)(x+2)}=\frac{1}{(x-0.5)^2-6.25}$$

complete square

x-int NA
y-int $-\frac{1}{6}$
V.A 3 and -2
H.A $y=0$
+/- chart

	$-\infty$		-2		3		∞
$x-3$		$-$		$-$	0	$+$	
$x+2$		$-$	0	$+$		$+$	
$f(x)$		$+$		$-$		$+$	

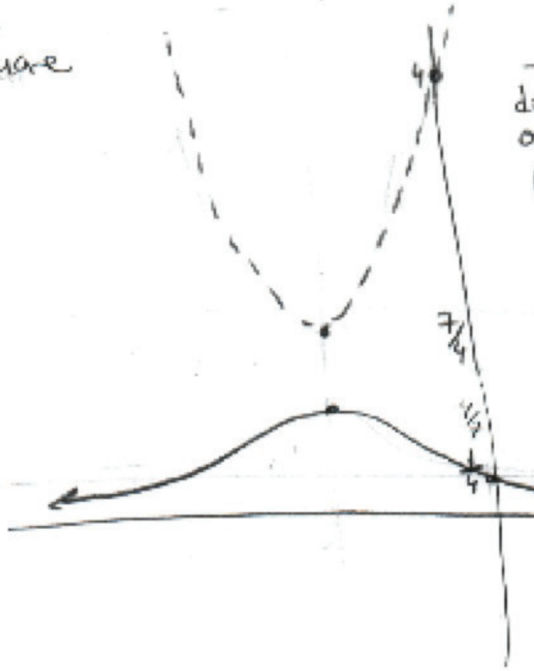


max happens when original parabola had a MIN at $x=0.5$
y-value was -6.25
now take reciprocal to get -0.16.

1 ⓗ $y = \dfrac{1}{x^2+3x+4}$

$y = \dfrac{1}{(x+\frac{3}{2})^2+\frac{7}{4}}$ complete square

x-int N/A
y-int 1/4
V.A N/A.
H.A $y=0$
+/- chart - not needed since always positive.



dotted line original parabola

remember to take all y-values of original and reciprocate them

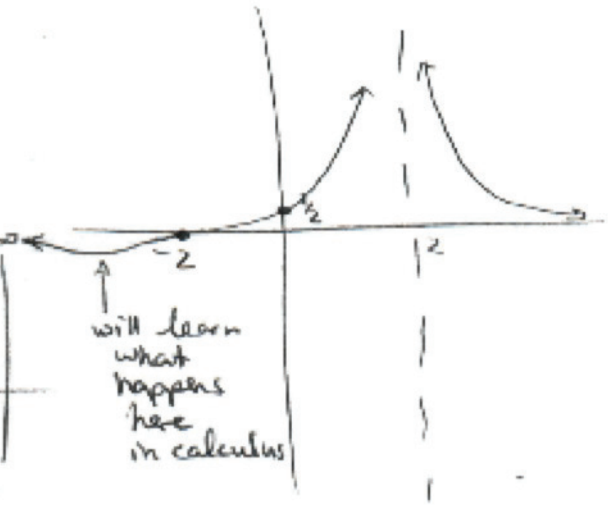
ⓘ $y = \dfrac{x+2}{x^2-4x+4}$

$y = \dfrac{x+2}{(x-2)(x-2)}$ Factor

x-int -2
y-int $\frac{1}{2}$
V.A 2
H.A $y=0$

+/- chart

	$-\infty$ to -2	-2 to 2	2 to ∞
$x+2$	−	+	+
$(x-2)^2$	+	+	+
	−	+	+



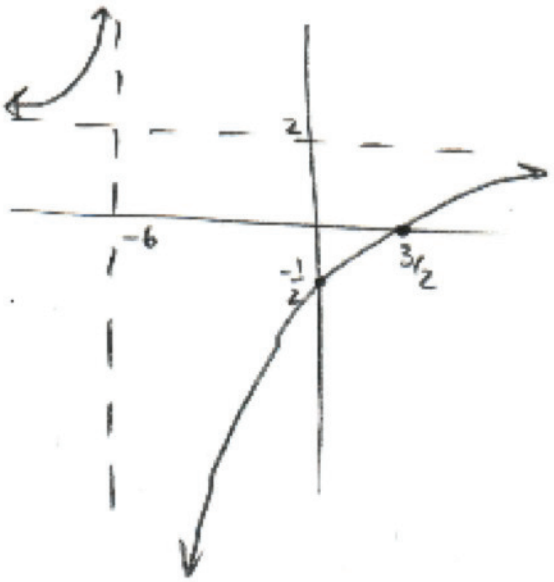
will learn what happens here in calculus

ⓙ $y = \dfrac{2x-3}{x+6}$

x-int 3/2
y-int -1/2
V.A -6
H.A $y=\frac{2}{1}$

+/- chart:

	$-\infty$ to -6	-6 to $3/2$	$3/2$ to ∞
$2x-3$	−	−	+
$x+6$	−	+	+
	+	−	+



1 ⓚ $y = \frac{5-x}{2x-8}$

x-int 5

y-int $-5/8$

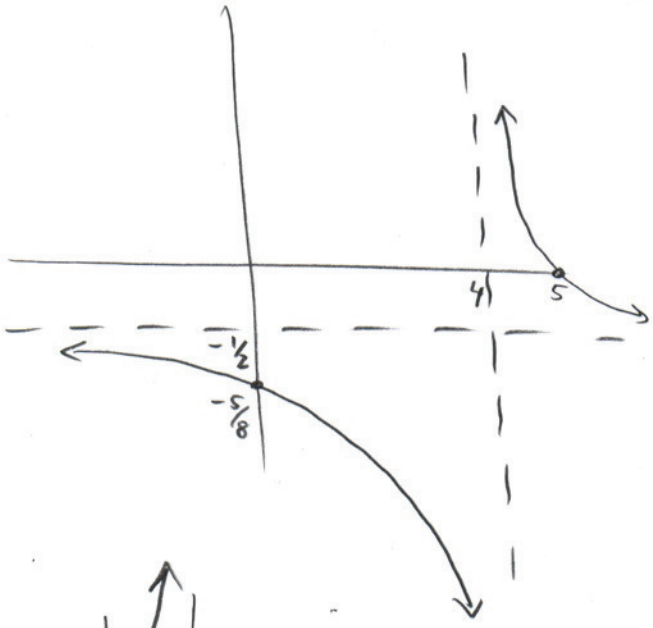
V.A 4

H.A $y = -\frac{1}{2}$

+/- chart:

$-\infty$ 4 5 ∞

$5-x$	+	+	−
$2x-8$	−	+	+
	−	+	−



ⓛ $y = \frac{x^2-9}{x-2}$

$y = \frac{(x+3)(x-3)}{x-2}$

x-int −3 and 3

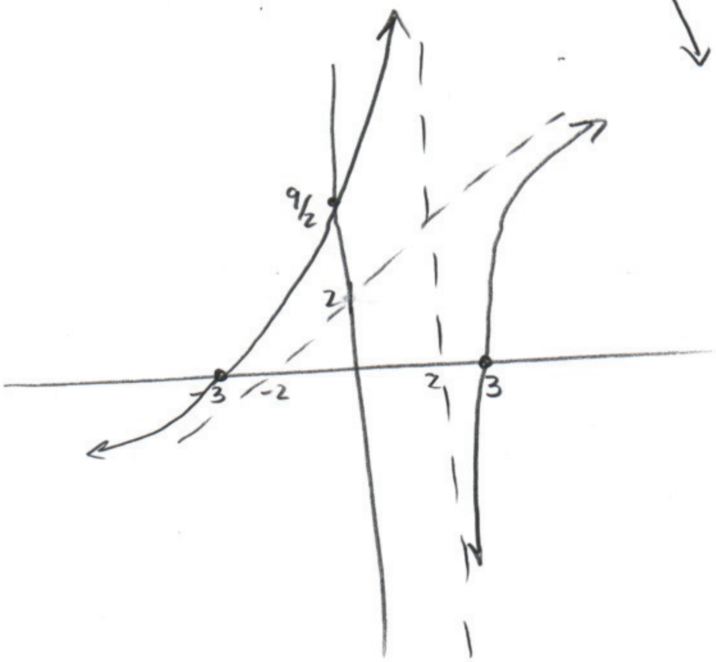
y-int $9/2$

V.A 2

O.A $$\begin{array}{r} x+2 \\ x-2\overline{)x^2+0x-9} \\ \underline{x^2-2x} \\ 2x \end{array}$$

etc
(can stop)

∴ OA $y = x+2$



ⓜ $y = \frac{2x^2-5x-3}{x+4}$

$y = \frac{(2x+1)(x-3)}{(x+4)}$

x-int $-1/2$ and 3

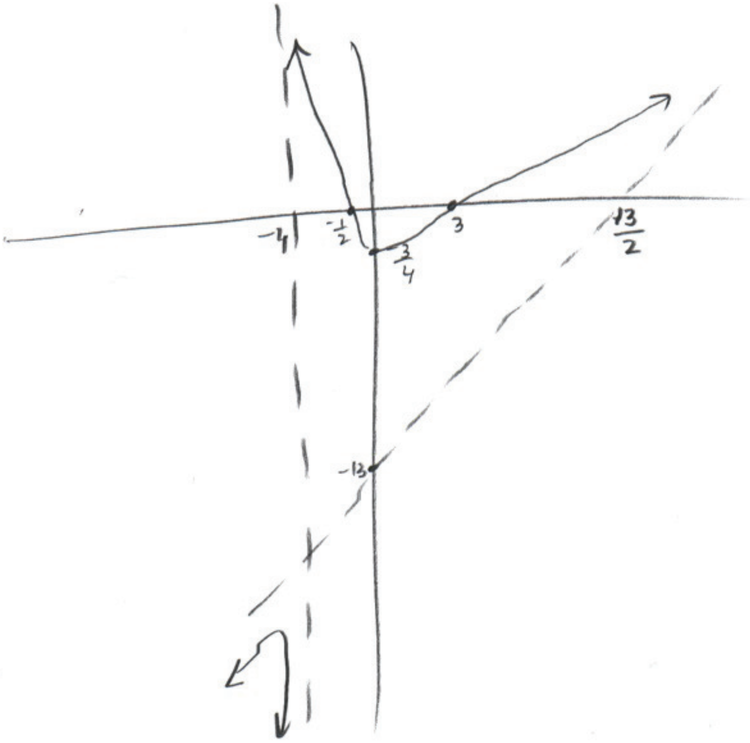
y-int $-3/4$

V.A −4

O.A $$\begin{array}{r} 2x-13 \\ x+4\overline{)2x^2-5x-3} \\ \underline{2x^2+8x} \\ -13x \end{array}$$

etc ...

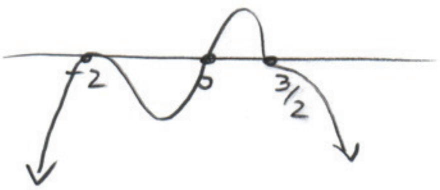
∴ OA $y = 2x-13$



1ⓝ $y=(x+2)^2(3-2x)^3(x)$

zeros at -2 order 2
at $\frac{3}{2}$ order 3
at 0 order 1

Degree 6
end beh:
$x \to \pm\infty, y \to -\infty$



ⓞ $f(x) = -2x^3 - 3x^2 + 11x + 6$

$f(2) = 0 \quad \therefore x-2$ is a factor

$$\begin{array}{r|rrrr} 2 & -2 & -3 & 11 & 6 \\ & \downarrow & -4 & -14 & -6 \\ \hline & -2 & -7 & -3 & 0 \end{array}$$

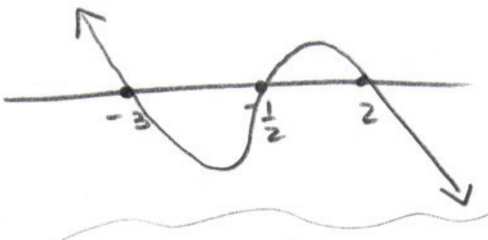
$\therefore f(x) = (x-2)(-2x^2-7x-3)$

$= (x-2)(-1)(2x^2+7x+3)$

$\begin{matrix}2\\1\end{matrix} \qquad \begin{matrix}3 & 1\\1 & 3\end{matrix}$

$= -(x-2)(2x+1)(x+3)$

zeros at $2, -\frac{1}{2}, -3$



2ⓐ $x = 0.5^y - 2$

$x+2 = 0.5^y$

$\log(x+2) = \log(0.5)^y$

$\log(x+2) = y\log(0.5)$

$\frac{\log(x+2)}{\log 0.5} = y$ OR $y = \log_{0.5}(x+2)$

2ⓑ $y = 3(1.5)^{-x}$

$x = 3(1.5)^{-y}$

$\frac{x}{3} = (1.5)^{-y}$

$\log\left(\frac{x}{3}\right) = \log(1.5)^{-y}$

$\log\left(\frac{x}{3}\right) = -y\log(1.5)$

$\frac{\log\left(\frac{x}{3}\right)}{-\log 1.5} = y$

$y = \frac{\log\left(\frac{x}{3}\right)}{0 - \log 1.5}$

OR another form

$y = \frac{\log\left(\frac{x}{3}\right)}{\log 1 - \log 1.5}$

$y = \frac{\log\left(\frac{x}{3}\right)}{\log\left(\frac{1}{1.5}\right)}$

$y = \frac{\log\left(\frac{x}{3}\right)}{\log\left(\frac{2}{3}\right)} = \log_{\frac{2}{3}}\left(\frac{x}{3}\right)$

2ⓒ $x = -2(5)^y + 6$

$x-6 = -2(5)^y$

$\frac{x-6}{-2} = 5^y$

$\log\left(\frac{x-6}{-2}\right) = \log(5)^y$

$\log\left(\frac{x-6}{-2}\right) = y\log 5$

$\frac{\log\left(\frac{x-6}{-2}\right)}{\log 5} = y$ OR $y = \log_5\left(\frac{x-6}{-2}\right)$

$y = \log_5\left(-\frac{1}{2}(x-6)\right)$

2ⓙ $x = \frac{2y-3}{y+6}$

$x(y+6) = 2y-3$

$xy + 6x = 2y - 3$

$xy - 2y = -6x - 3$

$y(x-2) = -3(2x+1)$

$y = \frac{-3(2x+1)}{x-2}$

2ⓚ $x = \frac{5-y}{2y-8}$

$x(2y-8) = 5-y$

$2xy - 8x = 5 - y$

$2xy + y = 8x + 5$

$y(2x+1) = 8x+5$

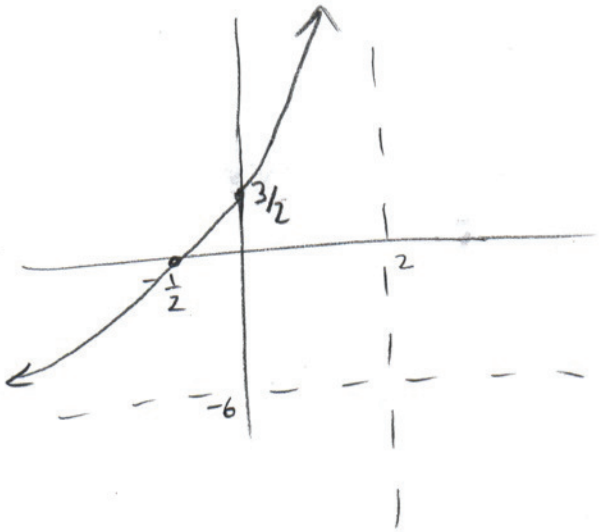
$y = \frac{8x+5}{2x+1}$

3ⓙ x-int $-\frac{1}{2}$

y-int $3/2$

V.A 2

H.A at $y=-6$

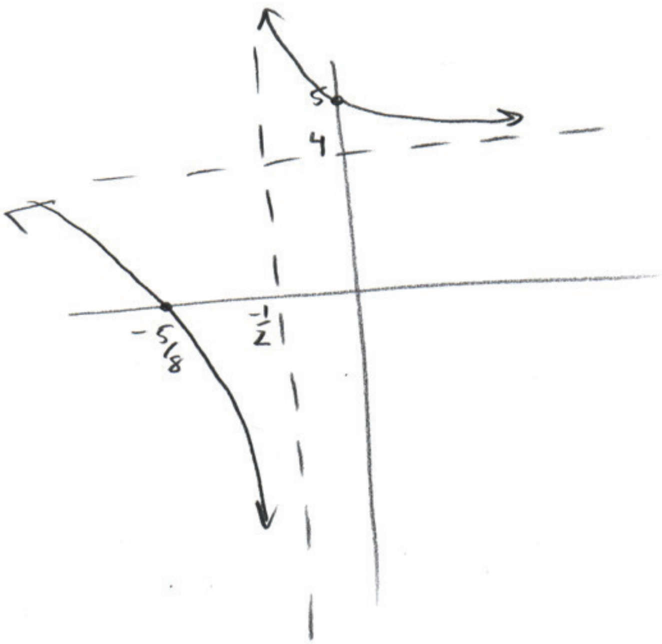


3ⓚ x-int $-5/8$

y-int 5

H.A at $y=4$

V.A at $-\frac{1}{2}$

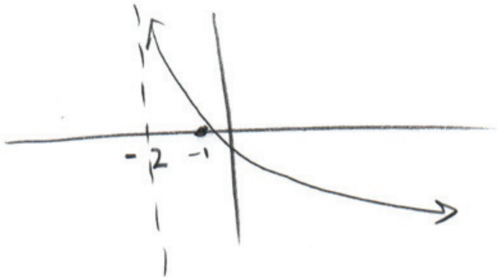


3ⓐ inverse of 1ⓐ $y = 0.5^x - 2$

is $y = \log_{0.5}(x+2)$

use the sketch of 1ⓐ

- HA at $y = -2$
  becomes V.A at $x = -2$
- y-int at $(0,-1)$
  becomes x-int at $(-1,0)$

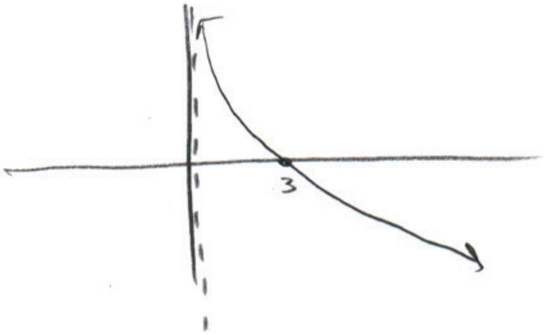


3ⓑ inverse of 1ⓑ $y = 3(1.5)^{-x}$

is $y = \log_{\frac{2}{3}}\left(\frac{x}{3}\right)$

use the sketch in 1ⓑ

- HA at $y=0$
  becomes V.A at $x=0$
- y-int at $(0,3)$
  becomes x-int at $(3,0)$



3ⓒ inverse of 1ⓒ $y = -2(5)^x + 6$

is $y = \log_5\left(-\frac{1}{2}(x-6)\right)$

use the sketch of 1ⓒ

- HA at $y=6$
  becomes V.A at $x=6$
- y-int at $(0,4)$
  becomes x-int at $(4,0)$

* make sure you check that the original function and your inverse function shape look like they are reflections in $y=x$ line.

4. ⓐ $(\cos x - \sin x)(\cos x - \sin x)$

FOIL

$\cos^2 x - 2\sin x\cos x + \sin^2 x$

$\cos^2 x + \sin^2 x - 2\sin x \cos x$

Pythag

$1 - 2\sin x\cos x$

$1 - \sin 2x$

double

$1 - 2\sin x\cos x$

LS=RS

ⓑ $2\cos x\cos y$

$\cos(x+y) + \cos(x-y)$

compound

$[\cos x\cos y - \cancel{\sin x\sin y}] + [\cos x\cos y + \cancel{\sin x\sin y}]$

simplify

$2\cos x\cos y$

LS=RS

ⓒ $(\csc x - \cot x)^2$

Foil

$\csc^2 x - 2\csc x\cot x + \cot^2 x$

reciprocals

$\frac{1}{\sin^2 x} - \frac{2\cos x}{\sin x\sin x} + \frac{\cos^2 x}{\sin^2 x}$

LCD

$\frac{1 - 2\cos x + \cos^2 x}{\sin^2 x}$

pythag

$\frac{\cos^2 x - 2\cos x + 1}{1 - \cos^2 x}$

Factor

$\frac{(\cos x - 1)(\cos x - 1)}{(1-\cos x)(1+\cos x)}$

pull out −1

$\frac{-1\cancel{(-\cos x + 1)}(\cos x - 1)}{\cancel{(1-\cos x)}(1+\cos x)}$

mult −1 in

$\frac{-\cos x + 1}{1 + \cos x}$

$1 - \frac{2\cos x}{1+\cos x}$

LCD

$\frac{1 + \cos x - 2\cos x}{1 + \cos x}$

simplify

$\frac{1-\cos x}{1+\cos x}$

LS=RS

5ⓐ $2\sec^2 x + \sec x - 1 = 0$

Factor $\rightarrow (2\sec x - 1)(\sec x + 1) = 0$

$2\sec x - 1 = 0$ or $\sec x + 1 = 0$

$\sec x = \frac{1}{2}$

$\frac{1}{\cos x} = \frac{1}{2}$

$\therefore \cos x = 2$

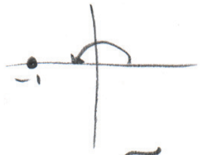
$x = \cos^{-1}(2)$

$x = N/A.$

$\sec x = -1$

$\frac{1}{\cos x} = \frac{-1}{1}$

$\therefore \cos x = -1$



$x = \pi$

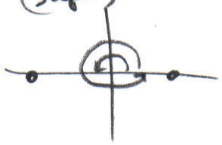
ⓑ $\tan x \cos^2 x - \tan x = 0$

common factor $\rightarrow \tan x(\cos^2 x - 1) = 0$

$\tan x(\cos x + 1)(\cos x - 1) = 0$

$\tan x = 0$ or $\cos x + 1 = 0$ or $\cos x - 1 = 0$

(slope)



$x_1 = 0$

$x_2 = \pi$

$x_3 = 2\pi$

$\cos x = -1$

(x-coordinate on unit circle)

$x_4 = \pi$

$\cos x = 1$

$x_5 = 0$

$x_6 = 2\pi$

$\therefore$ the only solutions are

$0, \pi, 2\pi$ on $[0, 2\pi]$

5ⓒ $2\cos^2 x + \sin x - 1 = 0$

pythag. $\rightarrow$

$2(1 - \sin^2 x) + \sin x - 1 = 0$

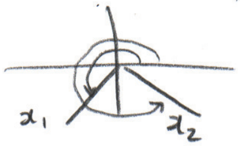
$2 - 2\sin^2 x + \sin x - 1 = 0$

$0 = 2\sin^2 x - \sin x - 1$) rearrange

$0 = (2\sin x + 1)(\sin x - 1)$) Factor

$2\sin x + 1 = 0$ or $\sin x - 1 = 0$

$\sin x = -\frac{1}{2}$



$x = \sin^{-1}(-\frac{1}{2}) = -\frac{\pi}{6}$

$x \doteq -0.542$ in radians!

$\therefore x_1 \doteq \pi + 0.542 \sim 3.665 = \frac{7\pi}{6}$

$x_2 \doteq 2\pi - 0.542 \sim 5.760 = \frac{11\pi}{6}$

$\sin x = 1$

$x_3 = \pi/2 \sim 1.571$

6ⓐ $\sin\frac{7\pi}{12} = \sin 105° = \sin(45°+60°)$

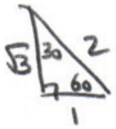
compound ↳ $= \sin 45° \cos 60° + \cos 45° \sin 60°$

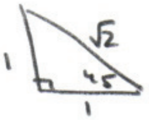
$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{1+\sqrt{3}}{2\sqrt{2}}$$ rationalize!

$$= \frac{(1+\sqrt{3})}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

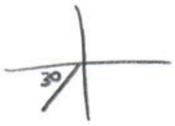
$$= \frac{\sqrt{2}+\sqrt{6}}{4}$$

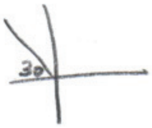




6ⓑ $\sin\frac{7\pi}{6}\cos\frac{4\pi}{3} - \cos\frac{5\pi}{4}\tan\frac{5\pi}{6}$

$= \sin 210° \cos 240° - \cos 225° \tan 150°$





$$= \left(-\frac{1}{2}\right)\left(\frac{-1}{2}\right) - \left(\frac{-1}{\sqrt{2}}\right)\left(\frac{-1}{\sqrt{3}}\right)$$

$$= \frac{1}{4} - \frac{1}{\sqrt{6}}$$

) LCD

$$= \frac{\sqrt{6}-4}{4\sqrt{6}}$$

) rationalize

$$= \frac{(\sqrt{6}-4)}{4\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$$

$$= \frac{6-4\sqrt{6}}{24}$$

) common

$$= \frac{2(3-2\sqrt{6})}{24}$$

) simplify.

$$= \frac{3-2\sqrt{6}}{12}$$

6ⓒ $\cos\frac{3\pi}{8}$

$= \cos 67.5°$ ~› like $\cos x$

if double the angle

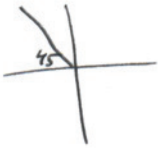
$\cos 135°$ ~› like $\cos 2x$

∴ use double angle

$$\cos 2x = 2\cos^2 x - 1$$

$$\cos 135° = 2\cos^2 67.5° - 1$$

isolate $\cos 67.5°$



$$-\frac{1}{\sqrt{2}} = 2\cos^2 67.5° - 1$$

add one ↳

$$1 - \frac{1}{\sqrt{2}} = 2\cos^2 67.5°$$

LCD ↳

$$\frac{\sqrt{2}-1}{\sqrt{2}} = 2\cos^2 67.5°$$

÷ by 2 ↳

$$\frac{\sqrt{2}-1}{2\sqrt{2}} = \cos^2 67.5°$$

rationalize ↳

$$\frac{2-\sqrt{2}}{4} = \cos^2 67.5°$$

sq. root. ↳

$$\pm\sqrt{\frac{2-\sqrt{2}}{4}} = \cos 67.5°$$

in quad I ⊕ $\frac{\sqrt{2-\sqrt{2}}}{2} = \cos 67.5° = \cos\frac{3\pi}{8}$

7ⓐ $\frac{500}{x} + x < 60$

all on one side ↳ $\frac{500}{x} + x - 60 < 0$

LCD ↳ $\frac{500 + x^2 - 60x}{x} < 0$

$\frac{x^2 - 60x + 500}{x} < 0$

Factor ↳ $\frac{(x-50)(x-10)}{x} < 0$

+/- chart:

	$-\infty$ 0	0 10	10 50	50 ∞
x	−	○ +	+	+
$x-50$	−	−	−	● +
$x-10$	−	−	● +	+
	$\boxed{-}$	+	$\boxed{-}$	+

∴ $x \in (-\infty, 0)$ and $(10, 50)$

7ⓑ $x^3 + 5x^2 + 2x - 8 \geqslant 16x - 8$

all on one side ↳ $x^3 + 5x^2 - 14x \geqslant 0$

common ↳ $x(x^2 + 5x - 14) \geqslant 0$

factor more ↳ $x(x+7)(x-2) \geqslant 0$

+/- chart:

	$-\infty$ -7	-7 0	0 2	2 ∞
x	−	− ●	+	+
$x+7$	−	● +	+	+
$x-2$	−	−	−	● +
	−	$\boxed{+}$	−	$\boxed{+}$

∴ $x \in [-7, 0]$ and $[2, \infty)$

7ⓒ $\frac{2x+1}{2x-3} \geqslant \frac{x+1}{x-5}$

all on one side ↲

$\frac{2x+1}{2x-3} - \frac{(x+1)}{x-5} \geqslant 0$

LCD ↲

$\frac{(2x+1)(x-5) - (x+1)(2x-3)}{(2x-3)(x-5)} \geqslant 0$

FOIL ↲

$\frac{2x^2 - 9x - 5 - (2x^2 - x - 3)}{(2x-3)(x-5)} \geqslant 0$

simplify ↲

$\frac{-8x - 2}{(2x-3)(x-5)} \geqslant 0$

common factor ↲

$\frac{-2(4x+1)}{(2x-3)(x-5)} \geqslant 0$

+/- chart:

	$-\infty$ $-1/4$	$-1/4$ $3/2$	$3/2$ 5	5 ∞
-2	−	−	−	−
$4x+1$	− ●	+	+	+
$2x-3$	−	− ○	+	+
$x-5$	−	−	− ○	+
	$\boxed{+}$	−	$\boxed{+}$	−

∴ $x \in (-\infty, -\frac{1}{4}]$ and $(3/2, 5)$

7ⓓ $5^{2x+2} - 3126(5)^x = -125$

rewrite using exponent law

$(5^{2x})(5^2) - 3126(5^x) = -125$

let $a = 5^x$

$a^2(5^2) - 3126(a) = -125$

this is a quadratic

$25a^2 - 3126a + 125 = 0$

by quad. formula

$$a = \frac{+3126 \pm \sqrt{(-3126)^2 - 4(25)(125)}}{2(25)}$$

$$a = \frac{3126 \pm 3124}{50}$$

$a = 125$ or $a = \frac{1}{25}$

put back 5^x as a

$5^x = 125$ or $5^x = \frac{1}{25}$

$5^x = 5^3$

$5^x = 5^{-2}$

$\boxed{\therefore x = 3}$

$\boxed{\therefore x = -2}$

7ⓔ $\log_5(x+1) + \log_5(x-3) = 1$

product law

$\log_5[(x+1)(x-3)] = 1$

change the form

$(x+1)(x-3) = 5^1$

FOIL

$x^2 - 2x - 3 - 5 = 0$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4$ or $\cancel{x = -2}$

not in the domain of log.

7ⓕ $2\tan x = \sqrt{3}\tan^2 x - \sqrt{3}$

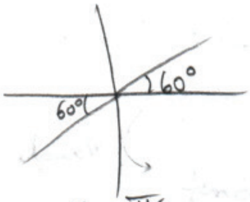
$0 = \sqrt{3}\tan^2 x - 2\tan x - \sqrt{3}$

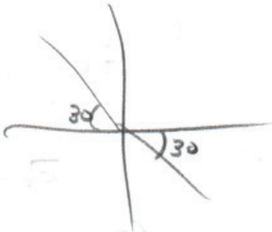
quad formula

$$\tan x = \frac{2 \pm \sqrt{16}}{2\sqrt{3}}$$

$$\tan x = \frac{2 \pm 4}{2\sqrt{3}} = \frac{1 \pm 2}{\sqrt{3}}$$

$\tan x = \frac{3}{\sqrt{3}}$ or $\tan x = \frac{-1}{\sqrt{3}}$





$x_1 = \pi/3$

$x_2 = \frac{4\pi}{3}$

$x_3 = \frac{5\pi}{6}$

$x_4 = \frac{11\pi}{6}$

7(g) $\sqrt{2x-1} + \sqrt{x+11} = 7$

the only way to get rid of square roots is to square both sides

* Note this question will not be on exam but you must know this for calculus class or for your university math classes.

$$\left(\sqrt{2x-1} + \sqrt{x+11}\right)^2 = (7)^2$$

but we then must FOIL since not monomials

$$\left(\sqrt{2x-1} + \sqrt{x+11}\right)\left(\sqrt{2x-1} + \sqrt{x+11}\right) = 49$$

FOIL

$$(2x-1) + 2\sqrt{2x-1}\sqrt{x+11} + (x+11) = 49$$

simplify

$$3x + 10 + 2\sqrt{(2x-1)(x+11)} = 49$$

isolate the radical.

$$2\sqrt{(2x-1)(x+11)} = 49 - 10 - 3x$$

$$2\sqrt{(2x-1)(x+11)} = 39 - 3x$$

square both sides to get rid of sq. root.

$$\left(2\sqrt{(2x-1)(x+11)}\right)^2 = (39-3x)^2$$

FOIL Binomials

square each factor of this monomial

$$4(2x-1)(x+11) = 1521 - 234x + 9x^2$$

$$4(2x^2 + 21x - 11) = 1521 - 234x + 9x^2$$

$$8x^2 + 84x - 44 = 1521 - 234x + 9x^2$$

$$0 = x^2 - 318x + 1565$$

$$x = \frac{+318 \pm \sqrt{(-318)^2 - 4(1)(1565)}}{2}$$

$$x = \frac{318 \pm 308}{2}$$

$\boxed{x = 313}$ or $\boxed{x = 5}$

* check if both x's work in the original.

only $x = 5$ works!!

7ⓗ $\log_9\left(\frac{9}{5}x\right) = \log_9\left(\frac{63}{10}\right) + \log_9(2)$

product law

$\log_9\left(\frac{9}{5}x\right) = \log_9\left(\frac{63}{10}\cdot 2\right)$

$\log_9\left(\frac{9}{5}x\right) = \log_9\left(\frac{63}{5}\right)$

equate inputs

(only can be done for some log bases and if one log on each side)

$\frac{9}{5}x = \frac{63}{5}$

$9x = 63$

$\boxed{x=7}$

7ⓘ $\log_5 x - 2\log_{\frac{1}{2}}(2^{-1}) = 2$

$\log_5 x - 2\log_{\frac{1}{2}}\left(\frac{1}{2}\right)^1 = 2$ neg. exponent rule

$\log_5 x - 2(1) = 2$ base of log and exponent match

$\log_5 x = 4$

change the form

$5^4 = x$

$\boxed{625 = x}$

7ⓙ $\left(\frac{1}{3}\right)^x = \sqrt[16]{81}$

exponent laws (neg. exp and rational exp.)

$3^{-x} = (81)^{1/16}$

rewrite 81 as 3^4

$3^{-x} = (3^4)^{1/16}$

power of power law.

$3^{-x} = 3^{1/4}$

equate inputs

$-x = \frac{1}{4}$

$\boxed{x = -\frac{1}{4}}$

7ⓚ $2^{x-1} = 6^x$

can't make bases match
∴ take log of both sides

$\log(2)^{(x-1)} = \log(6)^x$

power rule

$(x-1)\log 2 = x\log 6$

$x\log 2 - \log 2 = x\log 6$

$x\log 2 - x\log 6 = \log 2$

$x\log(2\div 6) = \log 2$

$x\log\frac{1}{3} = \log 2$

$x = \frac{\log 2}{\log\frac{1}{3}}$

OR $x = \log_{\frac{1}{3}}2$

7ⓛ $|2x+3| \geqslant 5$

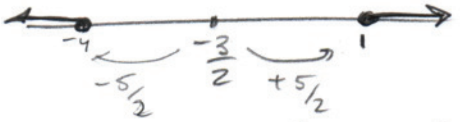
$|2||x+\frac{3}{2}| \geqslant 5$

$2|x+\frac{3}{2}| \geqslant 5$

$|x+3/2| \geqslant 5/2$

centre $-3/2$

radius



since $\geqslant$ shade outside

∴ $x \leq -4$ or $x \geqslant 1$

7ⓜ $\frac{600}{x-5} = \frac{600}{x} + 20$

LCD

$\frac{600}{x-5} = \frac{600+20x}{x}$

can cross multiply since not inequality but keep in mind that $x \neq 0,5$

OR move all to one side then LCD then ignore denominator since solutions come from zeros.

$600x = (600+20x)(x-5)$

$\cancel{600x} = \cancel{600x} + 3000 - 20x^2 + 100x$

$20x^2 - 100x - 3000 = 0$

$20(x^2 - 5x - 150) = 0$

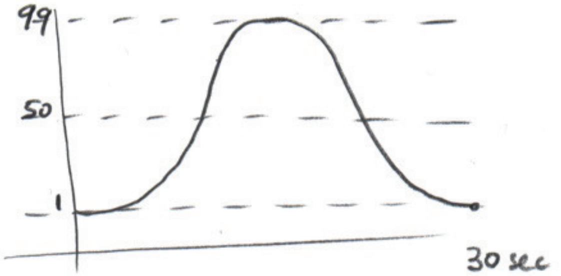
$x \quad -15$

$x \quad 10$

$20(x-15)(x+10) = 0$

$\therefore x = 15$ or $x = -10$

8



radius = 49 ∴ amplitude = 49

$\frac{2\text{ rev}}{1\text{ min}}$ means 1 cycle takes 0.5 min = 30 sec

ⓐ $y = -49\cos\left[\frac{2\pi}{30}x\right] + 50$

or

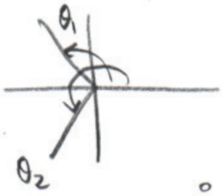
$y = 49\cos\left[\frac{\pi}{15}(x-15)\right] + 50$

or

$y = 49\sin\left[\frac{\pi}{15}(x-7.5)\right] + 50$

8ⓑ $90 = -49\cos\left[\frac{\pi}{15}x\right] + 50$

$\frac{40}{-49} = \cos\left[\frac{\pi}{15}x\right]$



$\theta = \cos^{-1}\left(-\frac{40}{49}\right)$

$\theta \doteq 2.526$

$\therefore \theta_1 \doteq 2.526$

and $\theta_2 = 2\pi - 2.526$

$\theta_2 \doteq 3.757$

but need x's not angles

$\therefore \theta_1 = 2.526 = \frac{\pi}{15}x_1$

$\boxed{12.06 = x_1}$

and

$\theta_2 = 3.757 = \frac{\pi}{15}x_2$

$\boxed{17.94 = x_2}$

(OR can also add/subtract period to get other answers)

But these answers are for the 1st cycle as the question asks.

8ⓒ $y = -49\cos\left[\frac{\pi}{15}x\right] + 50$

sub $x = 5$ min 20 sec

$x = 320$ sec

$y = -49\cos\left[\frac{\pi}{15}\cdot 320\right] + 50$

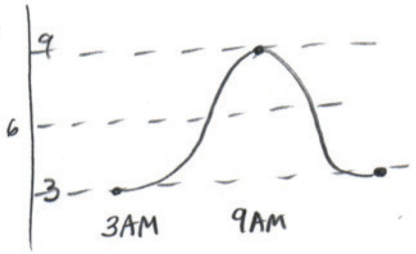
$y = 74.5$ meters

8ⓓ $\frac{\Delta y}{\Delta x} = \frac{y(10) - y(0)}{10 - 0}$

$= \frac{74.5 - 1}{10}$

$= 7.35$ m/sec

9.



period = 12 hours

ⓐ $y = 3\sin\left[\frac{2\pi}{12}(x-6)\right] + 6$

OR

$y = 3\cos\left[\frac{\pi}{6}(x-9)\right] + 6$

ⓑ sub $x = 14$ for 2 P.M.

$y = 3\cos\left[\frac{\pi}{6}(14-9)\right] + 6$

$y \doteq 3.402$ feet high

10ⓐ $x - x^3 = \frac{15}{8}$

$0 = x^3 - x + \frac{15}{8}$ (8×) } get rid of denom

$0 = 8x^3 - 8x + 15 = f(x)$

use factor theorem

$f(a) = 0$ { try for a $\pm 15, \pm 5, \pm 3, \pm 1$ over $\pm 8, \pm 4, \pm 2, \pm 1$ }

$f\left(-\frac{3}{2}\right) = 0$

$\therefore x + \frac{3}{2}$ is a factor

$$\begin{array}{r|rrrr} -1.5 & 8 & 0 & -8 & 15 \\ & \downarrow & -12 & 18 & -15 \\ \hline & 8 & -12 & 10 & 0 \end{array}$$

$\therefore f(x) = \left(x + \frac{3}{2}\right)\underbrace{(8x^2 - 12x + 10)}_{\text{try to factor more}}$

or quad. formula

$x = \frac{+12 \pm \sqrt{12^2 - 4(8)(10)}}{2(8)}$ can't

$\therefore \boxed{x = -\frac{3}{2}}$ is the only number that exceeds its cube by $\frac{15}{8}$

11 ⓐ $1200(0.5)^{w/4}$ in weeks

approx. → $\sim 1200(0.5)^{m/1}$ in months (approximately 4 weeks in one month) ↑ not always but oh well...

$1200(0.5)^{y/4/52}$ in years

or

$1200(0.5)^{13y}$

11 ⓒ sub $x = 2$days

$x = \frac{2}{7}$ weeks into weeks formula

$$M = 1200(0.5)^{2/7/4}$$

$$M = 1200(0.5)^{2/28}$$

$$M = 1200(0.5)^{1/14}$$

$$M = 1142{,}03 \text{ grams}$$

11 ⓑ sub $M = 0.5g$

$$0.5 = 1200(0.5)^{w/4}$$

$$\frac{1}{2400} = 0.5^{w/4}$$

$$\log\left(\frac{1}{2400}\right) = \log(0.5)^{w/4}$$

$$\log\frac{1}{2400} = \frac{w}{4}\log 0.5$$

$$4 \times \frac{\log\frac{1}{2400}}{\log 0.5} = w$$

$$44.9 \doteq w$$

weeks.

11 ⓓ $\frac{\Delta M}{\Delta w} = \frac{M(2) - M(0)}{2-0}$

$$\doteq \frac{848.53 - 1200}{2}$$

$= -175.7$ grams/week

* note if use a different formula you'd get:

-43.9 grams/month

-3.38 grams/year.

12. let I be initial amount (I should have given that)

$B = I(3)^{s/5}$ in seconds

$B = I(3)^{m/\frac{5}{60}}$ in minutes

or

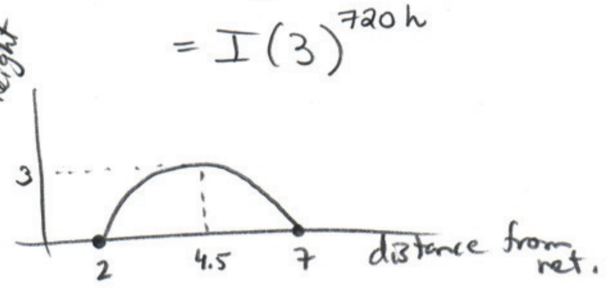
$= I(3)^{12m}$

$B = I(3)^{h/\frac{5}{3600}}$ in hours

or

$= I(3)^{720h}$

13 ⓐ



$\therefore y = a(x-2)(x-7)$ sub pt. $(4.5, 3)$

$$3 = a(4.5-2)(4.5-7)$$

$$3 = a(2.5)(-2.5)$$

$$\frac{3}{-6.25} = a$$

$$\therefore a = -0.48$$

$\therefore y = -0.48(x-2)(x-7)$

or

$y = -0.48(x-4.5)^2 + 3$

13ⓑ $\frac{\Delta y}{\Delta x} = \frac{\Delta \text{height}}{\Delta \text{distance}}$ at one point $x=5$

∴ instanteneous

$$h\to0 \quad \frac{y(5+h)-y(5)}{h}$$

use equation from ⓐ that has only one x in it for simplicity!

$$= \frac{-0.48(5+h-4.5)^2+3-2.88}{h}$$

$$= \frac{-0.48(h+0.5)^2+0.12}{h}$$

$$= \frac{-0.48(h^2+h+0.25)+0.12}{h}$$

$$= \frac{-0.48h^2-0.48h\cancel{-0.12}\cancel{+0.12}}{h}$$

$$= \frac{-0.48\cancel{h}[h+1]}{\cancel{h}}$$

$$= -0.48(h+1) \xrightarrow[\text{as } h\to0]{} -0.48(0+1) = -0.48 \text{ height/distance.}$$

14.ⓐ $(f-g)(x) = 3x-2-(x^2-6x)$

$= -x^2+9x-2$

on domain intersection of $[0,6]$ and $[-1,4]$

∴ on domain $[0,4]$

14ⓑ $(f\div g)(x) = \frac{3x-2}{x^2-6x} = \frac{3x-2}{x(x-6)}$

on domain intersection of $[0,6]$ and $[-1,4]$ AND exclude restrictions

∴ on domain $[0,4]$ but $x \neq 0, 6$

ie. $x \in (0,4]$

14 ⓒ $(f \circ g)(x) = f(g(x))$

$= f(x^2 - 6x)$

$= 3(x^2 - 6x) - 2$

$= 3x^2 - 18x - 2$

Domain is <u>NOT</u> intersection

Consider what goes on from input to output

input domain of g $[-9, 7]$

$[-1,4] \xrightarrow{g} [\cancel{-8,7}]$ → can't use this whole interval in $f(x)$

only $[0,6]$ is allowed. $\xrightarrow{f} [-2,16]$

↑ (try subbing in -1 and 4 into g and turning point of g $x = 3$ to see ABSOLUTE MIN)

∴ Domain of fog is $[-1, 4]$

Range of fog is $[-2, 16]$

ⓓ $(h+i)(x)$ can only add points with same x's

$= \{(3,9), (5,12)\}$

ⓔ $(h \circ i)(x) = h(i(x))$

(take all inputs of i, find outputs then put them into h to find new outputs)

$= \{(1,2), (2,1), (3,4)\ (4,3), (5,5)\}$

15. ⓐ $y = a(x+2)(x)^2(x-3)^3$

sub pt. $(-1,-30)$

$-30 = a(-1+2)(-1)^2(-1-3)^3$

$-30 = a(1)(1)(-4)^3$

$-30 = -64a$

$\frac{15}{32} = a$

$\therefore y = \frac{15}{32}(x+2)(x)^2(x-3)^3$

ⓒ • asymptote at $x=0$ and decreasing all the time, so this is

$\cot x$

• period usually π
here it is 8

$\therefore k = \pi/8$

$\therefore y = \cot(\pi/8\, x)$

16 ⓐ $f(x) < 0$ on

$x \in (-2,0) \quad (0,3)$

ⓑ $f(x) \geqslant -10$ on

$x \in (-\infty,-1.9] \quad [-0.6, 0.8] \quad [1.8, \infty)$

approx.

ⓒ <u>many answers</u>

ave. r.o.c. is negative on $\quad x \in (-\infty, -1.5)$

$x \in (0, 1.2)$

ⓓ instan. r.o.c is zero

at $x \doteq -1.5$ or 0 or 1.2 or 3

ⓑ • asymptote at $x=0$

so this is $\csc x$ no horizontal shift

(or $\sec x$ with horiz. shift)

• period usually 2π
here period is 4

$\therefore k = \frac{2\pi}{4} = \pi/2$

• MAX / min values usually at ±1
here at ±5

$\therefore a = 5$

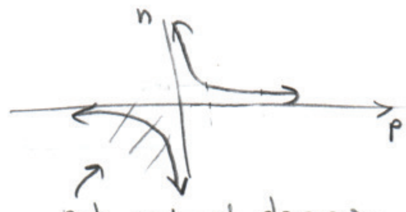
$\therefore y = 5\csc(\pi/2\, x)$

17.

	People	Lunches	Rate boxes/person
original	6	225	$\frac{225}{6} = 30$
hope to have	p	225	n

ⓐ $n = n(p) = \frac{225}{p}$

ⓑ this is a rational function with a vertical stretch



ⓒ not part of domain
can't have negative people

→ also p can't be decimals
since can't have a part of the person
but n can be since a person can put together part of the lunch.

$\therefore$ Domain $\{p \geqslant 1, p \in \mathbb{I}\}$

17 ⓓ sub $n=15$

$$15 = \frac{225}{p}$$

$$15p = 225$$

$$p = 15$$

∴ 15 people in total are needed

or 9 more people than the original 6 people who were there already.

18.

	D miles	T hrs	V mile/hr
up (against current)	30		$12-c$
down (with current)	27		$12+c$
Total		5	

$T = \frac{D}{V} = \frac{30}{12-c}$ and $\frac{27}{12+c}$

create equation using time column:

$$\frac{30}{12-c} + \frac{27}{12+c} = 5$$

LCD

$$\frac{30(12+c) + 27(12-c)}{(12-c)(12+c)} = 5$$

expand

$$\frac{360 + 30c + 324 - 27c}{144 - c^2} = \frac{5}{1}$$

cross mult.

$$3c + 684 = 720 - 5c^2$$

bring everything to one side

18. cont

$$5c^2 + 3c - 36 = 0$$

5	6	4	9	2	18	3	12
1	6	9	4	18	2	12	3

$$(5c - 12)(c + 3) = 0$$

$$\therefore c = \frac{12}{5} = 2.4 \text{ miles/hr}$$

or ~~c = -3~~ miles/hr

can't be negative current.

19.

	D km	V km/h	T hr
Ero	30	$1.4+x$	walk time see ★ stop time $\frac{1}{3}$ hr (chat) 2 hrs (wait at end for Jamal)
Jamal	30	x	$\frac{30}{x}$

★ 

walk time $T = \frac{D}{V} = \frac{30}{1.4+x}$

Create equation using time column

$$\frac{30}{1.4+x} + \frac{7}{3} = \frac{30}{x}$$

LCD

$$\frac{3(30) + 7(1.4+x)}{3(1.4+x)} = \frac{30}{x}$$

$$\frac{90 + 9.8 + 7x}{3(1.4+x)} = \frac{30}{x}$$

$$\frac{99.8 + 7x}{4.2 + 3x} = \frac{30}{x}$$

cross mult. other page

19 continued

$$99.8x + 7x^2 = 126 + 90x$$

$$7x^2 + 9.8x - 126 = 0$$

$$x = \frac{-9.8 \pm \sqrt{3624.04}}{2(7)}$$

$$x \doteq \frac{-9.8 \pm 60.2}{14}$$

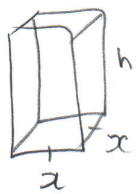
$x \doteq 3.6$ or $x \doteq \cancel{-35}$

can't be negative speed.

∴ Jamal's speed was 3.6 km/hr

and Ero's speed was 15 km/h

20.



$$V = 1000 = x^2 h$$

sub $h = \frac{1000}{x^2}$

ⓐ $\text{Cost} = C(x) = \frac{\$3}{100}(\text{Top} + \text{bottom}) + \frac{\$1.25}{100}(\text{sides})$

$$C(x) = 0.03(x^2 + x^2) + 0.0125(4xh)$$

$$C(x) = 0.06x^2 + 0.05x\left(\frac{1000}{x^2}\right)$$

$$C(x) = 0.06x^2 + \frac{50}{x}$$

ⓑ $39.50 = 0.06x^2 + \frac{50}{x}$

LCD

$$39.50 = \frac{0.06x^3 + 50}{x}$$

cross mult.

$$39.50x = 0.06x^3 + 50$$

20 cont

$$0 = 0.06x^3 - 39.50x + 50$$

mult. by 100 to get rid of decimals

$$0 = 6x^3 - 3950x + 5000$$

$$0 = 2\underbrace{(3x^3 - 1975x + 2500)}_{\text{let } f(x) \text{ equal this}}$$

$f(25) = 0$ ∴ $(x-25)$ is a factor

$$\begin{array}{r|rrrr} 25 & 3 & 0 & -1975 & 2500 \\ & \downarrow & 75 & 1875 & -2500 \\ \hline & 3 & 75 & -100 & 0 \end{array}$$

$$\therefore 0 = 2(x-25)(3x^2 + 75x - 100)$$

$x = 25$ or $x = \frac{-75 \pm \sqrt{6825}}{2(3)}$

$$x = \frac{-75 \pm 82.6}{6}$$

$x \doteq 1.27$ or $\cancel{x \doteq -26}$

∴ dimensions can be

$$\begin{cases} x = 25 \\ h = 1.6 \end{cases} \text{ or } \begin{cases} x \doteq 1.27 \\ h = 620. \end{cases}$$

21. ⓐ end behaviour

as $x \to \infty$ $y \to \infty$

as $x \to -\infty$ $y \to \infty$

Degree = 4
Lead. Coeff = 2

possible turning points

3 or 1

possible zeros

4 or 3 or 2 or 1 or none

ⓑ Degree = 5
Lead. Coeff = −2

end behaviour

as $x \to \infty$ $y \to -\infty$

as $x \to -\infty$ $y \to \infty$

possible turning points

4 or 2 or none

possible zeros

5 or 4 or 3 or 2 or 1

22. i) ⓐ exponential since 1st ratios $= \frac{\text{next}}{\text{prev}} = 2$ } true only if HA is $y=0$

ⓑ $y = ab^{x/c}$

$y = a(2)^{x/5}$

sub pt. $(-6, 0.3)$

$0.3 = a(2)^{-6/5}$

$\frac{0.3}{2^{-6/5}} = a$

$0.3(2)^{6/5} = a$

$0.69 \doteq a$

$\therefore y = 0.69(2)^{x/5}$

22. ii) ⓐ linear since 1st differences = 3

ⓑ $y = mx + b$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-2}{0.6-0.2}$

$= \frac{3}{0.4} = 7.5$

$y = 7.5x + b$

sub pt. $(0.2, 2)$

$2 = 7.5(0.2) + b$

$2 = 1.5 + b$

$0.5 = b$

$\therefore y = 7.5x + 0.5$

iii) ⓐ quadratic since 2nd differences = 2

$y = ax^2 + bx + c$ ↖ y-int $(0, 13)$

$y = ax^2 + bx + 13$

sub pt. $(1,8) \longrightarrow$ ① $8 = a + b + 13$

sub pt. $(2,5)$ ② $5 = 4a + 2b + 13$

①×2 $16 = 2a + 2b + 26$

subtract $-11 = 2a - 13$

$2 = 2a$

$\boxed{1 = a}$ → and

$8 = a + b + 13$

$8 = 1 + b + 13$

$-6 = b$

$\therefore y = 1x^2 - 6x + 13$

23. ⓐ $r = ?$

$$\omega = 80\pi \frac{rad}{min}$$

$$V = \frac{10\pi\ m}{15\ sec}$$

$$\omega = \frac{V}{r}$$

$$r = \frac{V}{\omega} = \frac{10\pi m}{\cancel{15 sec}} \times \frac{1\ \cancel{min}}{80\pi\ rad} \times \frac{60 \cancel{sec}}{1\ \cancel{min}}$$

not really there :)

$$r = \frac{600\pi}{1200\pi}\ m$$

$$r = 0.5\ m$$

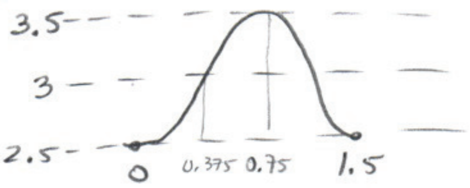
ⓑ period = ? reciprocal of ω in $\frac{rev}{sec}$

$$\omega = \frac{80\pi\ rad}{min} \times \frac{1 rev}{2\pi rad} \times \frac{1 min}{60 sec}$$

$$\omega = \frac{80\pi}{120\pi} \frac{rev}{sec} \quad \frac{2}{3}\ rev/sec$$

$\therefore$ period $= \frac{3}{2} = 1.5$ sec

ⓒ



$\therefore K = \frac{2\pi}{1.5} = \frac{4}{3}\pi$

$$y = -0.5 \cos\left[\frac{4\pi}{3}x\right] + 3$$

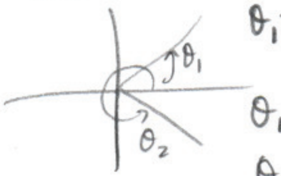
or

$$y = 0.5 \sin\left[\frac{4\pi}{3}(x - 0.375)\right] + 3$$

ⓓ $2.7 = -0.5 \cos\left[\frac{4\pi}{3}x\right] + 3$

$$-0.3 = -0.5 \cos\theta$$

$$\frac{0.3}{0.5} = \cos\theta$$



$$\theta_1 = \cos^{-1}(0.6)$$

$$\theta_1 = 53.13°$$

$\theta_1 = 0.9273$ radians

$$\therefore \frac{4\pi}{3}x_1 = 0.9273$$

$$\boxed{x_1 = 0.22\ sec}$$

$$\theta_2 = 306.87°$$

$\theta_2 = 5.3559$ radians

$$\therefore \frac{4\pi}{3}x_2 = 5.3559$$

$$\boxed{x_2 = 1.28\ sec}$$

to get others
add/subtract period = 1.5

$$\boxed{x_3 = 1.72}$$

$$\boxed{x_4 = 2.78}$$

$$\boxed{x_5 = 3.22}$$

~~$x_6 = 4.28$~~ ← too big

24. $r = 70\text{m}$

$\omega = \frac{1\text{rev}}{30\text{min}}$

ⓐ $\omega = \frac{1\,\cancel{\text{rev}}}{30\,\cancel{\text{min}}} \times \frac{2\pi\text{ rad}}{1\,\cancel{\text{rev}}} \times \frac{1\,\cancel{\text{min}}}{60\text{sec}}$

$\omega = \frac{2\pi\text{ rad}}{1800\text{ sec}}$

$\omega = \frac{\pi}{900}$ rad/sec

ⓑ $V = \omega r$

$V = \frac{\pi\text{ rad}}{900\,\cancel{\text{sec}}} \times 70\,\cancel{\text{m}} \times \frac{1\text{ km}}{1000\,\cancel{\text{m}}} \times \frac{3600\,\cancel{\text{sec}}}{1\text{ hr}}$

$V = \frac{252000\pi}{900\,000} \frac{\text{km}}{\text{h}}$

$V = \frac{7\pi}{25}$ km/h

ⓒ $a = ?$

$V = \frac{a}{t}$

$\therefore a = vt$

$a = \frac{7\pi\text{ km}}{25\,\cancel{\text{hr}}} \times 160\,\cancel{\text{min}} \times \frac{1\,\cancel{\text{hr}}}{60\,\cancel{\text{min}}}$

$a = \frac{1120\pi}{1500}$ km

$a = \frac{56\pi}{75}$ km

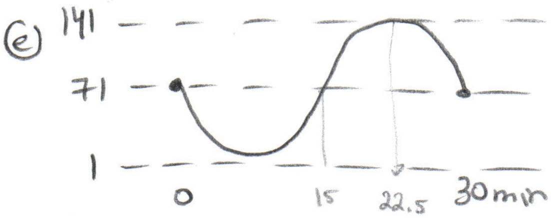
ⓓ period is reciprocal of ω in $\frac{\text{rev}}{\text{sec}}$

$\omega = \frac{1\text{rev}}{30\text{ min}}$ ∴ period = 30 min

$y \doteq 70\sin\left[\frac{\pi}{15}(x+23.9)\right]+67$

is one version of the equation.

ⓔ

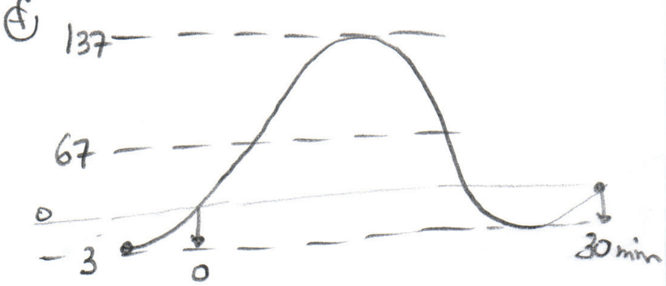


$y = 70\cos\left[\frac{2\pi}{30}(x-22.5)\right]+71$

or

$y = -70\cos\left[\frac{\pi}{15}(x-7.5)\right]+71$

ⓕ

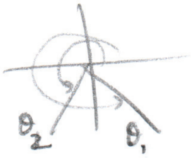


$y = 70\sin\left[\frac{\pi}{15}(x-d)\right]+67$

sub pt (0,0) to find d

$0 = 70\sin\left[\frac{\pi}{15}(0-d)\right]+67$

$\frac{-67}{70} = \sin\theta$



$\theta_1 = \sin^{-1}\left(\frac{-67}{70}\right)$

$\theta_1 = -73.17° = 286.8° = 5.0062$ radians

$\therefore \frac{\pi}{15}(-d) = 5.0062$

$\boxed{d_1 \doteq -23.9}$

OR $\theta_2 =$ ~~106.8~~° $=$ ~~[illegible]~~ radians and $\boxed{d_2 = \text{~~[illegible]~~}}$

253.2° 4.4186 -21.1

#25. ⓐ a.r.o.c = $\frac{f(4)-f(-2)}{4--2} = \frac{257-17}{6} = \frac{240}{6} = 40$

ⓑ a.r.o.c = $\frac{f(x+h)-f(x)}{h} = \frac{\left[(x+h)^4+1\right]-\left[x^4+1\right]}{h}$

$$= \frac{\cancel{x^4}+4x^3h+6x^2h^2+4xh^3+h^4+\cancel{1}-\cancel{x^4}-\cancel{1}}{h}$$

$$= \frac{\cancel{h}\left[4x^3+6x^2h+4xh^2+h^3\right]}{\cancel{h}}$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

$\xrightarrow[\text{as } h\to 0]{}$ i.r.o.c = $4x^3+6x^2(0)+4x(0)^2+0^3$

$= 4x^3$

∴ i.r.o.c at $x=3$

$= 4(3)^3$

$= 108.$

ANS review #26

January-09-13
8:34 AM

Find domain algebraically, then sketch and find range graphically

$a(x)=\sqrt{2x-5}+3$

radicand ≥ 0

$2x-5\geq 0$

$2x\geq 5$

$x\geq \frac{5}{2}$

$\therefore D=\{x\in\mathbb{R}, x\geq 5/2\}$

or $x\in[5/2,\infty)$

sketch:

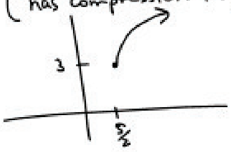
$\sqrt{2(x-5/2)}+3$

parent $y=\sqrt{x}$

shift right 5/2

up 3

(no reflections has compression too)



$\therefore R=\{y\in\mathbb{R}, y\geq 3\}$

or $y\in[3,\infty)$

$b(x)=3\log_5(14-7x)-4$

log input >0

$14-7x>0$

$-7x>-14$

$x<2$

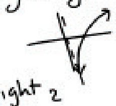
$\therefore D=\{x\in\mathbb{R}, x<2\}$

or $x\in(-\infty,2)$

sketch

$3\log_5(-7(x-2))-4$

parent $y=\log_5 x$

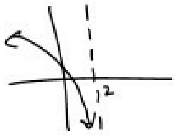


shift right 2

down 4

reflected in y-axis

(with stretch/compress but will not show these)



$\therefore R=\{y\in\mathbb{R}\}$

or $y\in(-\infty,\infty)$

$c(x)=\frac{x-11}{2x^2-15x-8}$

denom $\neq 0$

$2x^2-15x-8\neq 0$

$(2x+1)(x-8)\neq 0$

$\therefore D=\{x\in\mathbb{R}, x\neq -\frac{1}{2}, 8\}$

or $x\in(-\infty,-\frac{1}{2}),(-\frac{1}{2},8),(8,\infty)$

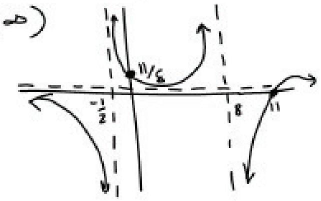
sketch:

VA $x=-\frac{1}{2}, 8$

HA $y=0$

zero $x=11$

y-int $y=11/8$



$\therefore R=\{y\in\mathbb{R}\}$

or $y\in(-\infty,\infty)$

$d(x)=2x^2-8x-10$

no restrictions

$\therefore D=\{x\in\mathbb{R}\}$

or $x\in(-\infty,\infty)$

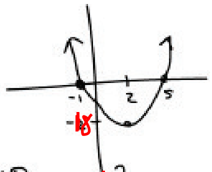
sketch:

$2(x^2-4x\quad)-10$

$2(x^2-4x+4-4)-10$

$2(x-2)^2-4(2)-10$

$2(x-2)^2-18$



$\therefore R=\{y\in\mathbb{R}, y\geq -18\}$

or $y\in[-18,\infty)$

Find domain algebraically, then sketch and find range graphically

$e(x)=-2(3.5)^x-6$

exponential, no restrictions

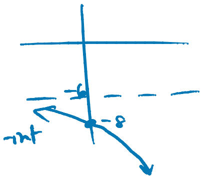
$D=\{x\in\mathbb{R}\}$

sketch:

parent $y=3.5^x$ growth

$a=-2$ reflected + distance HA to y-int

$c=-6$ HA.



$\therefore R=\{y\in\mathbb{R}, y<-6\}$

$f(x)=4-\sqrt{5-10x}$

radicand ≥ 0

$5-10x\geq 0$

$-10x\geq -5$

$x\leq 0.5$

$\therefore D=\{x\in\mathbb{R},\ x\leq 0.5\}$

or $x\in(-\infty,0.5]$

sketch

parent $y=\sqrt{x}$

$-\sqrt{-10(x-0.5)}+4$

reflected twice

compressed

right + up

$\therefore R=\{y\in\mathbb{R},\ y\leq 4\}$

or $y\in(-\infty,4]$

$g(x)=-2|x+5|$

abs.val, no restrictions

$D=\{x\in\mathbb{R}\}$

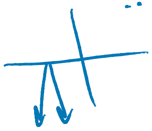
sketch:

parent $y=|x|$

reflected

stretched

left



$\therefore R=\{y\in\mathbb{R}, y\leq 0\}$

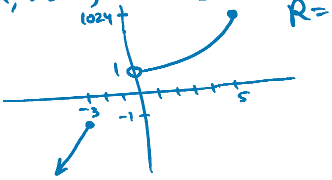
or $y\in(-\infty,0]$

$h(x)=\begin{cases}x+2, x\leq -3\\ 4^x, 0<x\leq 5\end{cases}$

piecewise domain is

$D=\{x\in\mathbb{R},\ x\leq -3,\ 0<x\leq 5\}$

sketch:



$R=y\in(-\infty,-1]$

$(1,1024]$

$i(x)=4\log_{0.5}(2x-1)$

log input >0

$2x-1>0$

$2x>1$

$x>\frac{1}{2}$

$\therefore D=x\in(\frac{1}{2},\infty)$

sketch:

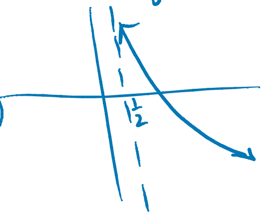
parent $y=\log_{0.5}x$ decay

$y=4\log_{0.5}(2(x-\frac{1}{2}))$

stretched + compressed

no reflection

right



$\therefore R=y\in(-\infty,\infty)$

$j(x)=\dfrac{3x^2}{2x^3+3x^2-18x+8}$

denom $\neq 0$

$f=2x^3+3x^2-18x+8\neq 0$

$\pm\frac{p}{q}=\pm\frac{8}{2},\frac{8}{1},\frac{4}{2},\frac{4}{1},\frac{2}{2},\frac{2}{1},\frac{1}{2},\frac{1}{1}$

$f(2)=0$ $\therefore (x-2)$ is a factor

$$\begin{array}{r|rrrr} 2 & 2 & 3 & -18 & 8 \\ & \downarrow & 4 & 14 & -8 \\ \hline & 2 & 7 & -4 & 0 \end{array}$$

$(x-2)(2x^2+7x-4)\neq 0$

$(x-2)(2x-1)(x+4)\neq 0$

$D=\{x\in\mathbb{R}, x\neq 2,\frac{1}{2},-4\}$

sketch:

$\dfrac{3x^2}{(x-2)(2x-1)(x+4)}$

VA $x=2,\frac{1}{2},-4$

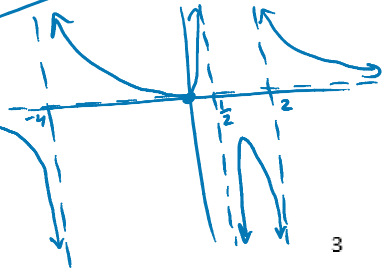
HA $y=0$

zero $x=0$

y-int $y=0$

HA gets crossed

$\therefore R=\{y\in\mathbb{R}\}$



Find domain algebraically, then sketch and find range graphically

$k(x)=\frac{2}{5}x+6$

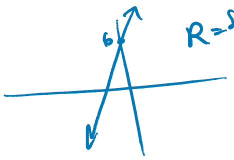
linear, no restrictions

$D=\{x\in\mathbb{R}\}$

sketch:

$m=2/5$

$b=6$



$R=\{y\in\mathbb{R}\}$

$l(x)=\sqrt[3]{2x+6}-8$

cube root, no restrictions

$D=\{x\in\mathbb{R}\}$

sketch:

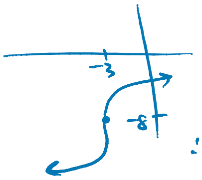
parent $y=\sqrt[3]{x}$

$y=\sqrt[3]{2(x+3)}-8$

compressed

left

down



$\therefore R=\{y\in\mathbb{R}\}$

$m(x)=\frac{4}{x^2+1}$

denom $\neq 0$

$x^2+1\neq 0$

can't factor

$\therefore$ no restrictions

$D=\{x\in\mathbb{R}\}$

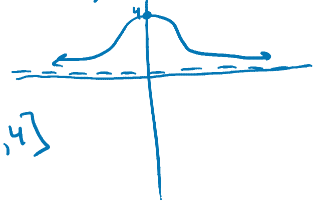
sketch:

no VA

HA $y=0$

zero none

y-int $y=4$



$R=y\in(0,4]$

$n(x)=-2\sin(\pi(x+4))+9$

sinusoidal, no restrictions

$D=\{x\in\mathbb{R}\}$

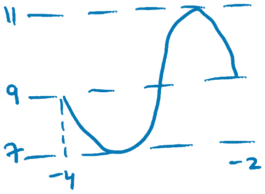
sketch:

$a=-2$

$c=9$

$\text{period}=\frac{2\pi}{\pi}=2$

$d=-4$



$\therefore R=y\in[7,11]$

$o(x)=\tan 0.25x+1$

has VA's !

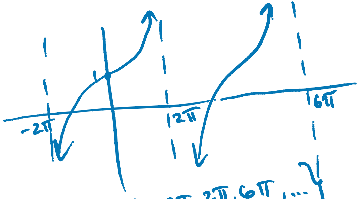
parent $y=\tan x$

period $=\pi$

horiz. stretch

$\therefore$ new period $=4\pi$

shift up 1



$D=\{x\in\mathbb{R},\ x\neq -2\pi, 2\pi, 6\pi, \ldots\}$

$R=\{y\in\mathbb{R}\}$

$p(x)=x^3(x-5)^2(x+1)$

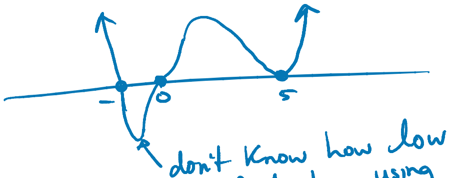
degree 6 polynomial, no restrictions

$D=\{x\in\mathbb{R}\}$

sketch: bend at $x=0$

bounce at $x=5$

cut at $x=-1$



don't know how low this is

can find t.p. using i.r.o.c.

or derivatives from calculus.

$\therefore R=\{y\in\mathbb{R},\ y\geq \text{Abs. Min}\}$

27. All customers pay a monthly customer fee of $8.91, plus a fee of 10.49 ¢ per kilowatt hour (kWhr) for the first 400 kWhr supplied in the month, plus a fee of 7.91¢ per kWhr for all usage over 400 kWhr. Write the piecewise equation for this relation.

$$C(x) = \begin{cases} 0.1049x + 8.91 & , 0 < x \le 400 \\ 0.0791(x-400) + 50.87 & , 400 < x \end{cases}$$

in dollars

28. A restaurant patron has decided to leave a 15% tip for meals costing up to $40, an 18% tip for meals costing at least $40 but less than $100, and a 20% tip for meals costing $100 or more. Write a piecewise function to describe the total amount, T, the patron will pay in terms of the meal cost c

$$T(c) = \begin{cases} 1.15c & , 0 \le c < 40 \\ 1.18(c-40)+46 & , 40 \le c < 100 \\ 1.20(c-100)+116.8 & , 100 \le c \end{cases}$$

(29.) The pH of water in a small lake in northern Quebec has dropped from 5.4 to 4.8 in the last three years. How many times as acidic as it was three years ago, is the lake now?

$$pH = -\log[H^+]$$

$$pH_2 - pH_1 = -\log\left[\frac{H_2}{H_1}\right]$$

$$4.8 - 5.4 = -\log\left[\frac{H_2}{H_1}\right]$$

$$-0.6 = -\log\left[\frac{H_2}{H_1}\right]$$

$$0.6 = \log\left[\frac{H_2}{H_1}\right] \longleftrightarrow 10^{0.6} = \frac{H_2}{H_1}$$

∴ about 4 times as acidic

(30.) Anna can scream at 56 db and Billy can yell at 48 db. How many more times intense is Anna's scream than Billy's yell?

Anna's $L = 56$ db

$$L = 10 \cdot \log\left(\frac{I_a}{10^{-12}}\right)$$

$$56 = 10 \cdot \log\left(\frac{I_a}{10^{-12}}\right)$$

$$5.6 = \log\left(\frac{I_a}{10^{-12}}\right)$$

$$10^{5.6} = \frac{I_a}{10^{-12}}$$

$$10^{5.6} \cdot 10^{-12} = I_a$$

$$10^{-6.4} = I_a$$ {Larger Intensity

Billy's $L = 48$ db

$$L = 10 \cdot \log\left(\frac{I_b}{10^{-12}}\right)$$

$$48 = 10 \cdot \log\left(\frac{I_b}{10^{-12}}\right)$$

$$4.8 = \log\left(\frac{I_b}{10^{-12}}\right)$$

$$10^{4.8} = \frac{I_b}{10^{-12}}$$

$$10^{4.8} \cdot 10^{-12} = I_b$$

$$10^{-7.2} = I_b$$

$$X \cdot I_b = I_a$$

$$X = \frac{I_a}{I_b} = \frac{10^{-6.4}}{10^{-7.2}} = 10^{.8} \approx 6.3$$

Anna's scream is 6.3 times as intense as billy's.