Review Gr.9-11 = Unit #0 Journal

1.(a) FACTORING METHODS

COMMON - divide out the greatest common factor (G.C.F) out of all terms and place the GCF outside of the bracket.

ex. $\frac{16x^3y}{4x^2y} - \frac{4x^2y^2}{4x^2y} + \frac{20x^4y^3}{4x^2y}$ G.C.F $= 4x^2y$

* if negative is 1st pull it out.

$= 4x^2y(4x - y + 5x^2y^2)$

DIFFERENCE of SQUARES

- You must have ① two terms ② one negative ③ even powers on variables only
- Create brackets $(\ +\ )(\ -\ )$ and square root each term

ex. $144m^2 - 49n^2$
$= (12m + 7n)(12m - 7n)$

ex. $x^8 - 6$
$= (x^4 + \sqrt{6})(x^4 - \sqrt{6})$

TRINOMIAL FACTORING (Mrs.K's favorite method)

ex. $9x^2 - 24x + 16$

$1x \quad 3x$
$9x \quad 3x$

2 8 -4 16 1
8 2 -4 1 16
or two neg.

$= (3x-4)(3x-4)$

• Must have variable pattern as follows

first	middle	last
x^2	x	
x^2	xy	y^2
x^4	x^2y^3	y^6 etc

* even powers on 1st and last and ½ of each power in middle

ex. $2x^2 + 3x - 20$

can drop variables in rough work

2
1

4 5 10 2 20 1
5 4 2 10 1 20
one neg.

$= (2x-5)(x+4)$

• List factor combinations for 1st and last terms

Positive #	Negative #
+ −	− +
+ −	+ −

ex. $4x^6 - 27x^3y + 35y^2$

4 2
1 2

7 5 1 35
-5 7 35 1
or two neg

$= (4x^3-7)(x^3-5)$

• Criss-cross multiply
Is there a combination that adds to middle?

$a \quad b$
$c \quad d$

• Record $(a+b)(c+d)$
(top with top)(bottom with bottom)

*check work by expanding
*check if can factor more (remember common factor 1st to make #'s smaller)

1(b) EXPANDING METHODS

PASCAL'S Δ

- helps with expanding BINOMIALS
- start and end row with ONE
- add two #'s above for other entries

$$
\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}
$$

ex. $(x-3)^5$

- Since power is 5, look at row with 5 in it for the coefficients of terms

$= 1(\ )^5(\ )^0 + 5(\ )^4(\ )^1 + 10(\ )^3(\ )^2 + 10(\ )^2(\ )^3 + 5(\ )^1(\ )^4 + 1(\ )^0(\ )^5$

- decrease powers of 1st term
- increase powers of 1st term

$= 1(x)^5\cancel{(-3)^0} + 5(x)^4(-3)^1 + 10(x)^3(-3)^2 + 10(x)^2(-3)^3 + 5(x)^1(-3)^4 + 1\cancel{(x)^0}(-3)^5$

- place terms inside brackets and simplify

$= x^5 - 15x^4 + 90x^3 - 270x^2 + 405x - 243$

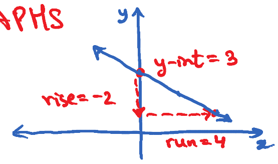
2 LINEAR RELATIONS

a) EQUATION with a slope $y = mx + b$ (slope m, y-int $(0,b)$)

horizontal $y = \#$

vertical $x = \#$

b) GRAPHS



$y = mx + b$

$y = -\frac{2}{4}x + 3$

$y = -\frac{1}{2}x + 3$

$\boxed{m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}}$

c) WORD PROBLEMS

Look for • initial value or flat rate number = y-int = b

• rate of change number = slope = m

ex. Pizza costs \$6.00 plus \$0.50 per topping.

flat = b   rate per topping = m

let C = cost

t = # of toppings

$y = mx + b$

$\therefore C(t) = 0.50t + 6.00$

d) SYSTEMS: SUBSTITUTION METHOD

- isolate one variable in one equation
- sub in other equation
- expand + solve for one variable
- sub in again + solve for other variable

ex. $x + 7y = 0$ ①

$2x - 8y = 22$ ②

①: $x = -7y$ sub in ②

②: $2(-7y) - 8y = 22$

$-14y - 8y = 22$

$-22y = 22$

$y = -1$

sub in ① $x = -7(-1)$

$x = 7$

$\therefore$ POI $(7, -1)$

ELIMINATION METHOD

- align like variables over top of each other (equals sign too)
- multiply the whole equation(s) to create the same coefficient on one variable that variable
- add/subtract to eliminate + proceed as before
- solve for the remaining variable

ex. ① $-17 - 5y - 11x = 0$

② $-15 = 9x + 4y$

① $-17 = 11x + 5y$

② x5: $-75 = 45x + 20y$

① x4: $-68 = 44x + 20y$

subtract $-7 = x$

sub in ② $-15 = 9(-7) + 4y$

$-15 = -63 + 4y$

$48 = 4y$

$12 = y$

$\therefore$ POI is $(-7, 12)$

3. QUADRATIC RELATIONS

a) EQUATIONS

Standard $y = ax^2 + bx + c$

direction of opening (a)

y-int $(0, c)$

Factored $y = a(x - r)(x - t)$

dir. (a)

x-int or zeros $(r, 0)$ and $(t, 0)$

* think, what # for x makes whole bracket zero

ex. $y = (2x - 3)(4x + 8)$

zeros $\left(\frac{3}{2}, 0\right)$ and $(-2, 0)$

Vertex $y = a(x - h)^2 + k$

dir (a)

vertex (h, k)

* this form also tells you transformations

$|a| > 1$ vertical stretch ex. $a = -2$ or $a = 2$

$|a| < 1$ vertical compression ex. $a = 0.3$ or $a = -0.3$

$h > 0$ right, $h < 0$ left (switch sign!!)

$k > 0$ up, $k < 0$ down

b) QUADRATIC FORMULA

used to find zeros of standard form of quadratic

must have equals to zero $0 = ax^2 + bx + c$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

* resort to this formula only if you can't factor to see zeros.

c) COMPLETING the SQUARE

- used to convert standard form to vertex form (when asked for tranformations, inverse or MAX/MIN)

ex. $y = -3x^2 + 4x + 1$

$y = -3\left(x^2 - \frac{4}{3}x \qquad\right) + 1$

- divide out "a" from first two terms (not common factor) x stays!

- find special # $\left(\frac{b}{2}\right)^2$

$\left(-\frac{4}{3} \div 2\right)^2 = \left(-\frac{4}{3} \times \frac{1}{2}\right)^2 = \left(-\frac{4}{6}\right)^2 = \left(-\frac{2}{3}\right)^2 = \frac{4}{9}$

- add and subtract this # in the bracket

$y = -3\left(x^2 - \frac{4}{3}x + \frac{4}{9} - \frac{4}{9}\right) + 1$

- move last term outside of bracket by multiplying by "a"

$y = -3\left(x^2 - \frac{4}{3}x + \frac{4}{9}\right) - \frac{4}{9}(-3) + 1$

will always be # you before square

- factor trinomial left + simplify

$y = -3\left(x - \frac{2}{3}\right)^2 + \frac{7}{3}$

opens ↷

$\therefore$ vertex $\left(\frac{2}{3}, \frac{7}{3}\right)$

$\therefore$ MAX val $= \frac{7}{3}$ at $x = \frac{2}{3}$ axis of symmetry

★ What makes this problem difficult?

- You need to remember the steps and be able to work with fractions
- There are many steps and a small mistake can lead to a wrong answer
- You must understand the steps + how to factor instead of memorizing the steps

d) GRAPHS

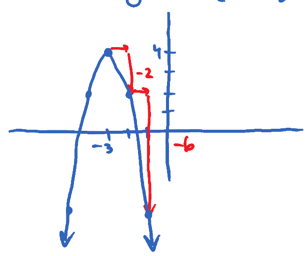
- use of vertex and step pattern is the fastest way of graphing vertex form

ex. $y = -2(x+3)^2 + 4$ vertex $(-3,4)$

step $(1,3,5) \times a$

$= -2, -6, -10$

think of this as $\frac{\text{rise}}{\text{run}}$

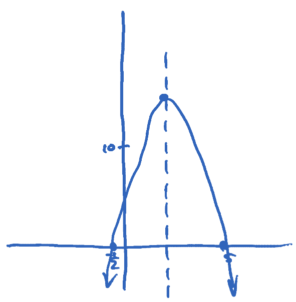


- use zeros and vertex for factored form

ex. $y = -(x-5)(2x+1)$

zeros $(5,0)$ and $(-\frac{1}{2},0)$

$\boxed{\text{a.o.s} = \frac{\text{add zeros}}{2}}$ $\boxed{\text{opt. val} = \text{sub a.o.s into equation}}$



$\text{a.o.s} = \frac{5 + -0.5}{2} = 2.25$

$\text{opt. val} = -(2.25-5)(2(2.25)+1)$

$= -(-2.75)(5.5)$

$= 15.125$ $\therefore$ vertex $(2.25, 15.125)$

4. EXPONENTIAL RELATIONS

a) EXPONENT RULES

$a^n \cdot a^m = a^{n+m}$

$\frac{a^n}{a^m} = a^{n-m}$ * record answer in numerator!

$(a^n b^m)^k = a^{nk} b^{mk}$ * allowed to distribute for MONOMIALS only

$\frac{a b^{-n} c^m}{d^{-k}} = \frac{a c^m d^k}{b^n}$ * move only the bases with negative exponents over the division line

$a^0 = 1$

$a^{n/m} = \sqrt[m]{a^n}$

ex. $\frac{(2x^0 y^2)^{-3} \cdot 2yx^3}{(2x^{-3})^2}$

power of power

$= \frac{2^{-3} x^0 y^{-6} \cdot 2^1 y x^3}{2^2 x^{-6}}$

mult/divide same bases can add/subtract exponents

$= 2^{-3+1-2} x^{0+3--6} y^{-6+1}$

$= 2^{-4} x^9 y^{-5}$

$= \frac{x^9}{2^4 y^5}$ negative exponent bases drop "downstairs"

$= \frac{x^9}{16y^5}$

* if m is even, a^n must be positive or will be undefined
∴ create restriction on variables

ex. $\sqrt[4]{81m^5}$

$= 3m^{5/4}$, $m>0$

* if m and n are both even and there is a variable in the radicand → use abs. val. if simplified power is odd to ensure the result is positive.

ex. $\sqrt{64x^3y^6z^4}$

$= 8x^{3/2}|y|^3z^2$, $x>0$

* when to use ± ?

→ if square root already written down then the sign is what's in front

ex. $-2\sqrt{x}$ or $\sqrt{8}$

→ if you must take sq. root to get the variable alone then use ±

ex. $\sqrt{x^2} = \sqrt{4}$
$x = \pm 2$

b) SOLVING FOR EXPONENT

* if $a^x = a^y$, $a \neq 1$
then $x = y$

this allows us to solve for exponent provided you can match bases.

* if $a^x = b^x$, $x \neq 0$
then $a = b$

ex. $\left(\frac{1}{6}\right)^{3x+2} \cdot 216^{3x} = \frac{1}{216}$

change to base 6 → $6^{-1(3x+2)} \cdot (6^3)^{3x} = 6^{-3}$

$\boxed{6^{-3x-2} \cdot 6^{9x}} = 6^{-3}$ add

$6^{6x-2} = 6^{-3}$ using above rule.

$\therefore 6x - 2 = -3$

$6x = -1$

$x = \frac{-1}{6}$

ex. $\boxed{3^{x+3}} - 3^x = 234$

mult. bases $\boxed{3^x \cdot 3^3} - 3^x = 234$ common factor

$3^x[27-1] = 234$

$3^x = 9$

$3^x = 3^2$ rule

$\therefore x = 2$

ex. $8(5^{\boxed{2x}}) + 8(5^x) = 6$ power of power

$8(5^{\boxed{x}})^2 + 8(5^x) = 6$

let $a = 5^x$ replace

$8a^2 + 8a - 6 = 0$

$2(4a^2 + 4a - 3) = 0$

$\left(\frac{2}{2}\right)^4_1 \quad \left(\frac{-1}{3}\right)^3_1$

$2(2a-1)(2a+3) = 0$

$a = \frac{1}{2}$ or $a = \frac{-3}{2}$

replace back

$5^x = \frac{1}{2}$ or $5^x = \frac{-3}{2}$

trial + error

$x \doteq -0.43$

N/A.

OR using log/ln

$x = \frac{\log(\frac{1}{2})}{\log(5)}$

$x = \frac{\ln(\frac{1}{2})}{\ln(5)}$

c) TRANSFORMED EQUATION
(with minimal # of constants used)

$y = ab^x + c$ — will change with reflections

parent $y = b^x$ — Growth $b>1$, Decay $b<1$

a - vertical stretch/compress
- reflect in x-axis

c - shift up/down

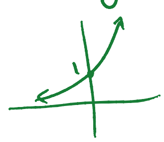
Horizontal Asympote $= c$

ex. $y = -5(4)^{\frac{x-1}{2}} + 3$
can be rewritten as
$y = -5\left((4)^{1/2}\right)^{x-1} + 3$
$y = -5(\sqrt{4})^{x-1} + 3$
$y = -5(2)^{x-1} + 3$
$y = -5(2)^x(2)^{-1} + 3$
$y = -5(2)^{-1}(2)^x + 3$
$y = -5\left(\frac{1}{2}\right)(2)^x + 3$
$y = -\frac{5}{2}(2)^x + 3$

$\therefore$ usual transformed form $y = af[k(x-d)] + c$
can always be reduced to
$y = a f(x) + c$
but only if f is exponential

d) GRAPHING

parent $y = 2^x$

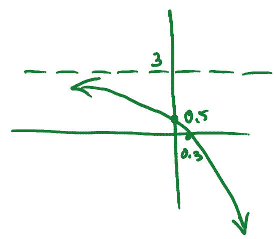


$a = -\frac{5}{2}$ → reflected in x-axis
→ vertically stretched

$c = 3$ → shift up

y-int $y = -\frac{5}{2}(2)^0 + 3 = 0.5$

x-int $0 = -\frac{5}{2}(2)^x + 3$
$-3 = -\frac{5}{2}(2)^x$
$1.2 = 2^x$
by trial + error
$x = 0.3$



e) EQUATION for Word Problems $y = ab^{\frac{x}{p}}$

Growth if $b>1$
Decay if $b<1$

% increase word prob: $b = 1 + \text{rate}$
% decrease word prob: $b = 1 - \text{rate}$
double: $b = 2$
half life: $b = \frac{1}{2}$ ... etc

a - initial value
y - final value
b - growth/decay factor
p - how long the growth/decay period is
$\frac{x}{p}$ = whole exponent = # of growth/decay periods
x - independent variable usually time

ex. Half life of a radioactive substance is 50 hours
Initially there was 300g of radioactive material
How much is left after 1 week?

let M = mass
h = hours

$M(t) = 300\left(\frac{1}{2}\right)^{h/50}$ ← function notation

1 week = 7 days × 24 hours
= 168 hours

$M(168) = 300\left(\frac{1}{2}\right)^{\frac{168}{50}} = 29\text{g}$

5. SINUSOIDAL RELATIONS – wavelike functions (sine or cosine)

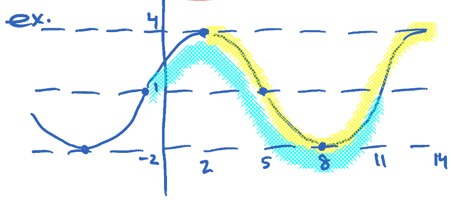
a) EQUATION from GRAPH

$y = a\sin[k(x-d)] + c$

or

$y = a\cos[k(x-d)] + c$

ex.



To find k

- find period = last point of cycle minus 1st pt

$= 14 - 2 = 12$

or $= 11 - (-1) = 12$

- $\boxed{k = \frac{360°}{\text{period}}} = \frac{360°}{12} = 30°$

To find |a| = amplitude (if positive)

$a = \text{MAX} - \text{axis} = 4 - 1 = 3$

or

$a = \text{axis} - \text{MIN} = 1 - (-2) = 3$

or $\boxed{a = \frac{\text{MAX} - \text{MIN}}{2}}$

To find c = axis $\boxed{c = \frac{\text{MAX} + \text{MIN}}{2}}$

$c = 1$

To find d = phase shift

look at the 1st point of the cycle (Horizontal value!)

Sine starts at axis and goes up → for sine $d = -1$

unless there are reflections

cosine starts at MAX → for cosine $d = 2$

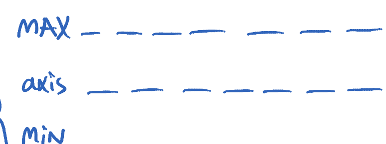
b) GRAPH from EQUATION

ex $y = -7\sin[15x - 165] + 3$

- Draw 3 horizontal lines

label vertical #'s

(don't worry about x + y axes)



$c = \text{axis}$

$|a| = \text{amplitude}$

MAX = axis + amplitude

positive "a"

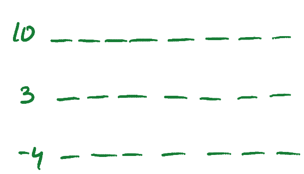
MIN = axis – amplitude

$|a| = 7$

$c = 3$

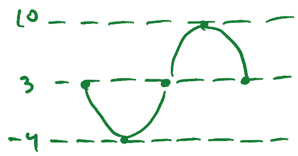
$\text{MAX} = 3 + 7 = 10$

$\text{MIN} = 3 - 7 = -4$



- Draw the shape of one cycle

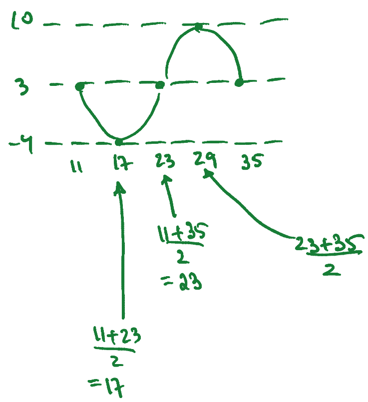
sine no reflections | sine with a reflection | cosine no reflections | cosine with a reflection



- Label the 5 points horizontally

$\boxed{\begin{array}{l} 1^{st} \text{ point} = d \quad \text{factor } k \text{ out!} \\ \text{last point} = d + \text{period} \end{array}}$ $\boxed{\text{period} = \frac{360°}{k}}$

For other points $\frac{\text{add two points}}{2}$ to find where the middle is



$y = -7\sin[15x - 165] + 3$

$y = -7\sin[15(x-11)] + 3$

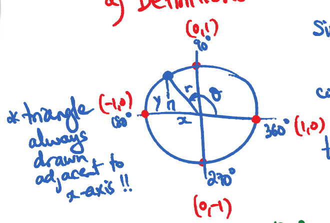
$k = \frac{360°}{15} = 24°$

$\therefore$ 1^{st} pt $= d = 11$

last pt $= d + \text{period} = 11 + 24 = 35$

6. UNIT CIRCLE

a) Definitions

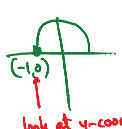


$\sin\theta = \frac{y}{r}$ or y-coordinate on unit circle (if r=1)

$\cos\theta = \frac{x}{r}$ or x-coordinate on unit circle

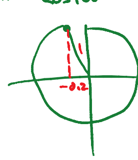
$\tan\theta = \frac{y}{x}$ or $\frac{\text{rise}}{\text{run}}$ = slope of the terminal arm

ex. $\sin 180° = 0$



look at y-coordinate of the point rotated 180° from positive x-axis

ex. $\cos 100° \doteq \frac{-0.2}{1}$

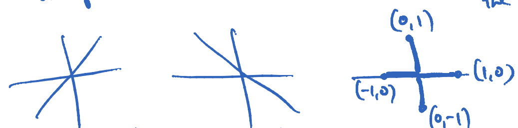


- use a calculator for non special angles
- x-coordinate is negative on the left side

b) Find ANGLES

- Draw a picture

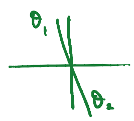
sine positive on top — sine negative on bottom — cosine positive on the right — cosine negative on the left



tangent positive for increasing slopes — tangent negative for decreasing slopes — terminal arm falls on axes for some ratios!!

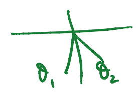
- Use calculator to find one answer (add period 360° if it's negative)

ex.1 $\tan\theta = -5$



$\theta = \tan^{-1}(-5)$
$\theta = -79° \quad +360°$
$\therefore \theta_2 = 281°$

ex.2 $\sin\theta = -0.7$



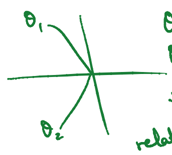
$\theta = \sin^{-1}(-0.7)$
$\theta = -44° \quad +360°$
$\therefore \theta_2 = 316°$

- Find the related acute angle OR use symmetry to find another angle

ex.1 related acute $= 79°$
$\therefore \theta_1 = 180 - 79°$
$\theta_1 = 101°$

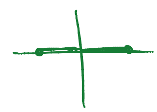
ex.2 related acute $= 44°$
$\therefore \theta_1 = 180 + 44$
$\theta_1 = 224°$

ex.3 $\cos\theta = -0.3$



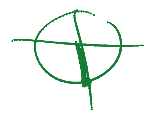
$\theta = \cos^{-1}(-0.3)$
$\theta = 107°$
$\therefore \theta_1 = 107$
related acute $= 73°$
$\therefore \theta_2 = 180° + 73°$
$\theta_2 = 253°$

ex.4 $\tan\theta = 0$ = slope when line horizontal



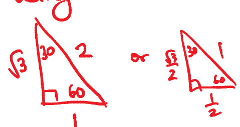
$\theta_1 = 0°, 360°$
$\theta_2 = 180°$

ex5 $\sin\theta = -1$ = y-coordinate at the very bottom

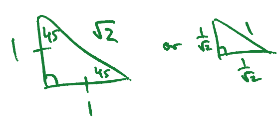


$\theta = 270°$

c) Using SPECIAL TRIANGLES

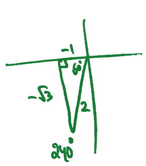


can be any size!!



Find RATIO Values

ex. $\sin 240°$

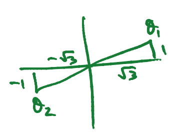


- Draw angle with special △ adjacent to x-axis
- label sides
- include any negatives
- use definition

$= \frac{y}{r}$
$= \frac{-\sqrt{3}}{2}$

Find ANGLE Values

ex. $\tan\theta = \frac{\sqrt{3}}{3}$ ← recognize this is the same as $\frac{1}{\sqrt{3}}$ or $\frac{-1}{-\sqrt{3}}$



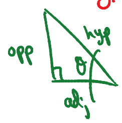
- find related acute + state angles

$\theta_1 = 30°$
$\theta_2 = 210°$

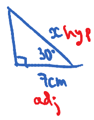
d) SOH CAH TOA *only used on Right △

$\sin\theta = \frac{opp}{hyp}$ $\quad \cos\theta = \frac{adj}{hyp}$ $\quad \tan\theta = \frac{opp}{adj}$

hyp hyp adj



ex. solve for side



$\cos 30^\circ = \frac{7}{x}$ • CAH

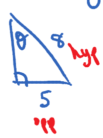
$x\cos 30 = 7$ • cross multiply

$x = \frac{7}{\cos 30^\circ}$ • solve

$x \doteq 8.1\text{ cm}$

make sure you're in DEGREE mode!!

ex. solve for angle



$\sin\theta = \frac{5}{8}$ • SOH

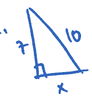
$\theta = \sin^{-1}\left(\frac{5}{8}\right)$ • inverse sine

$\theta \doteq 39^\circ$

e) Pythagorean Th * only used on Right Δ

$\boxed{a^2 + b^2 = c^2}$ ← hypotenuse

ex.



$7^2 + x^2 = 10^2$

$49 + x^2 = 100$

$x^2 = 100 - 49$

$x^2 = 51$

$x = \pm\sqrt{51}$

$x \doteq 7.1$

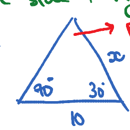
side length always positive

f) Sine Law * used on ANY Δ as long as you're given a pair of opposite side + angle

$$\boxed{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}}$$

can flip the formula ..

ex.



Find 3rd angle so you can have a pair of opposites

$180^\circ - 70^\circ - 30^\circ = 80^\circ$

$\frac{10}{\sin 80^\circ} = \frac{x}{\sin 70^\circ}$

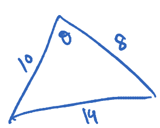
$\frac{10\sin 70^\circ}{\sin 80^\circ} = x$

$9.5 \doteq x$

g) Cosine Law * used on ANY Δ as long as you're given SSS or SAS

$$\boxed{\cos A = \frac{b^2 + c^2 - a^2}{2bc}} \qquad \boxed{a^2 = b^2 + c^2 - 2bc\cos A}$$

ex.



$\cos\theta = \frac{10^2 + 8^2 - 14^2}{2(10)(8)}$

$\cos\theta = \frac{-32}{160}$

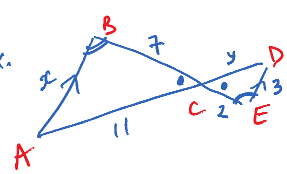
$\theta = \cos^{-1}\left(\frac{-32}{160}\right)$

$\theta \doteq 102^\circ$

h) Similar Triangles

- same shape/angles
- different size

ex.



order matters

$\Delta ABC \sim \Delta DEC$ } set up proportion statement

$\frac{AB}{DE} = \frac{BC}{EC} = \frac{AC}{DC}$

$\frac{x}{3} = \frac{7}{2} = \frac{11}{y}$ cross multiply to solve.

$2x = 21$ $\qquad$ $7y = 22$

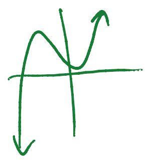
$x = \frac{21}{2}$ $\qquad$ $y = \frac{22}{7}$

7. FUNCTIONS

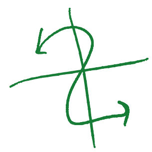
a) Functions vs. Non Functions

- For every input $= x$ there is only ONE output $= y$
- Graphs must pass a vertical line test

Function



Not a Function



- Equations can't have even power on output, y.

Function

$y^3 + x^4 = 10$

Not a Function

$y^4 + x = 5$ ← because if y is isolated you'd get many outputs

$y = \pm\sqrt[4]{5-x}$

b) FUNCTION NOTATION

- output y must be isolated and replaced with $f(x)$

$\underset{\text{any name}}{f}(\underset{\text{any input variable}}{x})$

ex. $\underset{\text{name}}{\sin}(\underset{\text{input}}{\theta})$

- evaluating:

ex. $f(x) = \sqrt{x^2 - 4}$

Find $f(10)$

$f(10) = \sqrt{10^2 - 4}$

$= \sqrt{96}$

$= 4\sqrt{6}$

don't think brackets are multiplication!

just replace x with new input

Find $3f(n)$

$3f(n) = 3\sqrt{n^2 - 4}$

Find $2 + f(\sqrt{x+4})$

$2 + f(\sqrt{x+4}) = 2 + \sqrt{(\sqrt{x+4})^2 - 4}$

$= 2 + \sqrt{x+4-4}$

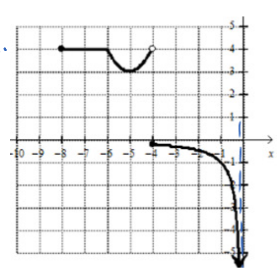
$= 2 + \sqrt{x}$

c) DOMAIN + RANGE

Domain – all possible x values that are defined
"shadow" of graph on x-axis
★ Solve algebraically $\boxed{\text{radicand} \geq 0}$ and $\boxed{\text{denom} \neq 0}$

Range – all resulting y-values from given domain
"shadow" of graph on y-axis
★ HARD to find algebraically – easy from graph

ex.



D: $\{x \mid x \in \mathbb{R},\ -8 \leq x < 0\}$

R: $\{y \mid y \in \mathbb{R},\ y < -\frac{1}{4},\ 3 \leq y \leq 4\}$

ex.

$$y = \frac{\sqrt{x-5}}{x-10} + x$$

radicand ≥ 0

$x - 5 \geq 0$

$x \geq 5$

denom $\neq 0$

$x - 10 \neq 0$

$x \neq 10$

$\therefore$ D: $\{x \mid x \in \mathbb{R},\ x \geq 5,\ x \neq 10\}$

R: too hard without graphing

d) INVERSES

★ for inverses to be functions the original function should be ONE-to-ONE (pass both vertical + horizontal line tests)

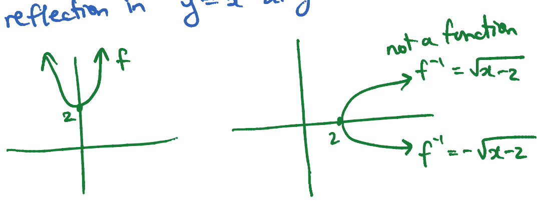
★ Equations → switch x and y
→ isolate y to use function notation $f^{-1}(x)$

ex. $y = x^2 + 2$ ← original
$x = y^2 + 2$ ← inverse
} completely different now! Label!

$\pm\sqrt{x-2} = y$ $\quad \therefore f^{-1}(x) = \sqrt{x-2}$ or $f^{-1}(x) = -\sqrt{x-2}$

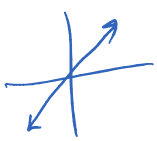
★ Graphs → switch x and y for several pts
→ reflection in $y = x$ diagonal line

ex.

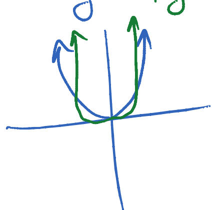


e) BASIC SHAPES

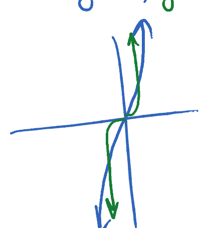
Linear $y = x$



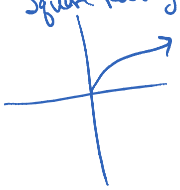
Even powers $y = x^2$, $y = x^4$



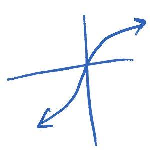
Odd Powers $y = x^3$, $y = x^5$



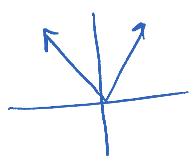
Square Root $y = \sqrt{x}$



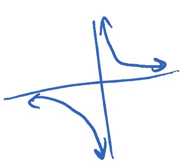
Cube Root $y = \sqrt[3]{x}$



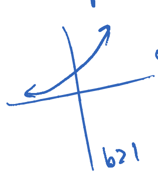
Absolute Value $y = |x|$



Rational $y = \frac{1}{x}$

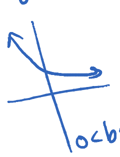


Exponential $y = b^x$



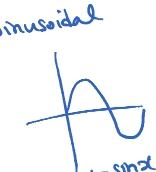
$b > 1$

or



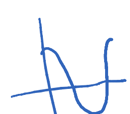
$0 < b < 1$

Sinusoidal



$y = \sin x$

or



$y = \cos x$

8. RATIONAL EXPRESSIONS

a) Restrictions

- can not divide by zero
- factor everything! otherwise can't simplify or see restrictions
- solve each denominator factor $\neq 0$

ex. $\frac{3x}{2x^2-3x-5} = \frac{3x}{(2x-5)(x+1)}$ $\quad x \neq -1, \frac{5}{2}$

$\binom{2}{1}$ $\binom{-5}{1}$ $\frac{1}{5}$

b) Multiplying / Dividing

- factor everything + state restrictions from ALL denominators (before + after flip)

$\frac{a}{b} \times \frac{c}{d}$ multiply across (not cross mult!)

$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$ reciprocate the second then multiply

ex. $\frac{14x^2y}{2x^2-5x-3} \div \frac{21xy^3}{4x^2-1}$

$\binom{2}{1}$ $\binom{3}{-3}$ $= \frac{14x^2y}{(2x+1)(x-3)} \times \frac{(2x+1)(2x-1)}{21xy^3}$

$= \frac{2\cancel{14}x\cancel{x}\cancel{y}\cancel{(2x+1)}(2x-1)}{\cancel{(2x+1)}(x-3)\cancel{(21)}_3\cancel{x}yy\cancel{y}}$

- cancel only identical polynomials
  MONOMIAL cancels MONOMIAL
  BINOMIAL cancels BINOMIAL
  etc...

$= \frac{2x(2x-1)}{3y^2(x-3)}$ $\quad , x \neq -\frac{1}{2}, 3, 0$

$y \neq 0, \frac{1}{2}, -\frac{1}{2}$

c) Adding / Subtracting

- factor only the denominator (for now, later will do numerator too)
- find LCD. (make sure you take the lowest otherwise question will be hard)
- simplify numerator (don't cancel until everything is factored and has multiplication between polynomials.)
- factor numerator to see cancellations (if possible)
- state restrictions

ex. $\frac{5x+5}{5x^2+35x-40} - \frac{7}{3x^2-3x}$

$= \frac{\cancel{5}(x+1)}{\cancel{5}(x^2+7x-8)} - \frac{7}{3x(x-1)}$

$= \frac{(x+1)}{(x+8)(x-1)} - \frac{7}{3x(x-1)}$

$= \frac{3x(x+1) - 7(x+8)}{3x(x+8)(x+1)}$

$= \frac{3x^2+3x-7x-56}{3x(x+8)(x+1)}$

$= \frac{3x^2-4x-56}{3x(x+8)(x+1)}$

$x \neq 0, -8, -1$

9. RADICAL EXPRESSIONS

a) Simplifying

ex. $2\sqrt{8p^2q^3r}$

- split up radicand parts into perfect squares

$= 2\sqrt{2^2 \cdot 2p^2q^2 \cdot qr}$

- cancel

$= 2\sqrt{2^2}\sqrt{p^2}\sqrt{q^2}\sqrt{2qr}$

- remember p could have been neg or pos so use abs. value
- restrict q and r, still under root
  $q \geq 0$, $r \geq 0$

$= 4|p|q\sqrt{2qr}$

b) Adding + Subtracting

ex. $\sqrt{54}+\sqrt{24}+\sqrt{72}-\sqrt{32}$

$= \sqrt{9}\sqrt{6} + \sqrt{4}\sqrt{6} + \sqrt{36}\sqrt{2} - \sqrt{16}\sqrt{2}$ — reduce, using perfect squares.

$= 3\sqrt{6} + 2\sqrt{6} + 6\sqrt{2} - 4\sqrt{2}$ — collect together LIKE radicals (only coefficient changes!)

$= 5\sqrt{6} + 2\sqrt{2}$

c) Multiplying + Dividing — do coefficients separately from radicands

ex. $4\sqrt{15}(-3\sqrt{6}+5)$ expand

$= -12\sqrt{90} + 20\sqrt{15}$

reduce

$= -12\sqrt{9}\sqrt{10} + 20\sqrt{15}$

$= -36\sqrt{10} + 20\sqrt{15}$

ex. $\dfrac{-12\sqrt{24}}{3\sqrt{2}}$ do separately

$= -4\sqrt{12}$ reduce.

$= -4\sqrt{4}\sqrt{3}$

$= -8\sqrt{3}$

d) Rationalizing the denominator.

- it's not proper form to leave radicals in the denominator

for MONOMIALS

→ multiply by identical radical (no coefficient) on top + bottom

for BINOMIALS

→ multiply by denominator's conjugate (like diff. of sq.) on top + bottom

ex. $\dfrac{\sqrt{3x^2y^3}}{4\sqrt{5xy^3}}$ reduce 1st

$= \dfrac{\sqrt{3x}}{4\sqrt{5}} \times \dfrac{\sqrt{5}}{\sqrt{5}}$

$= \dfrac{\sqrt{15x}}{4\sqrt{25}}$

$= \dfrac{\sqrt{15x}}{20}$

ex. $\dfrac{(\sqrt{5}+3)}{(4-\sqrt{5})} \times \dfrac{(4+\sqrt{5})}{(4+\sqrt{5})}$ conjugates

$= \dfrac{4\sqrt{5}+\sqrt{25}+12+3\sqrt{5}}{16+\underbrace{4\sqrt{5}-4\sqrt{5}}_{\text{cancel}}-\sqrt{25}}$ FOIL

$= \dfrac{8\sqrt{5}+17}{9}$ Simplify