nNOTESfixed2012

see below for last years solutions

Functions Unit 1

Tentative TEST date_______________

Big idea/Learning Goals

In this unit you will learn about key properties of basic functions studied in grade 11. You will also learn about two new functions: piecewise and absolute value functions. The key characteristics you will study here will enable you to learn about more complex type functions in the next units: polynomial, rational, trigonometric, exponential, and logarithmic functions.

Corrections for the textbook answers:
Section 1.2 #4d $x \in R$
Section 1.3 #10 typo-extra neg in the interval $(-\infty, 2)$
Section 1.4 #3 (-4, -10), ignore the other points
Section 1.6 #6 one part is wrong $0.02x + 5$
Review #15 piecewise: 3/2x-1 if x<=2 and -1/2x if x>2
Review #17 piecewise: 30 if 0<=m<=200 and 30 + 0.03(m-200) if m>200
Ch. Test #7a) (-2, 17)

Success Criteria

- ☐ I am ready for this unit if I am confident in the following review topics
  - ☐ Factoring & Expanding (Pascal)
    TWO Handouts
  - ☐ Linear Relations (find equations, sketch, word problems) & Solve Equations & Linear Systems
    THREE Handouts
  - ☐ Quadratics (formula, complete square, find equations, sketching, word problems)
  - ☐ Exponentials
    TWO Handouts
  - ☐ Trigonometry (SOH CAH TOA, sine law, cosine law, unit circle, special triangles, identities, sinusoidals)
    TWO Handouts
  - ☐ Functions (notation, inverses, transformations, domain & range, sketching, rational expressions)
    THREE Handouts

- ☐ I understand the new topics for this unit if I can do the practice questions in the textbook/handouts
  Specific questions will not be assigned, since it will depend on your knowledge and skill (everyone is at a different level). The goal is to do all types of questions quickly and without reference to notes or back of textbook or another individual. BUT you may not have time to do every single question available… so… If you are a strong student you may just concentrate on harder TIPS or APP questions, while if you are a weak student you may want to use all your time practicing the basic KU or COMM questions. The number of questions done should also be proportional to your mark so far. If you have very low scores, more practice is required.

Date	pg	Topics	# of quest. done? You may be asked to show them
	1-4	Review graphs from previous grades Getting started pg 2 in textbook	
	5-7	Review functions vs. non-functions Section 1.1	
	8-9	Review transformations of functions Section 1.4	
	10-11	Review inverses of functions Section 1.5 & Handout	
	12-13	Piecewise Functions Section 1.6	
	14-15	Absolute value Section 1.2 & Handout	
	16-18	Properties of functions Section 1.3 & TWO Handouts	
		REVIEW	

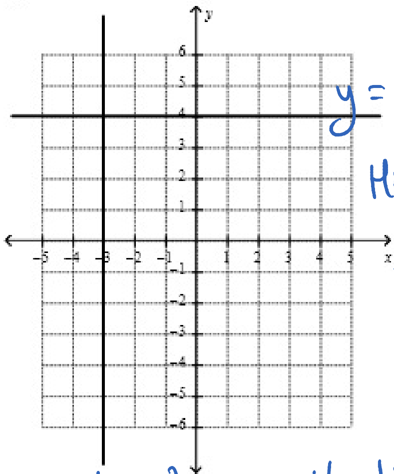
Reflect – DIAGNOSTIC TEST mark ___________, Overall mark now___________.

Looking back, what can you improve upon?

Functions studied in grade 9-11

For each of the following state the parent function and the equation of the graphed function. Summarize any key information about this type of relation.

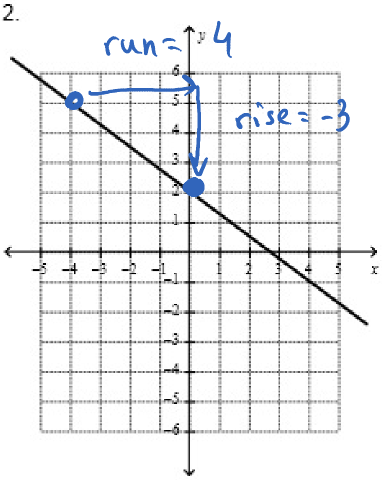
1.



Horizontal lines have the equation of the form $\boxed{y = \#}$

Vertical lines $\boxed{x = \#}$

2.



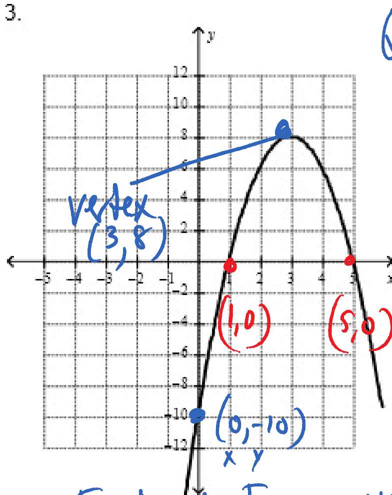
$y = mx + b$

slope y-int

$m = \frac{rise}{run} = \frac{-3}{4}$

$y = -\frac{3}{4}x + 2$

3.



Quadratic parent $y = x^2$

zeros

Factored Form: $y = a(x-r)(x-t)$

$y = a(x-1)(x-5)$

$-10 = a(0-1)(0-5)$

$-10 = a(-1)(-5)$

$\frac{-10}{5} = \frac{5a}{5}$

$-2 = a$

$\therefore \boxed{y = -2(x-1)(x-5)}$

Vertex Form: $y = a(x-h)^2 + k$

vertex $(h,k) = (3,8)$

$\boxed{y = -2(x-3)^2 + 8}$

Standard Form:

$y = ax^2 + bx + c$ y-int

$y = -2x^2 + bx - 10$

sub pt. for "b" OR expand factored form

$-2(x^2 - 5x - 1x + 5)$

$-2x^2 + 10x + 2x - 10$

$\boxed{y = -2x^2 + 12x - 10}$

$y = a\sin[k(x-d)] + c$

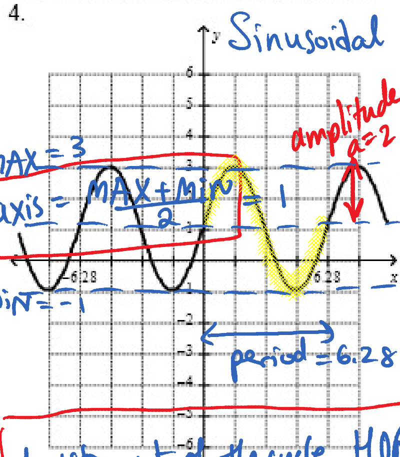
$y = ab^x + c$

For each of the following state the parent function and the equation of the graphed function. Summarize any key

$y = ab^x + c$; d

For each of the following state the parent function and the equation of the graphed function. Summarize any key information about this type of relation.

4\. Sinusoidal parents $y = \sin x$ or $y = \cos x$



MAX = 3

$c = $ axis $= \frac{MAX + MIN}{2} = 1$

MIN = −1

amplitude $a = 2$

$K = \frac{360°}{period}$

period = 6.28

$K = \frac{360°}{6.28} \doteq 57°$

d = 1st point of the cycle HORIZONTALLY!

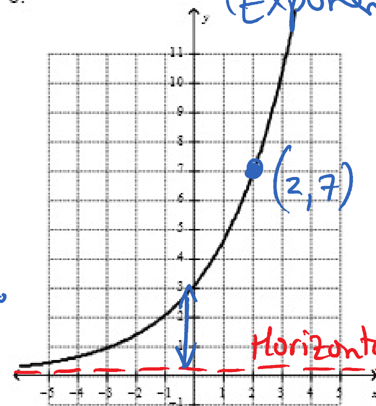
sine ∩ cosine ∪

$y = 2\sin[57°(x-0)] + 1$

6\. Exponential parent $y = b^x$

$b > 1$ grow

$0 < b < 1$ decay



(2, 7)

Horizontal asymptote = c = 0

3 = a = Distance between asymptote and y-int

b = sub in point

$y = 3b^x + 0$

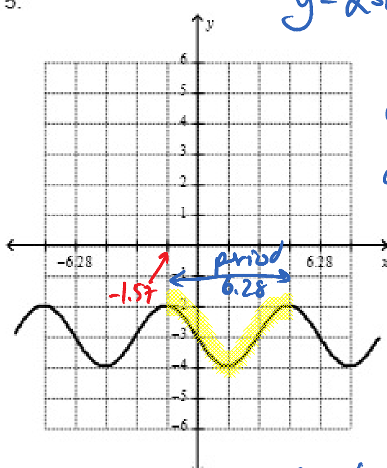
$7 = 3(b)^2$

$\sqrt{\frac{7}{3}} = \sqrt{b^2}$

$1.5 \doteq b$

$\therefore y = 3(1.5)^x$

5\.



−1.57

period 6.28

$c = -3$

$a = 1$

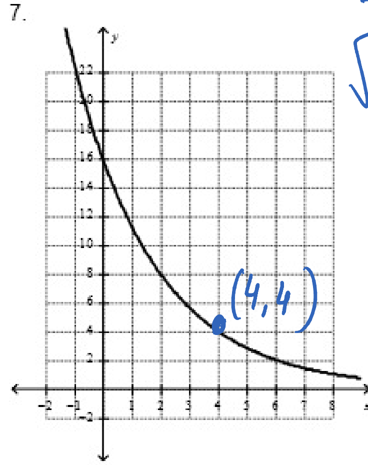
period = 6.28

$k = \frac{360°}{6.28} = 57°$

$d = -1.57$

$y = 1\cos[57°(x + 1.57)] - 3$

7\.



(4, 4)

$c = 0$

$a = 16$

$y = 16b^x$

$\frac{4}{16} = \frac{16}{16}b^4$

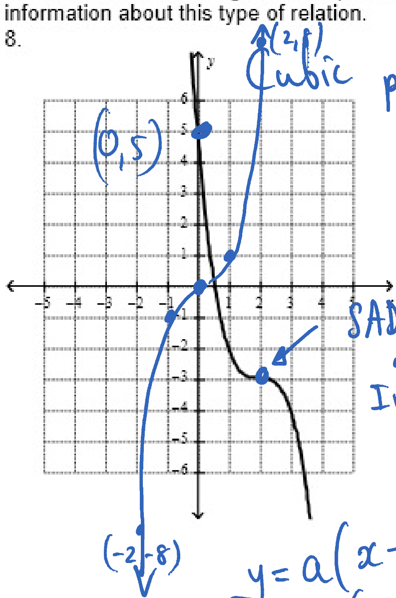
$\sqrt[4]{0.25} = \sqrt[4]{b^4}$

$0.7 \doteq b$

$\therefore y = 16(0.7)^x$

For each of the following state the parent function and the equation of the graphed function. Summarize any key information about this type of relation.

8.



Cubic parent $y=x^3$

SADDLE point or Inflection pt. (h,k)

$y=a(x-h)^3+k$

$y=a(x-2)^3-3$

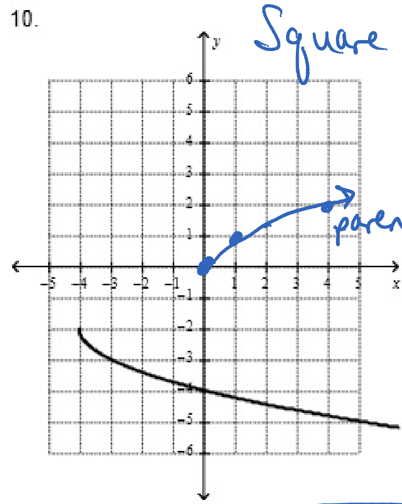
$5=a(0-2)^3-3$

$5=-8a-3$

$8=-8a$ $a=-1$

$\therefore y=-1(x-2)^3-3$

10.



Square Root/Radical

parent $y=\sqrt{x}$

$y=a\sqrt{x-d}+c$

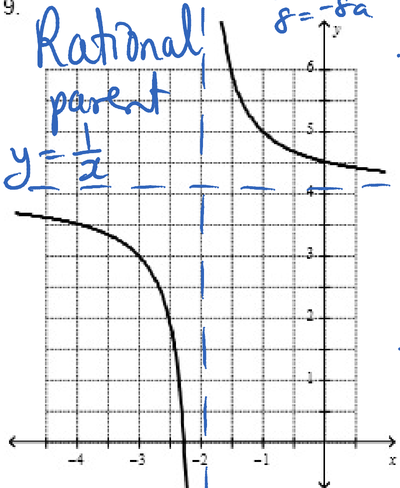
$y=-1\sqrt{x+4}-2$

left 4

down 2

reflected in y-axis.

9.

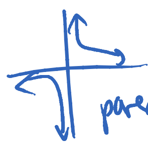


Rational

parent $y=\frac{1}{x}$

HA. horiz. asympt.

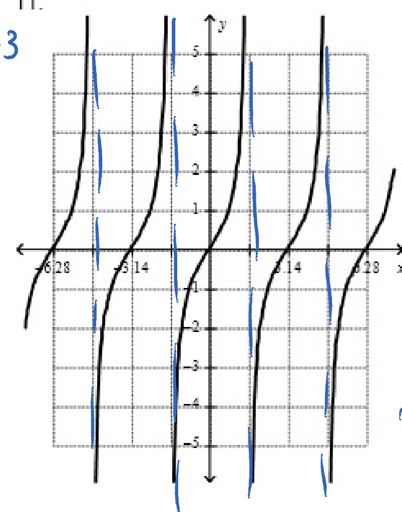
V.A vertical asympt.



$y=\frac{1}{(x+2)}+4$ ← shift up/down

shift left/right

11.



parent

$\boxed{y=\tan x}$

4

Functions vs. Non-Functions

1. What characterizes a relationship that is a function, and what may cause it to be a non function?

Functions have only one output (y) for every input (x)

NON-function if 1.) graph does not pass a vertical line test
2.) There are two/more outputs for a single input
3.) Equations have $y^2, y^4, y^6 \dots$

2. Identify if each of the following is a function or not.

a. The input is the postal code and the output is a person's address.

code → Apt #1, Apt #1 NOT

b. The input is button pressed on a vending machine and the output is the item that you get.

YES.

c. $x^2 + y = 5$ YES $y = 5 - x^2$ quadratic

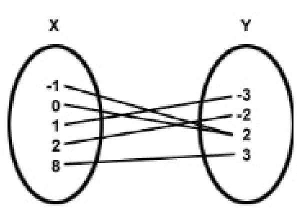
d. $x + y^2 = 5$ NO

e.

x	y
0	1
3	-5
6	1

YES

f.



YES

g. $\{(-1,4),(0,7),(5,-2),(10,7)\}$

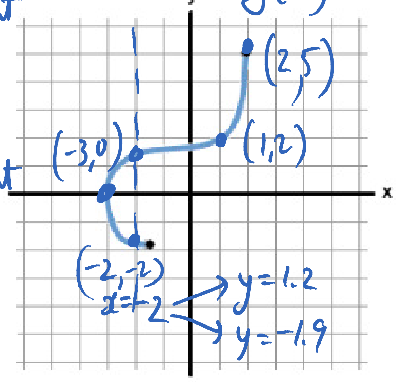
YES

h. $\{(-3,5),(2,8),(-3,9),(0,-4)\}$

$-3 \to 5$, $-3 \to 9$ NOT a function

i. is y a function of x? $y(x)$

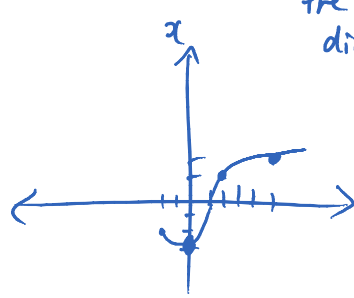
output $\boxed{f(x)}$ name of function, input



NOT a function

j. is x a function of y for the above? $x(y)$

inverse = reflection of the graph in diagonal $y=x$



YES

5

SYMBOLS
→ approaching
{…} set of
∈ element of
ℝ real #'s (decimals, neg, pos, fraction)
I or ℤ integers
< less than
≥ greater than + equal

3. What is the domain and range of a relation?

Domain = all defined values of independent variable, x.
Range = all outputs y from the given domain.

4. State if the following are functions or not and then state the domain and range.

a. You are investigating the average height of a grade 9 student in ajax.

Function

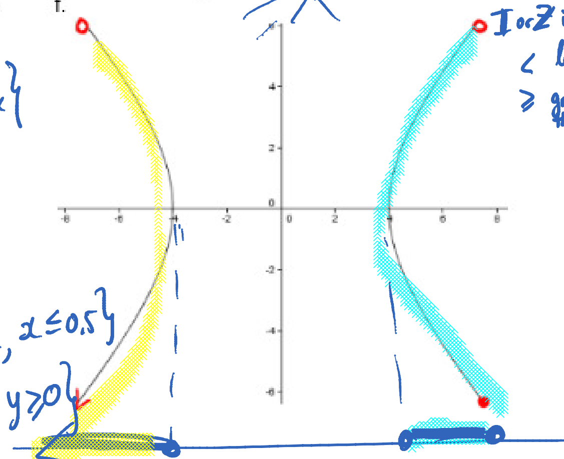
$D = \{\text{all grade 9 students in Ajax}\}$
$R = \{\text{their heights}\}$

b. $y = \sqrt{3-6x}$ sq. root $\sqrt{x}$

$y = \sqrt{-6x+3}$
$y = \sqrt{-6(x-0.5)}$ ← right
reflect in y-axis
horiz. compressed

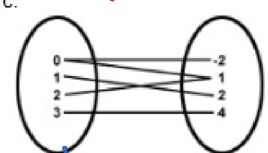
$D = \{x \in \mathbb{R}, x \leq 0.5\}$
$R = \{y \in \mathbb{R}, y \geq 0\}$

f.



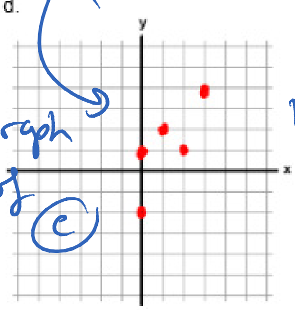
$D = \{x \in \mathbb{R}, x \leq -4, 4 \leq x \leq 8\}$
$R = \{y \in \mathbb{R}, y < 6\}$

c.



$D = \{x = 0, 1, 2, 3\}$
$R = \{y = -2, 1, 2, 4\}$

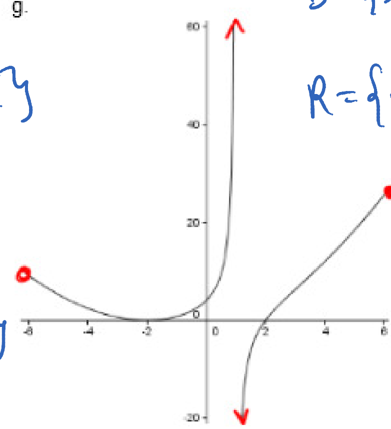
d.



graph of (c)

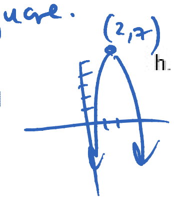
$D = \{x = 0, 1, 2, 3\}$
$R = \{y = -2, 1, 2, 4\}$

g.



e. $y = -2x^2 + 8x - 1$ complete square.

$y = -2(x^2 - 4x) - 1$
$y = -2(x^2 - 4x + 4 - 4) - 1$
$\left(\frac{b}{2}\right)^2 \quad \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$



$y = -2(x^2 - 4x + 4) - 4(-2) - 1$
$y = -2(x-2)(x-2) + 8 - 1$
$y = -2(x-2)^2 + 7$

$D = \{x \in \mathbb{R}\}$
$R = \{y \in \mathbb{R}, y \leq 7\}$

h. $y = \dfrac{1}{4-2x} = \dfrac{1}{-2(x-2)}$

$D = \{x \in \mathbb{R}, x \neq 2\}$
$R = \{y \in \mathbb{R}, y \neq 0\}$

i. $x^2 + y^2 = 25$

circle of radius $r = 5$

$D = \{x \in \mathbb{R}, -5 \leq x \leq 5\}$
$R = \{y \in \mathbb{R}, -5 \leq y \leq 5\}$

5. What is function notation? What are the benefits and disadvantages of using the function notation?

Function Notation $= f(x)$ ← output $f(x) = y$

name of function; input

Disadvantage – brackets do not mean to multiply

6. For each of the following identify the function that operates on the given input to produce the given output.

a. $f: x \to 5x$ linear $f(x) = 5x$

b. $g: \begin{pmatrix} 9 \\ 49 \\ 4 \end{pmatrix} \to \begin{pmatrix} 3 \\ 7 \\ 2 \end{pmatrix}$ sq. root $g(x) = \sqrt{x}$

c. $h: \begin{pmatrix} 0^\circ \\ 90^\circ \\ 180^\circ \end{pmatrix} \to \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ Cos. Cosine $h(x) = \cos x$

d. $i: \begin{pmatrix} 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} \to \begin{pmatrix} 5 \\ 10 \\ 17 \\ 26 \end{pmatrix}$ 2^2+1, 3^2+1 quadratic $i(x) = x^2 + 1$

e. $j: \begin{pmatrix} 0 \\ 4 \\ 8 \\ 12 \\ 16 \end{pmatrix} \to \begin{pmatrix} 1024 \\ 128 \\ 16 \\ 2 \\ 0.25 \end{pmatrix}$ $\times\frac{1}{8}$, $\times\frac{1}{8}$, $\times\frac{1}{8}$

exponential with HA $y = 0$

$y = ab^x + c$

$y = 1024\left(\frac{1}{8}\right)^x + 0$

f. $k: \begin{pmatrix} -2 \\ 0 \\ 2 \\ 3 \\ 4 \end{pmatrix} \to \begin{pmatrix} 18 \\ 12 \\ 6 \\ 3 \\ 0 \end{pmatrix}$ $-3(-2)+12 = 18$; $-1 \to 15$, $1 \to 9$; $)-3$, $)-3$ linear

$k(x) = -3x + 12$

g. $l: \begin{pmatrix} -1 \\ 2 \\ 5 \\ 8 \\ 11 \end{pmatrix} \to \begin{pmatrix} 0 \\ 0 \\ -54 \\ -162 \\ -324 \end{pmatrix}$ zeros; $)0$, $)-54$, $)-108$, $)-162$; $)-54$, $)-54$, $)-54$ quad

$l(x) = a(x+1)(x-2)$

$-54 = a(5+1)(5-2)$

$-54 = 18a$

$-3 = a$

$l(x) = -3(x+1)(x-2)$

7. For the function $f(x) = \sqrt{x+5} + 3$

a. Find $f(20)$ $= \sqrt{20+5} + 3$

$= \sqrt{25} + 3$

$= 5 + 3$

$= 8$

ie. $f(20) = 8$ if $x = 20$, $y = 8$

b. Find $f(x^2)$

$f(x^2) = \underbrace{\sqrt{x^2+5} + 3}_{\text{can't simplify.}}$

8. Write the area of a circle as a function of its radius

$A(r) = \pi r^2$

9. Write the perimeter of a circle as a function of its radius

$P(r) = 2\pi r$

10. Write the area of a circle as a function of its perimeter.

$P = 2\pi r$ $\quad r = \frac{P}{2\pi}$

$A = \pi r^2$

$A = \pi\left(\frac{P}{2\pi}\right)^2$

$= \frac{\pi P^2}{4\pi^2}$

$\therefore A(P) = \frac{P^2}{4\pi}$

Review Transformation of Functions

1. Describe what each constant in $y = af(k(x-d))+c$ controls

Parent $y=x$ $y=x^2$ $y=x^3$ $y=\sqrt{x}$ $y=\frac{1}{x}$

a → vertical stretch ex. $a=2$ or compress $a=0.1$
→ reflect in x-axis. ex. $a=$ neg.

k → horizontal compress $k=3$ or stretch $k=\frac{1}{3}$
→ reflect in y-axis

d → shift left $d=-3$ equation $(x+3)$ or right $d=2$ $(x-2)$

c → shift up $c=5$ or down $c=-5$

2. Describe the most effective method of applying transformations.

Steps

(1.) Factor out the "k" so that d is visible.

(2.) Create these tables.

x	$y=$parent	$x \div k$	$y \cdot a$	$x \pm d$	$y \pm c$
any x values					

3. Sketch each of the following

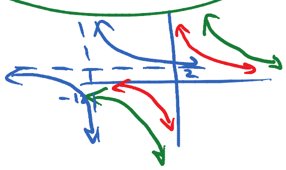
a. $h(x) = \dfrac{4}{6+0.5x} + 2$

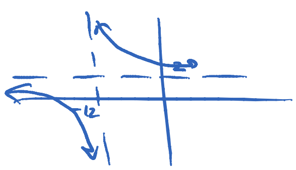
parent $y=\frac{1}{x}$

$= 4\left[\dfrac{1}{0.5(x+12)}\right] + 2$

$a=4$ vertical stretch, $k=0.5$ horiz. stretch

$d=-12$ move left, $c=2$ move up. perform these at the end.





b. $i(x) = 2\sqrt{4-x} - 1$

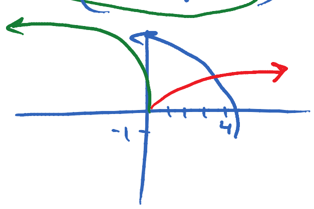
$\sqrt{-x+4}$

$= 2\sqrt{-1(x-4)} - 1$

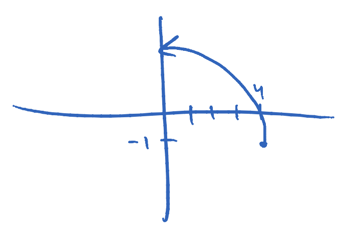
parent $y=\sqrt{x}$

$a=2$ vertical stretch, $k=-1$ reflect in y-axis (no stretch or compression!)

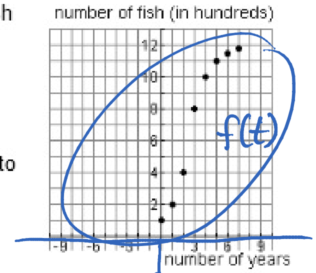
$d=4$ move right, $c=-1$ move down



Final one

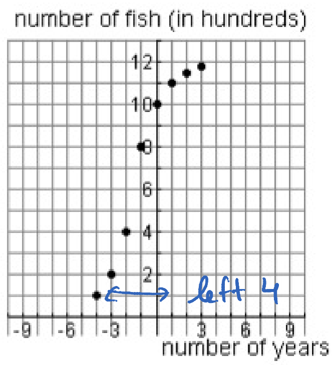


4. Daniel has a lake in his backyard. He plans to raise fish in his lake. He released 100 fish in his lake in 1996. The graph ~~below~~ represents the number of fish in Daniel's lake in terms of the number of years since 1996. Let the function f(t) represent the number of fish in Daniel's lake in terms of the number of years since 1996.



The graphs shown below are a transformation of the function f(t) shown above. For each of the following explain the real-world meaning of the transformation and use function notation to represent it.

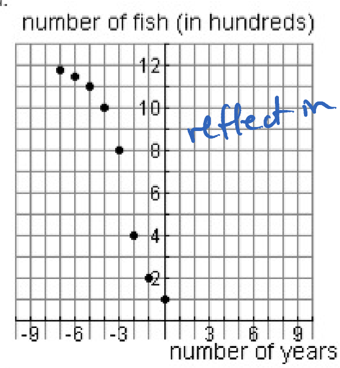
a.



$f(t+4)$

He released 100 fish 4 years earlier

d.

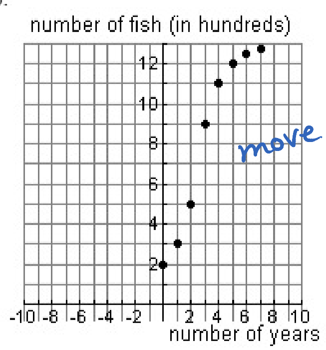


reflect in y-axis

$f(-t)$

released ~ 12 hundred fish and they're dying.

b.

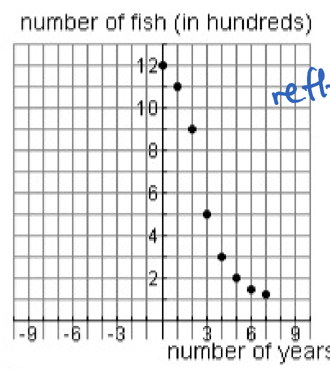


move up $c=1$

$f(t)+1$

released 200 fish instead of 100

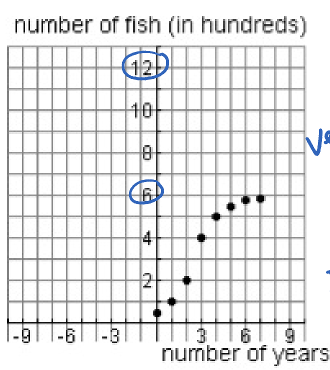
e.



reflect in x-axis and shift up by 13

$-f(t)+13$

c.



vertical compression $a=\frac{1}{2}$

$\frac{1}{2}f(t)$

f. If more points were added on either side of this type of graph it will seem to have two horizontal asymptotes, what type of function can model this?

$\frac{\#}{\text{exponential}}$

See this in Unit #9

ex. $\frac{1}{1+2^x}$

as $x \to -\infty$ $f(x) \to 1 = HA$

as $x \to \infty$ $f(x) \to 0 = HA$

1 — — — —

0 — — — —

Review of Inverses of Functions

1. Clarify the meaning of the words opposite, reciprocal and inverse.

Opposite – switch sign; ex. $x-4$ opposite $-x+4$

Reciprocal – flip fraction; ex. $\frac{2x}{1}$ reciprocal is $\frac{1}{2x}$

inverse – undo operations (input switches with output. SAMDEB); ex. square inverse sq. root; $+$ $\sin(\ )$ — $\sin^{-1}(\ )$

2. Find the algebraic inverse for each of the following functions

a. $f(x) = 2x - 6$

$y = 2x-6$ ← original

$x = 2y - 6$ ← inverse

$\frac{x+6}{2} = \frac{2y}{2}$

$\frac{x+6}{2} = f^{-1}(x)$

Steps
1. Switch x and y
2. Solve for y

b. $f(x) = \frac{x}{x-3}$

$y = \frac{x}{x-3}$ ← original

$\frac{x}{1} = \frac{y}{y-3}$ ← inverse

$x(y-3) = y$

$xy - 3x = y$ collect y's on one side

$-3x = y - xy$ common factor y

$-3x = y(1-x)$

$\frac{-3x}{1-x} = y = f^{-1}(x)$

c. $f(x) = -2(x+5)^2 + 8$

$y = -2(x+5)^2+8$ ← original

$x = -2(y+5)^2 + 8$ ← inverse

$x - 8 = -2(y+5)^2$

$\frac{x-8}{-2} = (y+5)^2$

$\pm\sqrt{\frac{x-8}{-2}} = y+5$

$\pm\sqrt{\frac{x-8}{-2}} - 5 = y = \cancel{f^{-1}(x)}$ can't use function notation unless it is a function. (one output)

$f^{-1}(x) = +\sqrt{\frac{x-8}{-2}} - 5$

or

$f^{-1}(x) = -\sqrt{\frac{x-8}{-2}} - 5$

Show a way of finding the inverse for the following:

d. $f(x) = -x^2 + 2x - 9$

$y = -x^2 + 2x - 9$ ← original

$x = -1y^2 + 2y - 9$ ← inverse

can't do SAMDEB since y appears twice, not like terms!

Complete the square

$x = -1(y^2 - 2y + 1 - 1) - 9$ ← $\left(\frac{b}{2}\right)^2$

$x = -1(y^2-2y+1) - 1(-1) - 9$ times "a"

$(y-1)(y-1)$

$x = -1(y-1)^2 - 8$ SAMDEB

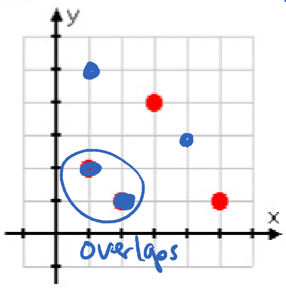
$\boxed{y = f^{-1}(x)} = \sqrt{-1(x+8)} + 1$

or

$f^{-1}(x) = -\sqrt{-1(x+8)} + 1$

3. Find the graphical inverse of each of the following → switch x and y.

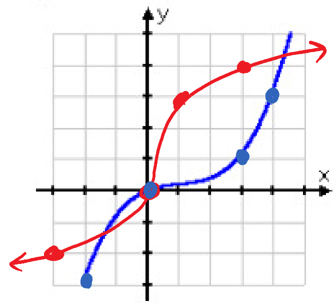
a.



$f(x)$	$f^{-1}(x)$
(1,2)	(2,1)
(2,1)	(1,2)
(3,4)	(4,3)
(5,1)	(1,5)

plot

b.



$f(x)$	$f^{-1}(x)$
(-2,-3)	(-3,-2)
(0,0)	(0,0)
(3,1)	(1,3)
(4,3)	(3,4)

4. Can you think of applications of inverse functions?

2 ⓓ $f(x) = -x^2 + 2x - 9$

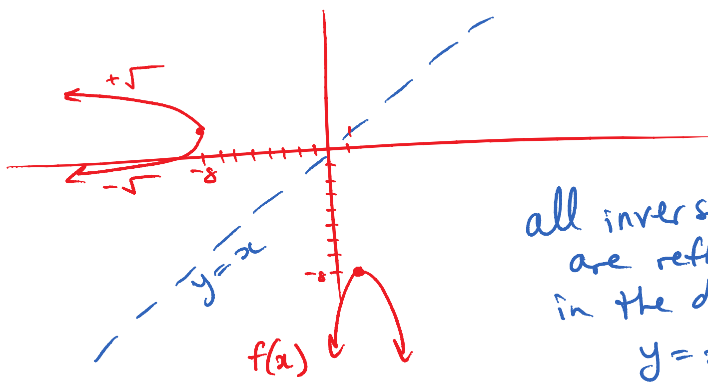
$= -(x-1)^2 - 8$

vertex$(1,-8)$

$f^{-1}(x) = \sqrt{-1(x+8)} + 1$

or

$f^{-1}(x) = -\sqrt{-(x+8)} + 1$



all inverses are reflections in the diagonal $y=x$ line.

Solving for the input (exponentials → logs, trig → trig inverse...)

find Range algebraically

Piecewise Functions

function that's in pieces

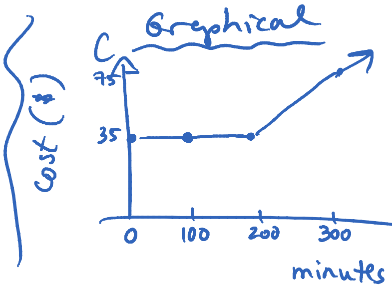
1. Some relationships in real life cannot be represented by one single function. You can define a relationship using different pieces/functions. Can you make up a real life problem where cost of something changes with time? Write out the problem and show an example of algebraic and graphical model for it.

Cost of cell phone is \$35 per month for the 1st 200 minutes then 40 cents per minute for overcharge.

Algebraic

$$C(m)=\begin{cases}35, & \text{if } 0\le m\le 200\\ 35+0.4(m-200), & \text{if } m>200\end{cases}$$

Graphical



100(0.4)
\$40
+35

2. Sketch each of the following piecewise graphs.

a.

$$f(x)=\begin{cases}x+1, & x\le 2\\ (x-3)^2+1, & 2<x\le 6\\ -\dfrac{1}{2}x+5, & 6<x\end{cases}$$

line
quadratic
line

$-4+5$

sub pt's in

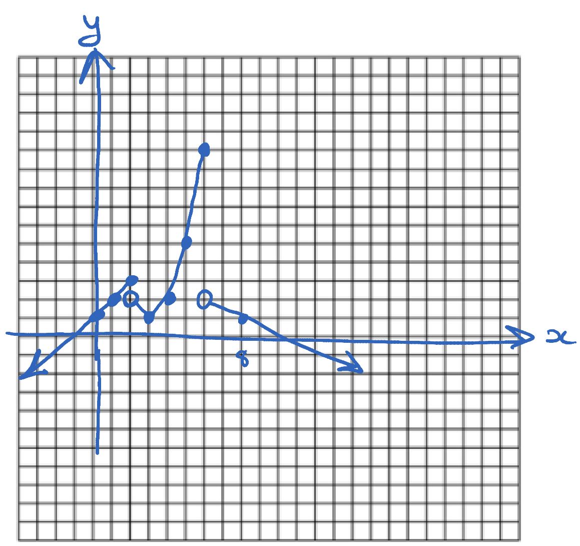
b.

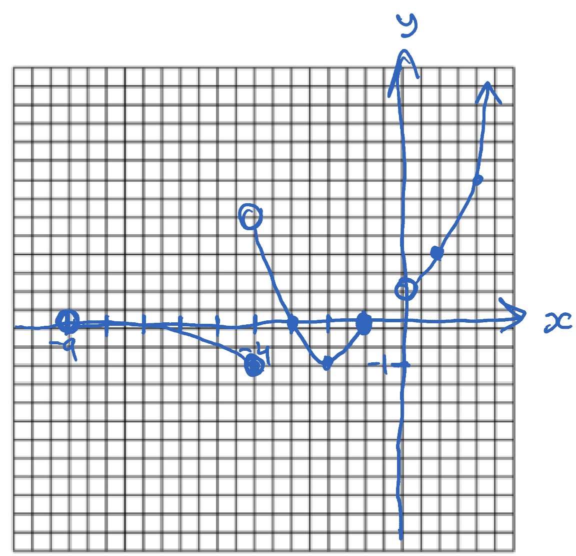
$$f(x)=\begin{cases}-3+\sqrt{-x}, & -9<x<-4\\ -1, & x=-4\\ x^2+4x+3, & -4<x\le -1\\ 2^x, & 0<x\end{cases}$$

sq. root
point
quad
exp.

$x^2+4x+4-4+3$

$(x+2)^2-1$





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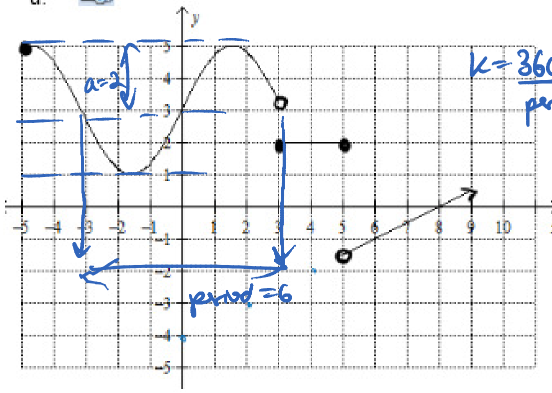
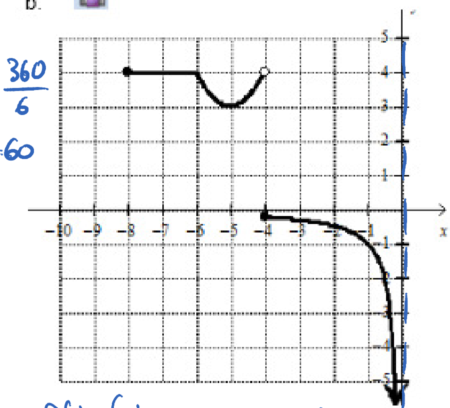
3. Give an algebraic representation for each of the following graphs.

a.



$k = \frac{360°}{\text{period}} = \frac{360}{6} = 60$

b.

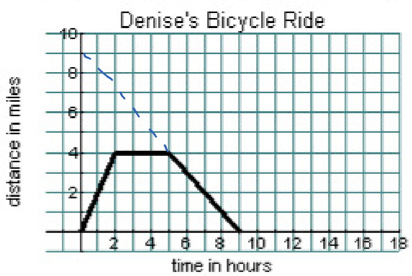


$$f(x) = \begin{cases} 2\sin(60°x) + 3, & \text{if } -5 \le x < 3 \\ 2, & \text{if } 3 \le x \le 5 \\ -1.5 + \frac{1}{2}(x-5), & \text{if } x > 5 \end{cases}$$

initial value, slope, with shift.

$$f(x) = \begin{cases} 4 & \text{if } -8 \le x \le -6 \\ (x+5)^2 + 3 & \text{if } -6 < x < -4 \\ \frac{1}{x} & \text{if } -4 \le x < 0 \end{cases}$$

4. Denise took a bicycle ride away from her home today. She left home at 12 p.m. and arrived back at home at 9 pm. The graph shown represents Denise's distance from home during her ride. Let the function d(t) represent Denise's distance in miles from home during her trip in terms of the time in hours. Create an algebraic model for this graph.



$$d(t) = \begin{cases} 2t & \text{if } 0 \le t \le 2 \\ 4 & \text{if } 2 < t \le 5 \\ 4 - 1(t-5) & \text{if } 5 < t \le 9 \end{cases}$$

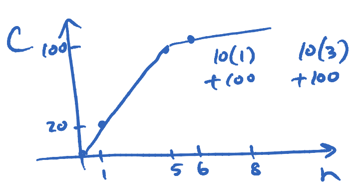
(or $-t+9$)

5. Model the following real life situation with an algebraic model

a. Buy 5, get each one after that at half ~~price~~. \$20. per item

let C = total cost
n = number of items.

$$C(n) = \begin{cases} 20n, & \text{if } 0 < n \le 5 \\ 100 + 10(n-5), & \text{if } n > 5 \end{cases}$$



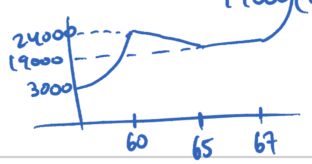
b. Bacteria divides into two every 20 minutes. If you start with a 3000 sample and after 1 hour apply an antibecterial solution, the bacteria population decreases at 1000 bacterium per minute for the first 5 minutes and then remains constant for 2 minutes before it starts to divide again.

$y = ab^{\frac{x}{c}}$

a = initial value
b = grow/decay factor
c = how long it takes

let B = number of Bacteria
t = time in minutes

$$B(t) = \begin{cases} 3000(2)^{\frac{t}{20}}, & \text{if } 0 \le t \le 60 \\ 24000 - 1000(t-60), & \text{if } 60 < t \le 65 \\ 19000, & \text{if } 65 < t \le 67 \\ 19000(2)^{\frac{(t-67)}{20}}, & \text{if } t > 67 \end{cases}$$



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Absolute Value Functions

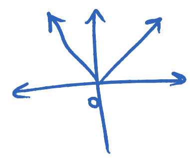
1. What is an absolute value of a number? Where is this concept used?

$|-5| = 5$ $\quad$ • absolute value of a number, makes the number positive

$|6| = 6$ $\quad$ • used in formulas $\sqrt{|x-y|}$

2. What is the parent function of the absolute value?

$f(x) = |x|$



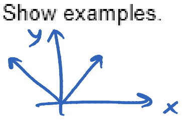
3. How can this function be represented using piecewise notation?

$$f(x) = |x| = \begin{cases} -x & , \text{if } x \leq 0 \\ x & , \text{if } x > 0 \end{cases}$$

4. What is the difference between 1 and 2 dimensional representations of an absolute value? Show examples.

$\boxed{\text{2-D}}$ - has 2 variables

ex $y = |x|$



$\boxed{\text{1-D}}$ - have 1 variable

ex. $|x+3| = 2$

5. Show how one dimensional absolute value equations can be solved.

ex. $|x+3| = 2$

$x+3 = 2$ $\quad$ or $\quad$ $-(x+3) = 2$

$x = -1$ $\qquad\qquad$ $-x-3 = 2$

$-x = 5$

$x = -5$

ex. $|x+3| < 2$

6. What is the difference between set notation of a solution and interval notation of the solution?

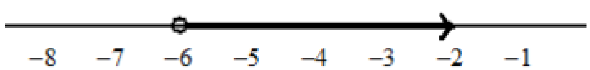
Set notation - uses $\{\ldots\}$ set brackets ex. $\{x \in \mathbb{R}, -5 < x \leq -1\}$

Interval notation - uses (, [brackets ex. $x \in (-5, -1]$

must write $x \in$ in front since $(0,1)$ looks like point (x, y)

7. Write the following in both set notation as well as interval notation.

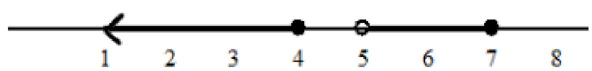
a.



Set: $\{x \in \mathbb{R}, x > -6\}$

Interval: $x \in (-6, \infty)$

b.



Set: $\{x \in \mathbb{R}, x \leq 4, 5 < x \leq 7\}$

Interval: $x \in (-\infty, 4]$ and $x \in (5, 7]$

8. What type of solutions can be represented by an absolute value?

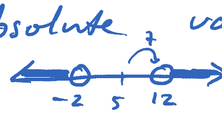
Only symmetric solutions can be represented by absolute value.

$|x - c| \underset{<}{\overset{>}{=}} r$

ex.

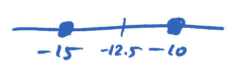
$|x-9| \leq 3$

ex.



$|x-5| > 7$

ex



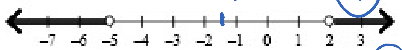
$|x+12.5| = 2.5$

9. Express the following solutions with a few algebraic representations:

a. 1.) $x=3$ and $x=9$

2.) $|x-6|=3$

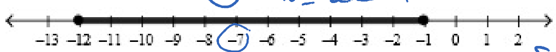
b.



1.) $x>2$, $x<-5$

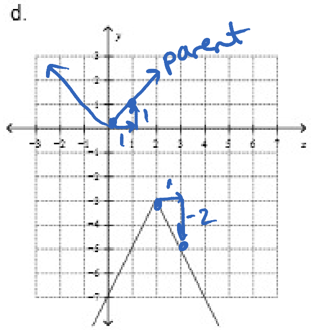
2.) $|x--1.5|>3.5$ 3.) $x\in(-\infty,-5)\cup(2,\infty)$

c. 1.) $-12\le x\le -1$



2.) $|x--7|\le 5$ 3.) $x\in[-12,-1]$

d.



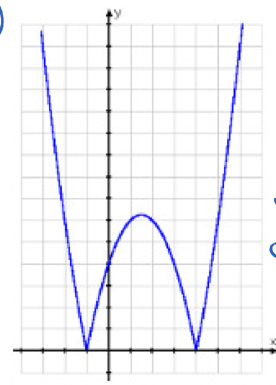
$y=-2|x-2|-3$

e. 1.) $-2\le x<7$

2.) $x\in[-2,7)$

can't do abs. val
not symmetrical.

f.



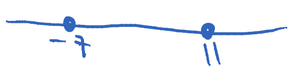
$y=a|(x+1)(x-4)|$

find by subbing in a point.

10. Solve the following absolute values and graph the solution set on a number line. ~~Show two methods of solving it for at least one of these questions.~~

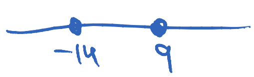
a. $|x-2|=9$

$x-2=9$ or $-(x-2)=9$
$x=11$ $\quad x-2=-9$
$\quad x=-7$



b. $3|2x+5|=69$

$3(2x+5)=69$ or $-3(2x+5)=69$
$2x+5=23$ $\quad 2x+5=-23$
$2x=18$ $\quad 2x=-28$
$x=9$ $\quad x=-14$



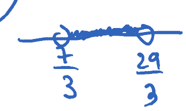
c. $|5x-15|\le 45$

$5x-15\le 45$ or $-(5x-15)\le 45$
$5x\le 60$ $\quad -5x+15\le 45$
$\frac{5x}{5}\le\frac{60}{5}$ $\quad \frac{-5x}{-5}\le\frac{30}{-5}$
$x\le 12$ $\quad x\ge -6$

* if mult/div by negative sign > flips to <

d. $5|18-3x|>55$

$5(18-3x)>55$ or $-5(18-3x)>55$
$18-3x>11$ $\quad 18-3x<-11$
$-3x>-7$ $\quad -3x<-29$
$x<\frac{7}{3}$ flip! $\quad x>\frac{29}{3}$



$4 \le x \le 8$ $\quad$ $4 < x \le 8$ $\quad$ $4 \le x$

$x \in [4, 8]$ $\quad$ $x \in (4, 8]$ $\quad$ $x \in [4, \infty)$

New Properties of Functions

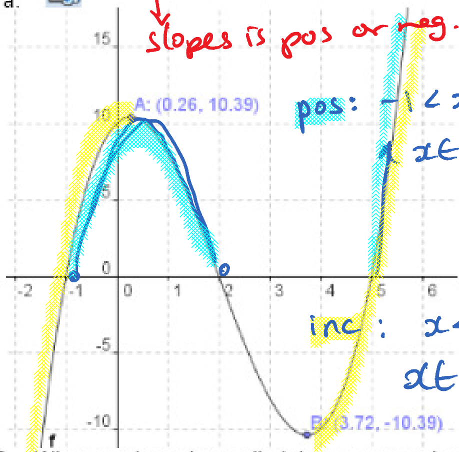
order from smallest to largest

1. Functions can be described using intervals. There are ~~**positive and negative**~~ intervals, as well as **increasing and decreasing intervals**. For the following graphs state both.

slopes is pos or neg.

y is pos or neg

a.



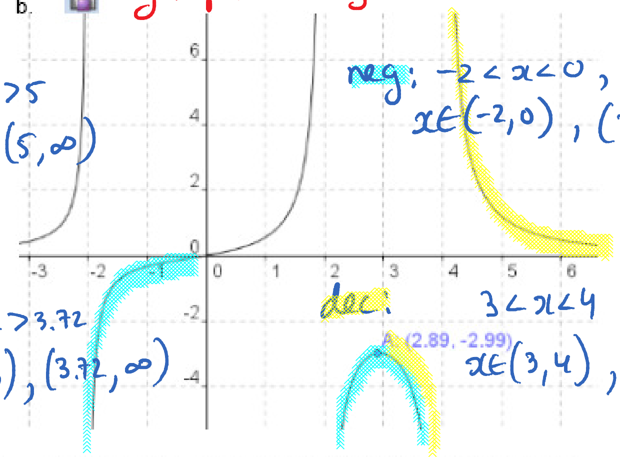
pos: $-1 < x < 2$, $x > 5$

$x \in (-1, 2), (5, \infty)$

inc: $x < 0.26$, $x > 3.72$

$x \in (-\infty, 0.26), (3.72, \infty)$

b.



neg: $-2 < x < 0$, $2 < x < 4$

$x \in (-2, 0), (2, 4)$

dec: $3 < x < 4$ or $x > 4$

$x \in (3, 4), (4, \infty)$

2. What are the points called that separate the positive and negative intervals?

zeros / x-int

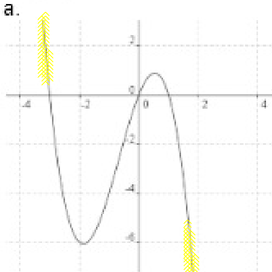
3. What are the points called that separate the increasing decreasing intervals.

turning / critical points

left most + right most edge of graph

4. Functions can also be decribed using **end behaviour**. This helps you recognize what the output values are approaching on the left-most and right-most sides of the graph. For the functions below state the end behaviour in proper notation.

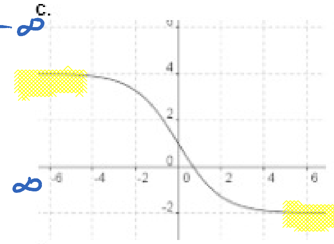
a.



as $x \to \infty$, $y \to -\infty$ on the right side

as $x \to -\infty$ on left side, $y \to \infty$

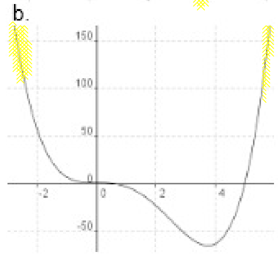
c.



as $x \to \infty$, $y \to -2$

as $x \to -\infty$, $y \to 4$

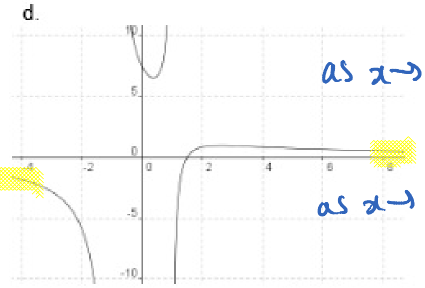
b.



as $x \to \infty$, $y \to \infty$

as $x \to -\infty$, $y \to \infty$

d.



as $x \to \infty$, $y \to 0$

as $x \to -\infty$, $y \to 0$

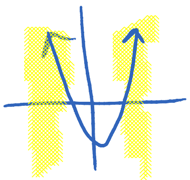
5. Vertical aymptotes cannot be crossed however notice in the last question the horizontal asymptote is crossed. How are horizontal asymptotes related to the end behaviour?

HA show end behaviour not where it's undefined (like VA's)

6. Functions can be described using **symmetry**. Describe what is meant by **even**, **odd**, and **neither** symmetry. Show graphical representations and algebraic.

EVEN - symmetry around y-axis
ie. reflect in y-axis and get back the original.

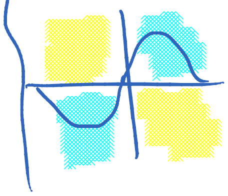
$\boxed{f(-x)=f(x)}$



if you put a neg. input and get exactly original function → even

ODD - rotational symmetry of 180° around the origin
OR. Two reflections in x and y-axis gives the same as original.

$\boxed{f(-x)=-f(x)}$



if input is neg and you get the opposite of original → ODD.

7. Identify whether the following are odd, even or neither in their symmetry.

a. $y=2x-5$ $=f(x)$

$f(-x)=2(-x)-5=-2x-5$

$-f(x)=-(2x-5)=-2x+5$

NEITHER

b. $y=x^2-5$ $=f(x)$

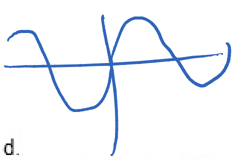
$f(-x)=(-x)^2-5=x^2-5$

$-f(x)=-(x^2-5)=-x^2+5$

$f(x)=f(-x)$ EVEN

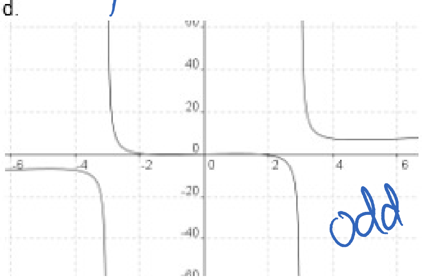
c. $y=\sin x$

draw a pic



odd
$\therefore \sin(-x)=-\sin x$

d.



odd

e. $y=\sqrt{4-x}$

$f(-x)=\sqrt{4+x}$

$-f(x)=-\sqrt{4-x}$

Neither

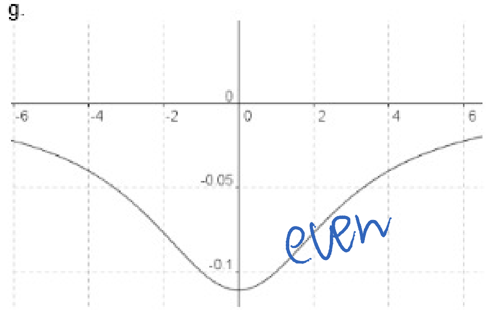
f. $y=\dfrac{2}{x}$

$f(-x)=\dfrac{2}{-x}=-\dfrac{2}{x}$

$-f(x)=-\dfrac{2}{x}$

$\therefore$ odd
$f(-x)=-f(x)$

g.



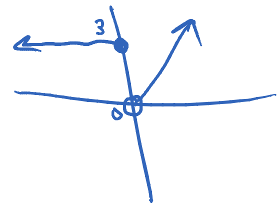
even

8. Finally, functions can also be described as **continuous** or **discontinuous**. What can cause a discontinuity in a function? Show examples.

Gaps can be created by

- Restrictions ex. $y = \frac{1}{x-1}$ is not continuous at $x=1$

- Piecewise functions not connecting ex. $y = \begin{cases} 3 & x \le 0 \\ x & x > 0 \end{cases}$



9. Find the values of each constant that would make this function continuous.

$$f(x) = \begin{cases} 2x + a, & x \le -1 \\ 22, & -1 < x \le 3 \\ -bx^2 + 31, & 3 < x \le 5 \\ \sqrt{cx} + b, & 5 < x \end{cases}$$

(1.) 1st check if any individual piece has any restrictions on given domain

No!

(2.) Then solve for constants to ensure outputs line up.

at $x=-1$
LS = RS
$2x + a = 22$
$2(-1) + a = 22$
$a = 24$

sub

at $x = 3$
LS = RS
$22 = -bx^2 + 31$
$22 = -b(9) + 31$
$-9 = -9b$
$1 = b$

at $x = 5$
LS = RS
$-bx^2 + 31 = \sqrt{cx} + b$
$-1(5)^2 + 31 = \sqrt{c(5)} + 1$
$-25 + 31 = \sqrt{5c} + 1$
$6 = \sqrt{5c} + 1$
$5 = \sqrt{5c}$
$25 = 5c$
$5 = c$

$\therefore \left.\begin{array}{l} a = 24 \\ b = 1 \\ c = 5 \end{array}\right\}$ will make $f(x)$ continuous

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