Functions Unit 1

Tentative TEST date_______________

Big idea/Learning Goals

In this unit you will learn about key properties of basic functions studied in grade 11. You will also learn about two new functions: piecewise and absolute value functions. The key characteristics you will study here will enable you to learn about more complex type functions in the next units: polynomial, rational, trigonometric, exponential, and logarithmic functions.

Corrections for the textbook answers:

Section 1.2 #4d $x \in R$

Section 1.3 #10 typo-extra neg in the interval $(-\infty, 2)$

Section 1.4 #3 (-4, -10), ignore the other points

Section 1.6 #6 one part is wrong $0.02x + 5$

Review #15 piecewise: 3/2x-1 if x<=2 and -1/2x if x>2

Review #17 piecewise: 30 if 0<=m<=200 and 30 + 0.03(m-200) if m>200

Ch. Test #7a) (-2, 17) #9a) 11500 b) $\begin{cases} 0.05x \\ 0.12x - 6000 \end{cases}$

Success Criteria

Specific questions will not be assigned, since it will depend on your knowledge and skill (everyone is at a different level). The goal is to do all types of questions quickly and without reference to notes or back of textbook or another individual. BUT you may not have time to do every single question available… so… If you are a strong student you may just concentrate on harder TIPS or APP questions, while if you are a weak student you may want to use all your time practicing the basic KU or COMM questions. The number of questions done should also be proportional to your mark so far. If you have very low scores, more practice is required.

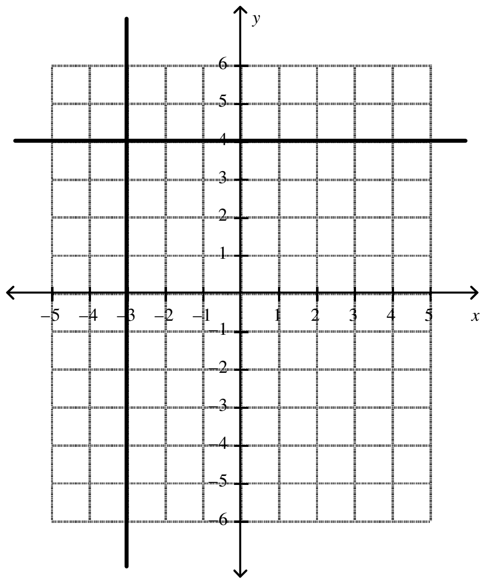
Date	pg	Topics	**# of quest. done?** You may be asked to show them	**Questions I had difficulty with** ask teacher before test!
	1-5	Review graphs from previous grades Getting started pg 2 in textbook		
	6-8	Review functions vs. non-functions Section 1.1		
	9-10	Review transformations of functions Section 1.4		
	11-12	Review inverses of functions Section 1.5 & Handout		
	13-15	Piecewise Functions Section 1.6		
	16-17	Absolute value Section 1.2 & Handout		
	18-20	Properties of functions Section 1.3 & TWO Handouts		
		REVIEW		

Reflect – DIAGNOSTIC TEST mark __________.

Functions studied in grade 9-11

For each of the following state the parent function and the equation of the graphed function. Summarize any key information about this type of relation.

1.



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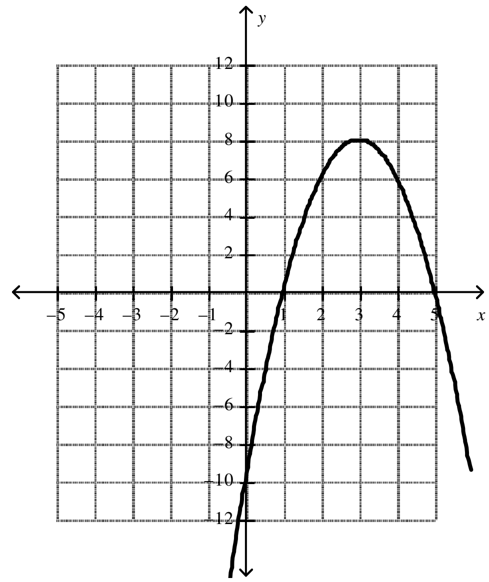

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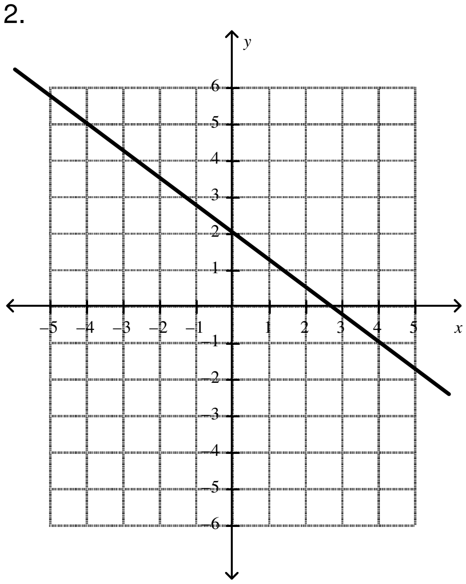

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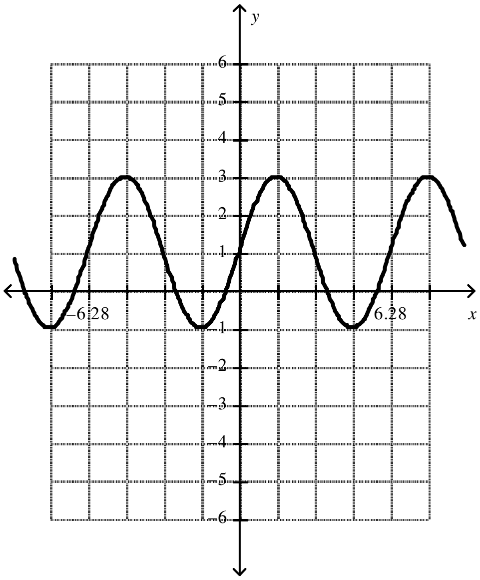
4.



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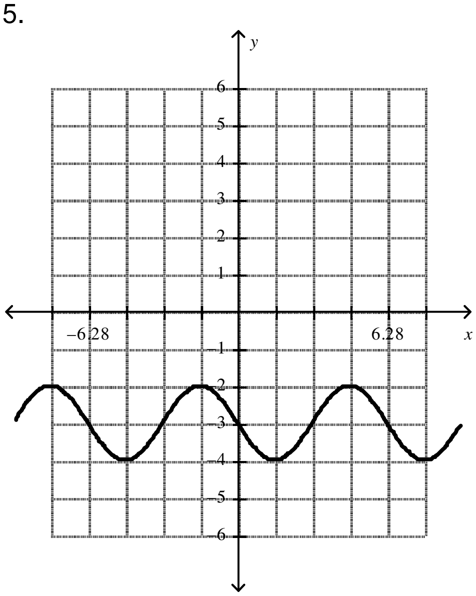

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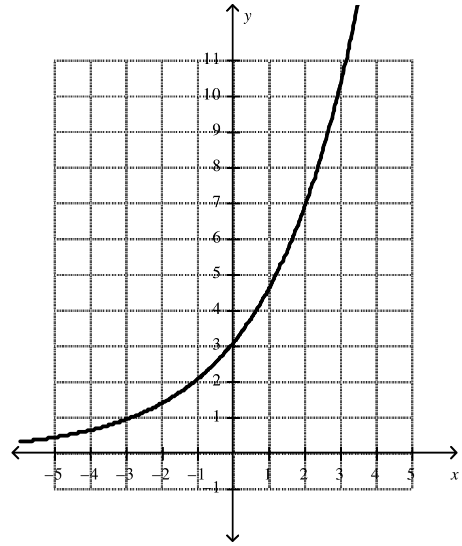

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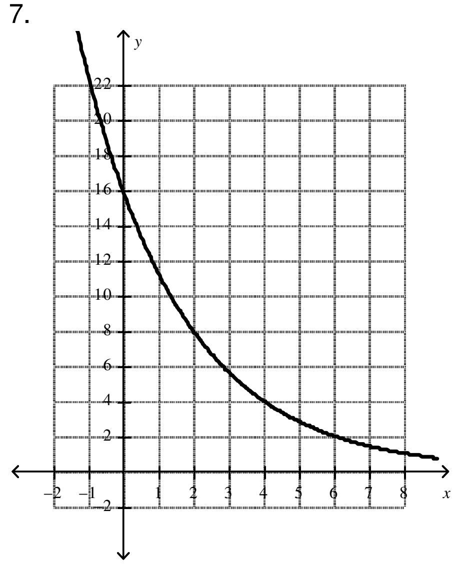
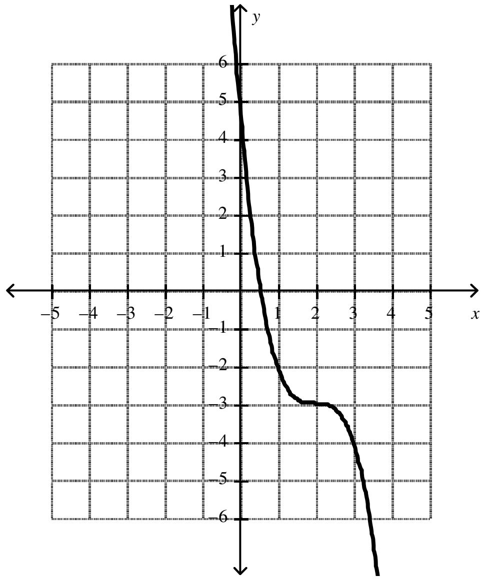


For each of the following state the parent function and the equation of the graphed function. Summarize any key information about this type of relation.

8.



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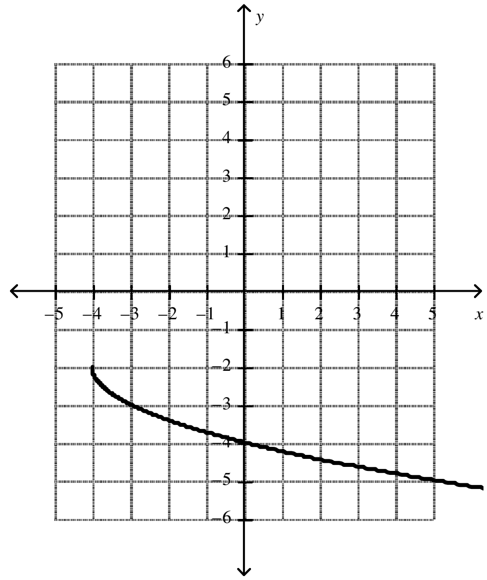

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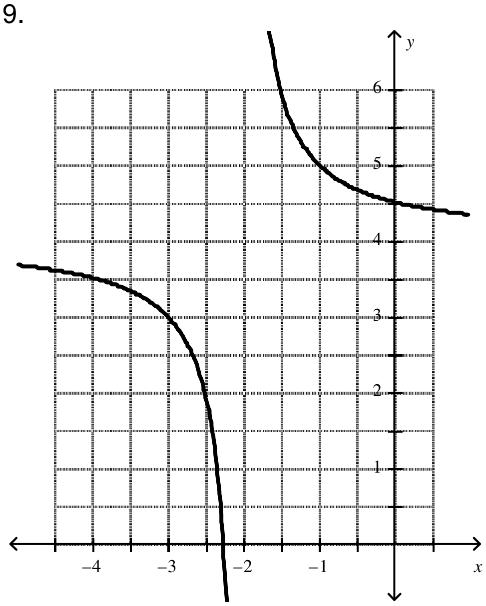

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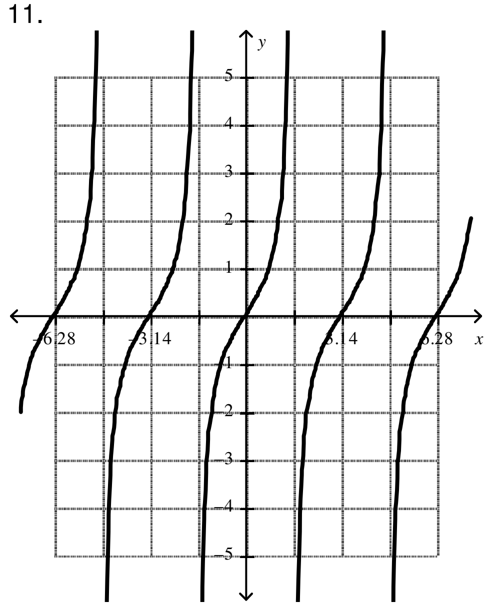


12. For each of the following identify the function that operates on the given input to produce the given output.

a. $f: x \to 5x$

b. $g: \begin{pmatrix} 9 \\ 49 \\ 4 \end{pmatrix} \to \begin{pmatrix} 3 \\ 7 \\ 2 \end{pmatrix}$

c. $h: \begin{pmatrix} 0^o \\ 90^o \\ 180^o \end{pmatrix} \to \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

d. $i: \begin{pmatrix} 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} \to \begin{pmatrix} 5 \\ 10 \\ 17 \\ 26 \end{pmatrix}$

e. $j: \begin{pmatrix} 0 \\ 4 \\ 8 \\ 12 \\ 16 \end{pmatrix} \to \begin{pmatrix} 1024 \\ 128 \\ 16 \\ 2 \\ 0.25 \end{pmatrix}$

f. $k: \begin{pmatrix} -2 \\ 0 \\ 2 \\ 3 \\ 4 \end{pmatrix} \to \begin{pmatrix} 18 \\ 12 \\ 6 \\ 3 \\ 0 \end{pmatrix}$

g. $l: \begin{pmatrix} -1 \\ 2 \\ 5 \\ 8 \\ 11 \end{pmatrix} \to \begin{pmatrix} 0 \\ 0 \\ -54 \\ -162 \\ -324 \end{pmatrix}$

13. Review some mathematical symbols to be used in this course

Functions vs. Non-Functions

1. What characterizes a relationship that is a function, and what may cause it to be a non function?

2. Identify if each of the following is a function or not.

   a. $\{(-3,5),(2,8),(-3,9),(0,-4)\}$

   b. The input is button pressed on a vending machine and the output is the item that you get.

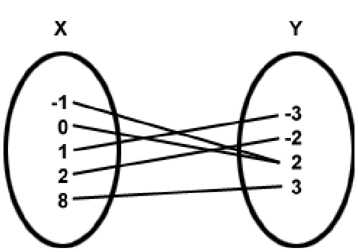
   c. $x^2 + y = 5$

   d. $x + y^2 = 5$

   e.

x	y
0	1
3	-5
6	1

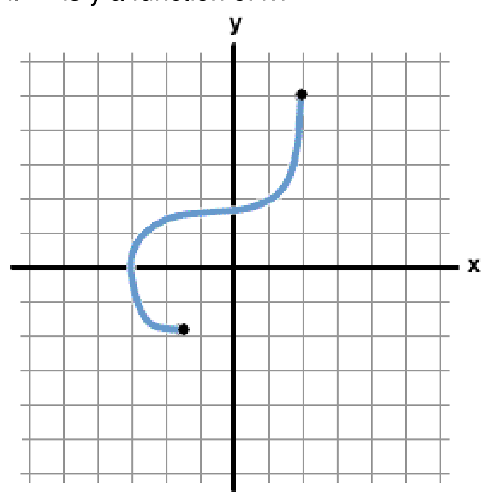
   f.



   g. $\{(-1,4),(0,7),(5,-2),(10,7)\}$

   h. The input is the postal code and the output is a person's address.

   i. is y a function of x?



   j. is x a function of y for the above?

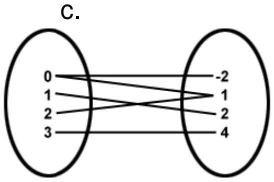
3. What is the domain and range of a relation?

4. State if the following are functions or not and then state the domain and range.

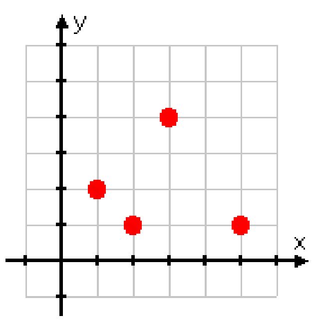
a. You are investigating the average height of a grade 9 student in ajax.

b. $y = \sqrt{3-6x}$

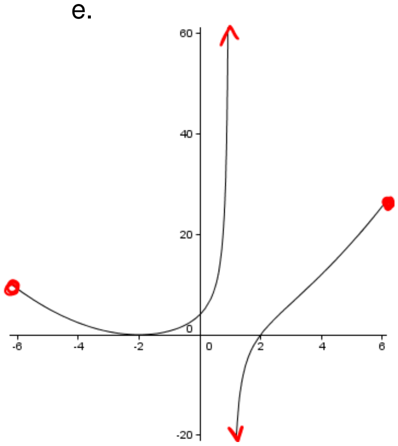
c.



d.



e.



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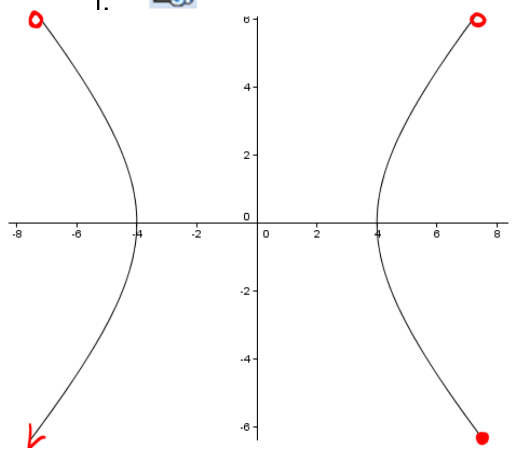


g. $y = -2x^2 + 8x - 1$

h. $y = \dfrac{1}{4-2x}$

i. $x^2 + y^2 = 25$

5. What is function notation? What are the benefits and disadvantages of using the function notation?

6. Find the following for the functions $f(x)=2x^4-3x^2$, $h(a)=\sqrt{a-5}$, $g(k)=\dfrac{6k}{k+1}$

a. $f(2)$

b. $h(69)$

c. $f(\sqrt{5})-h(30)$

d. $f(-3)+4g(3)$

e. $2g\left(\frac{1}{2}\right)f(-1)$

f. $g(3)^{h(6)}$

7. Write down the formula, in function notation for the perimeter of a rectangle, considering perimeter as input and length as output.

8. Write the area of a circle as a function of its perimeter.

9. Write the area of a circle as a function of its radius

10. Write the perimeter of a circle as a function of its radius

11. Write down the formula, in function notation for the area of the triangle, considering area as input and height as output.

Review Transformation of Functions

1. Describe what each constant in $y = af(k(x-d))+c$ controls

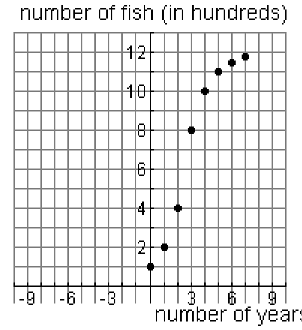
2. Describe the most effective method of applying transformations.

3. Sketch each of the following

a. $h(x) = \dfrac{4}{6+0.5x} + 2$

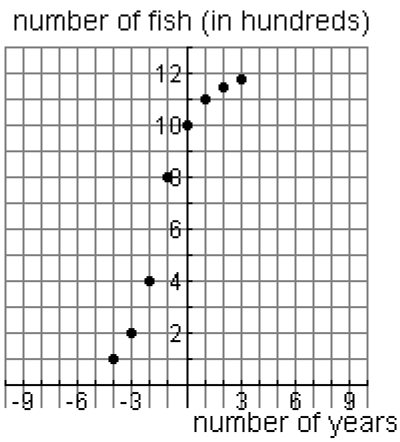
b. $i(x) = 2\sqrt{4-x} - 1$

4. Daniel has a lake in his backyard. He plans to raise fish in his lake. He released 100 fish in his lake in 1996. The graph on the side represents the number of fish in Daniel's lake in terms of the number of years since 1996. Let the function f(t) represent the number of fish in Daniel's lake in terms of the number of years since 1996.



The graphs shown below are a transformation of the function f(t) shown above. For each of the following explain the real-world meaning of the transformation and use function notation to represent it.

a.



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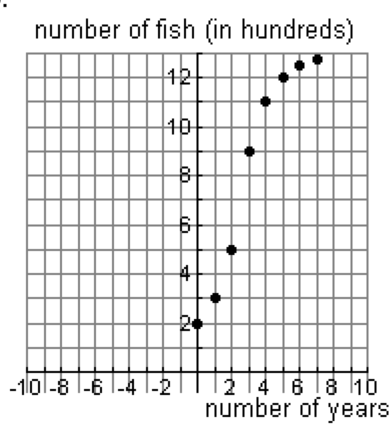
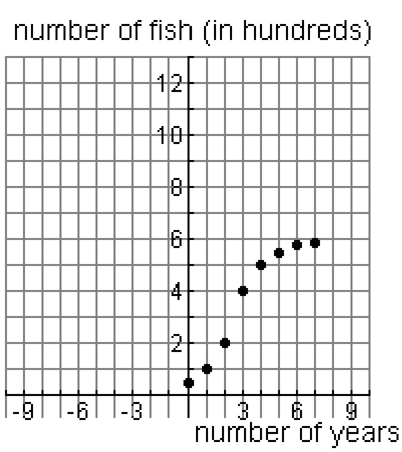
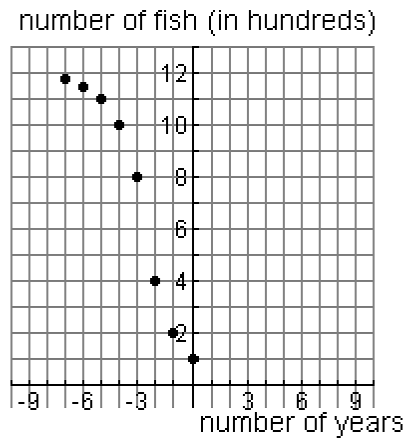


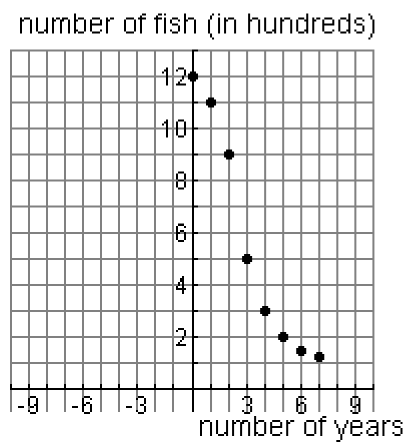
c.



d.



e.



f. If more points were added on either side of this type of graph it will seem to have two horizontal asymptotes, what type of function can model this?

Review of Inverses of Functions

1. Clarify the meaning of the words opposite, reciprocal and inverse.

2. Find the algebraic inverse for each of the following functions

   a. $f(x) = 2x - 6$

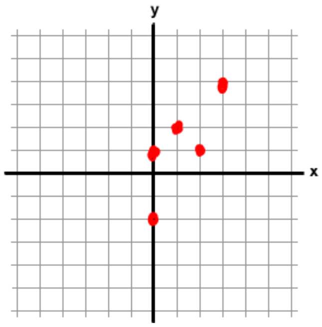
   b. $f(x) = \dfrac{x}{x-3}$

   c. $f(x) = -2(x+5)^2 + 8$

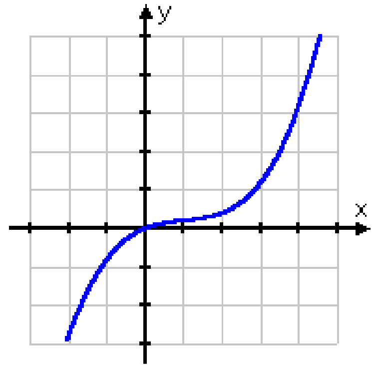
   d. $f(x) = -x^2 + 2x - 9$

3. Find the graphical inverse of each of the following

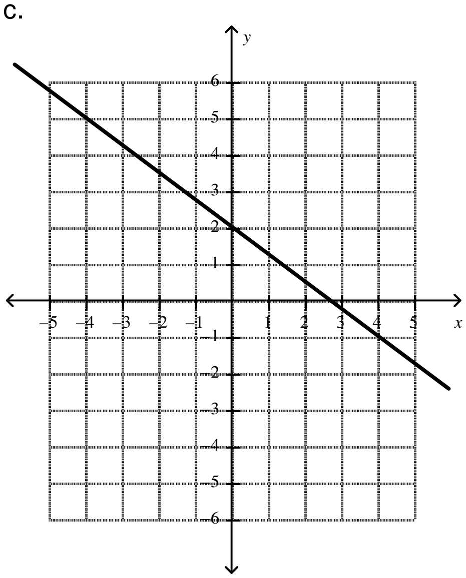
a.



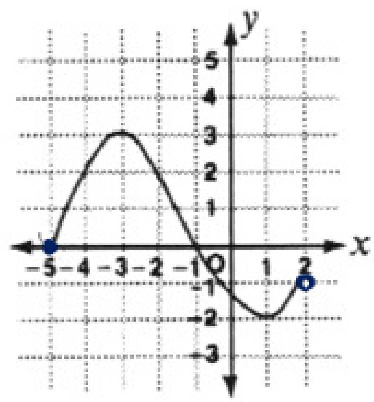
b.



c.



d.



e. Sketch $f(x) = -x^2 + 2x - 9$ (same question as 2d) then sketch $f^{-1}(x)$ on the same grid

4. Can you think of applications of inverse functions?

Piecewise Functions

1. Some relationships in real life cannot be represented by one single function. You can define a relationship using different pieces/functions. Can you make up a real life problem where cost of something changes with time? Write out the problem and show an example of algebraic and graphical model for it.

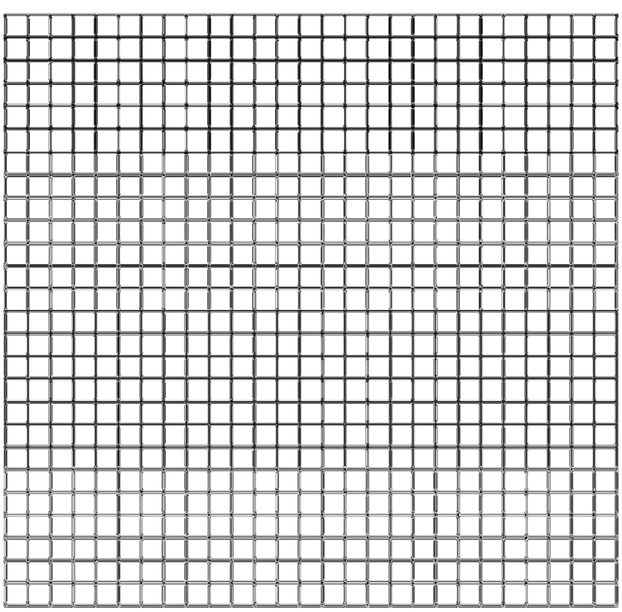
2. Sketch each of the following piecewise graphs.

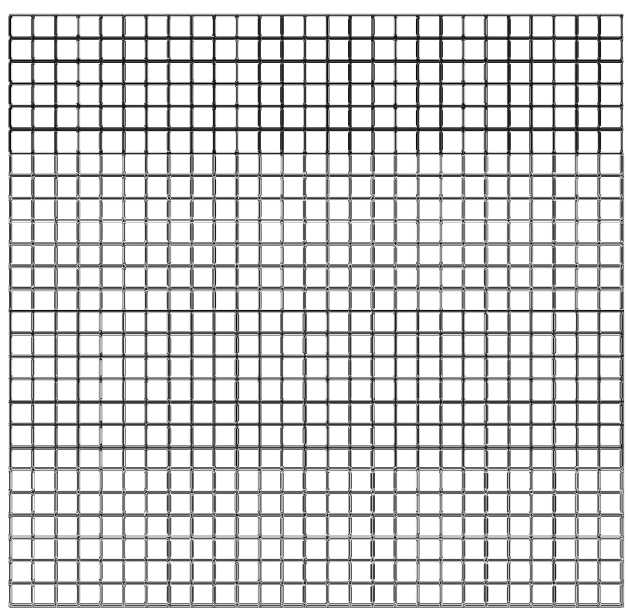
a.

$$f(x)=\begin{cases} x+1 & x\le 2 \\ (x-3)^2+1 & 2<x\le 6 \\ -\dfrac{1}{2}x+5 & 6<x \end{cases}$$

b.

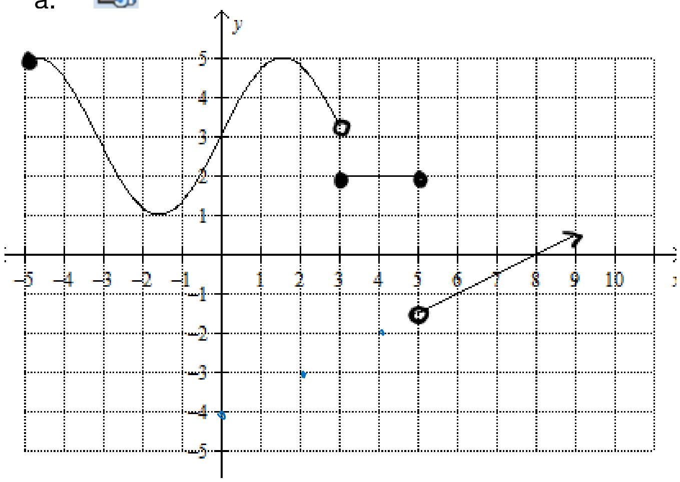
$$f(x)=\begin{cases} -3+\sqrt{-x} & -9<x<-4 \\ -1 & x=-4 \\ x^2+4x+3 & -4<x\le -1 \\ 2^x & 0<x \end{cases}$$





3. Give an algebraic representation for each of the following graphs.

a.



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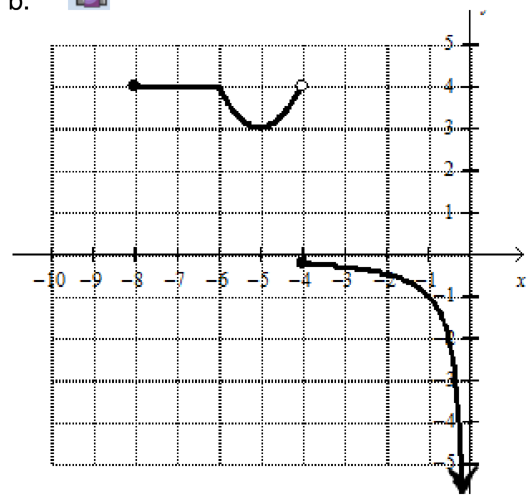
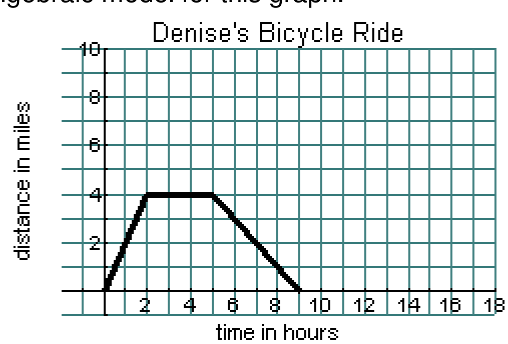


4. Bacteria divides into two every 20 minutes. If you start with a 3000 sample and after 1 hour apply an antibecterial solution, the bacteria population decreases at 1000 bacterium per minute for the first 5 minutes and then remains constant for 2 minutes before it starts to divide again.
   a. Sketch the function.
   b. Give an equation that models this.

5. Denise took a bicycle ride away from her home today. She left home at 12 p.m. and arrived back at home at 9 pm. The graph shown represents Denise's distance from home during her ride. Let the function d(t) represent Denise's distance in miles from home during her trip in terms of the time in hours. Create an algebraic model for this graph.



6. Model the following real life situations with an algebraic model

a.

Provincial tax rates for 2011 are:

5.05% on the first $37,774 of taxable income, +

9.15% on the next $37,776 (on the portion of taxable income between $37,776 and $75,552), +

11.16% on the amount over $75,552

b. Buy 5, get each one after that at half price. Original price is $20.

c.

An airport parking garage costs $20 per day for the first week. After that, the cost decreases to $17 per day.

d.

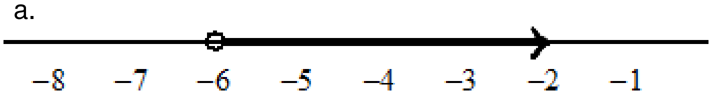
You have a summer job that pays time and half for overtime. That means, if you work more than 40 hours in a week, your hourly wage for the extra hours in 1.5 times your normal rate of $7 per hour.

Absolute Value Functions

1. What is the difference between set notation of a solution and interval notation of the solution?

2. Write the following in both set notation as well as interval notation.

a.



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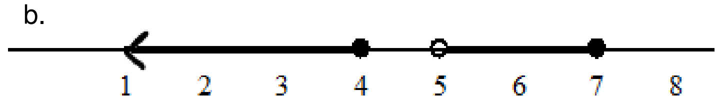


3. What is an absolute value of a number?

4. What is the parent function of the absolute value?

5. How can this function be represented using piecewise notation?

6. What is the difference between 1 and 2 dimensional representations of an absolute value? Show examples.

7. Things to know when solving inequalities:

Solve the following:

8. $4-4\left|8x-4\right|>-76$

9. $-4\left|-3+7v\right|+9\le -59$

10. $3+2\left|9+n\right|\le -1$

11. $-1+4\left|6r\right|>-97$

12. Solve the following absolute values and graph the solution set on a number line

a. $|x-2|=9$

b. $|5x-15|-3\le 42$

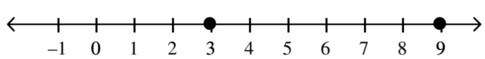
c. $-3|2x+5|>69$

d. $5|18-3x|>55$

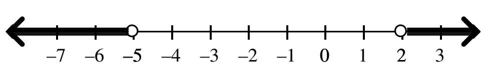
13. Express the following with a few algebraic representations:

$$|x - \text{ centre }| \begin{matrix} = \\ < \\ > \end{matrix} \text{ radius}$$

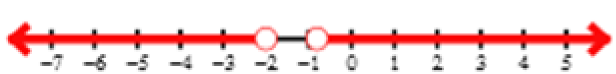
a.



b.



c.



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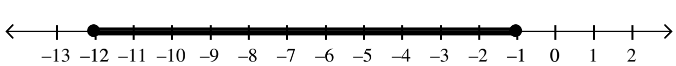

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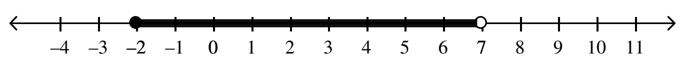
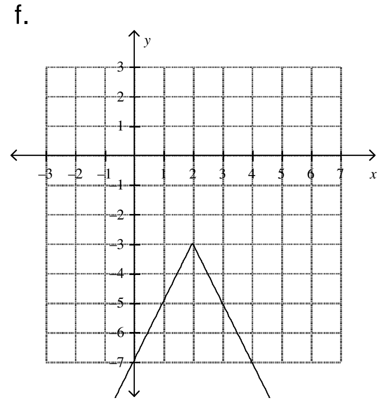
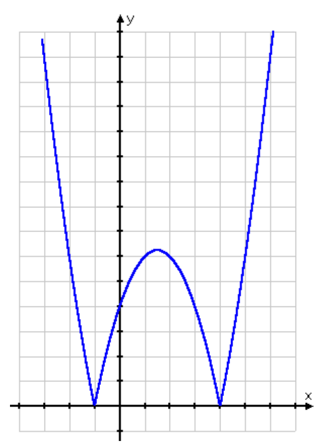


f.



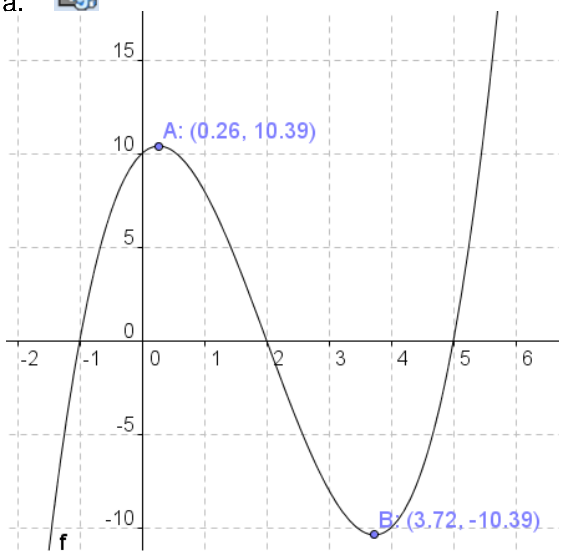
g.



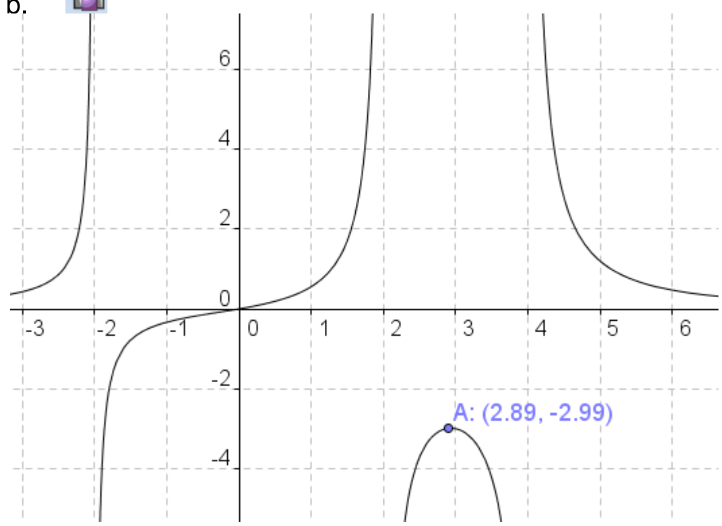
New Properties of Functions

1. Functions can be described using intervals. There are **positive and negative intervals**, as well as **increasing and decreasing intervals**. For the following graphs state both.

a.



b.

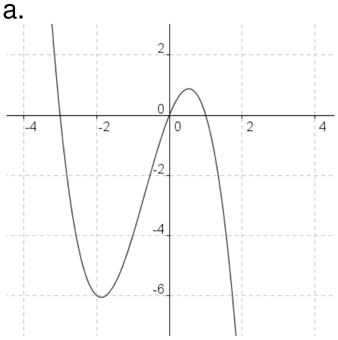


2. What are the points called that separate the positive and negative intervals?

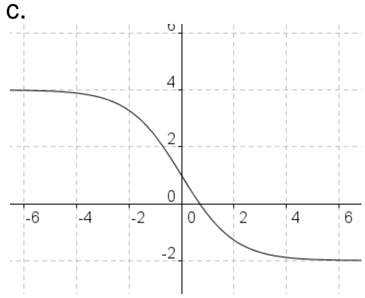
3. What are the points called that separate the increasing decreasing intervals.

4. Functions can also be decribed using **end behaviour.** This helps you recognize what the output values are approaching on the left-most and right-most sides of the graph. For the functions below state the end behaviour in proper notation.

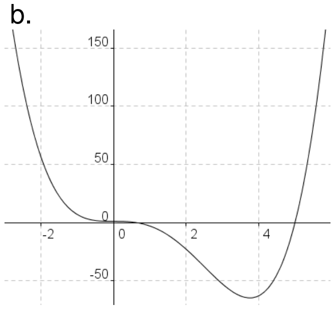
a.



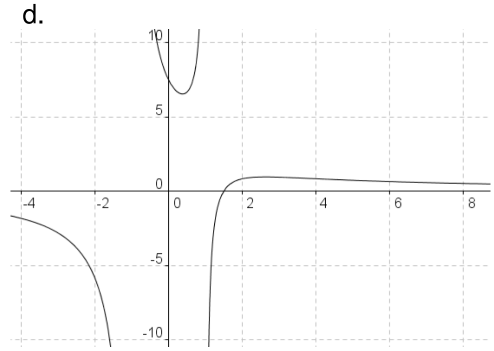
c.



b.



d.



5. Vertical aymptotes cannot be crossed however notice in the last question the horizontal asymptote is crossed. How are horizontal asymptotes related to the end behaviour?

6. Functions can be described using **symmetry**. Describe what is meant by **even**, **odd**, and **neither** symmetry. Show graphical representations and algebraic.

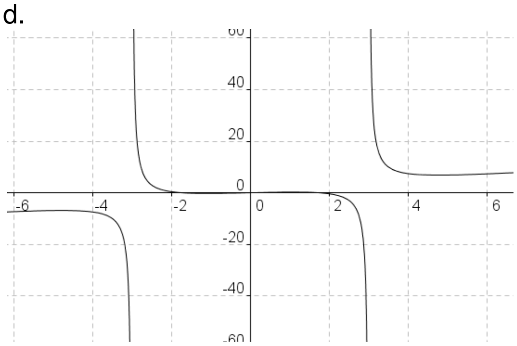
7. Identify whether the following are odd, even or neither in their symmetry.

a. $y = 2x^3 - 5x$

b. $y = x^6 - 5x^4 + 2$

c. $y = 4x^5 - x^3 + 10$

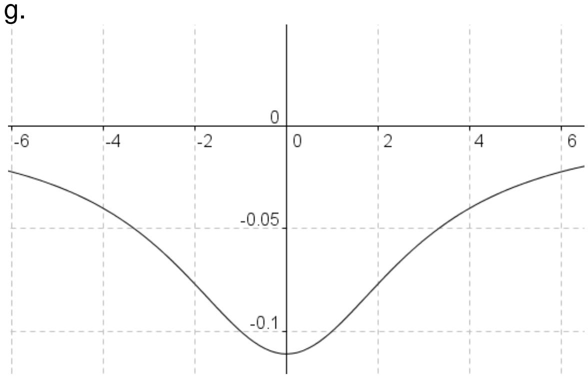
d.



e. $y = \sqrt{4 - x^3}$

f. $y = \dfrac{2}{x^2 - 1}$

g.



8. Finally, functions can also be described as **continuous** or **discontinuous**. What can cause a discontinuity in a function? Show examples.

9. Find the values of each constant that would make this function continuous.

$$f(x)=\begin{cases}2x+a, & x\le -1\\ 22, & -1<x\le 3\\ -bx^2+31, & 3<x\le 5\\ \sqrt{cx}+b, & 5<x\end{cases}$$