Combination of Functions Unit 9

Tentative TEST date_______________

Big idea/Learning Goals

In this unit you will learn how to combine functions by different operations like addition, subtraction, multiplication, division as well as compose one function as an input of another. You will learn how the combined functions are used in real life - they describe more complicated but more realistic situations than you've seen so far. (ex. a spring wobbling up and down will eventually stop its movement due to friction which we have ignored in this type of question when dealing with sinusoidals – we assumed the amplitude will remain constant – not very realistic.) You will see that solving equations is not always possible with algebraic methods when several types of functions are involved and so the use of approximation methods and technology are needed.

Corrections for the textbook answers:

Success Criteria

☐ I understand the new topics for this unit if I can do the practice questions in THIS BOOKLET

Date	pg	Topics	# of quest. done? You may be asked to show them	**Questions I had difficulty with** ask teacher before test!
	2-6	Explore Combined Functions – 2 days Section 9.1 – 9.5 & Handout		
	7-9	Techniques of Solving & Real Life Models Section 9.6 & Handout Section 9.7 & TWO Handouts		
	10	REVIEW – answers are online for this -the answers to the rest of the booklet are not provided		

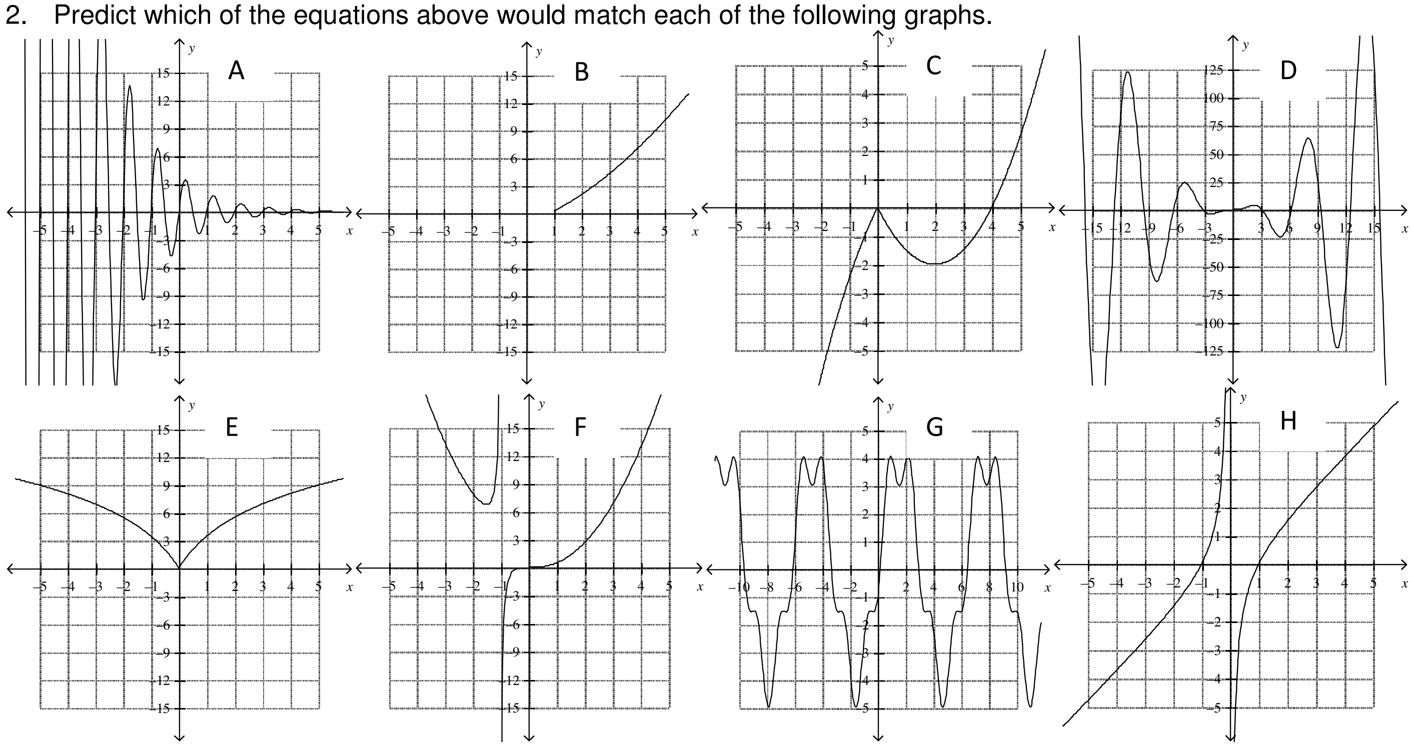
Reflect – previous TEST mark __________, Overall mark now__________.

Explore Combined Functions

1. Identify the OPERATION (product, quotient, sum, difference, piecewise, composition – one function as input of another) with which the functions are combined in each of the following AND identify what shape each separate function has.

$y = x\sqrt{x-1}$	$y = 4\sin x - \cos 4x$	$y = x - \dfrac{1}{x}$	$y = 5\log(\lvert x\rvert + 1)$
$y = (x^2)(\sin(x))$	$y = \begin{cases} -0.5(x-2)^2 + 2, x < 0 \\ 0.5(x-2)^2 - 2, x \geq 0 \end{cases}$	$y = (0.5^x)(4\sin(2\pi x))$	$y = x^3 \div (x+1)$

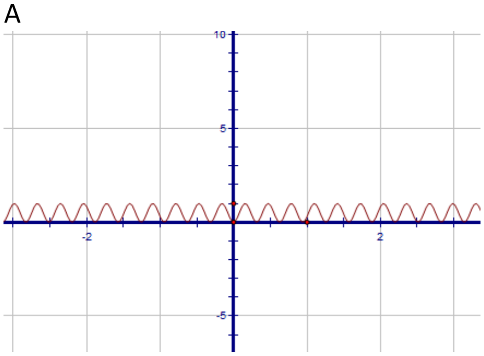
2. Predict which of the equations above would match each of the following graphs.



3. The above functions you had to match were different enough that you probably didn't have that much trouble distinguishing the graphs of each. But if there are only two functions and they were combined in different ways, it may be harder to tell them apart. Write the equations for the following if $f(x)=\sin 10x$ and $g(x)=x^2$ then match to the given graphs. USE PENCIL in case you have to adjust your result after learning some more material.

____________ $g(x)+f(x)=$	____________ $g(x)-f(x)=$
____________ $g(x)f(x)=$	____________ $\dfrac{f(x)}{g(x)}=$
____________ $f(g(x))=$	____________ $g(f(x))=$

A



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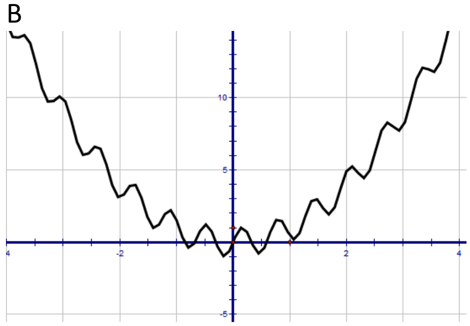

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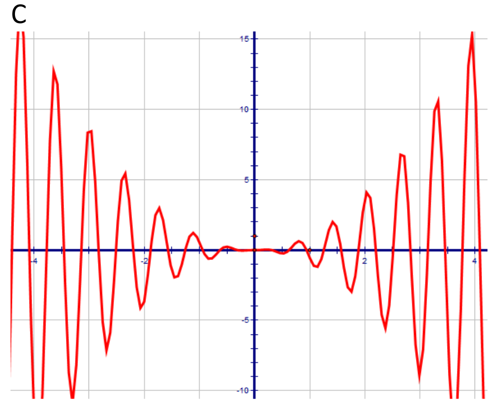

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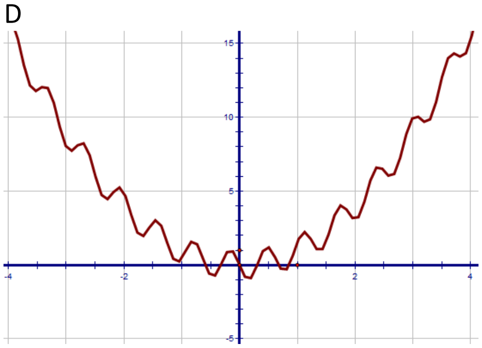

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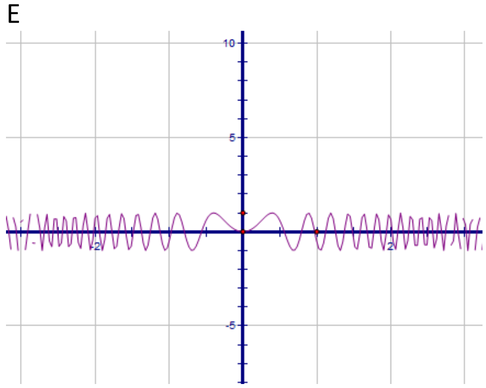

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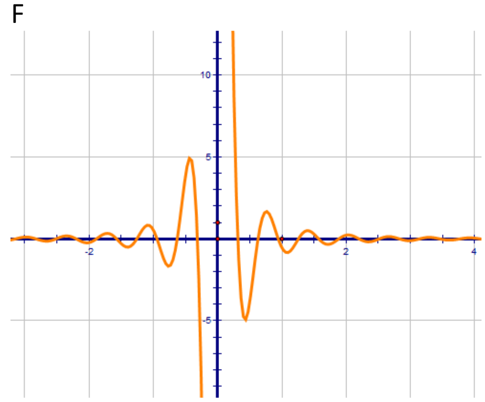
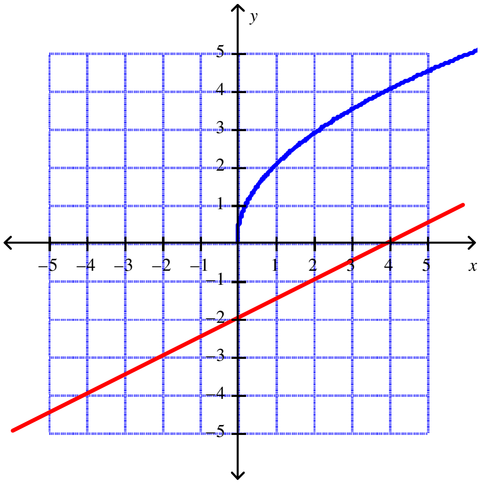
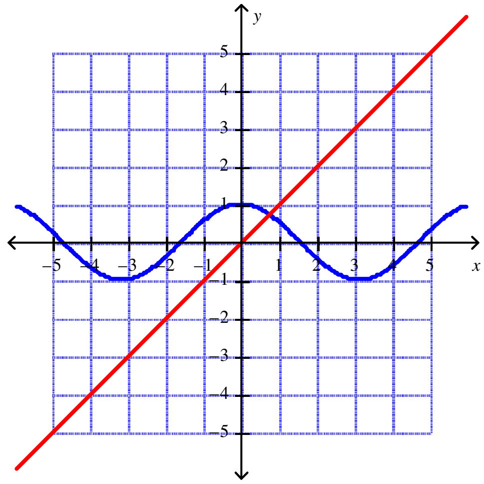


4. Sketch the COMBINED version of the following graphs under the given operation, right onto the picture below, then come up with the equations for the two graphs shown and check your sketch with your combined equation.

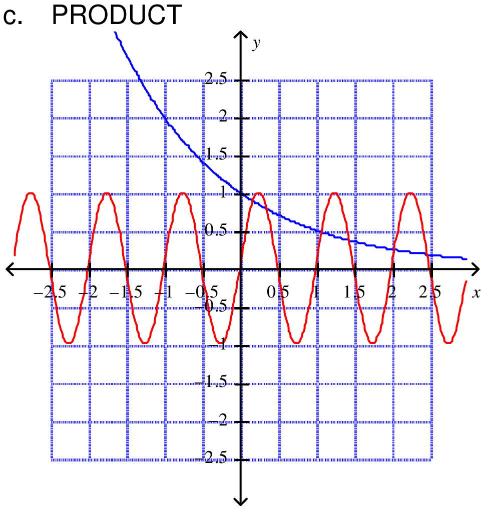
a. PRODUCT



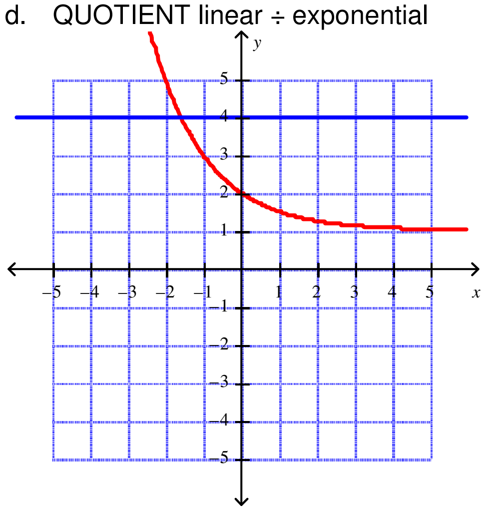
b. SUM



c. PRODUCT

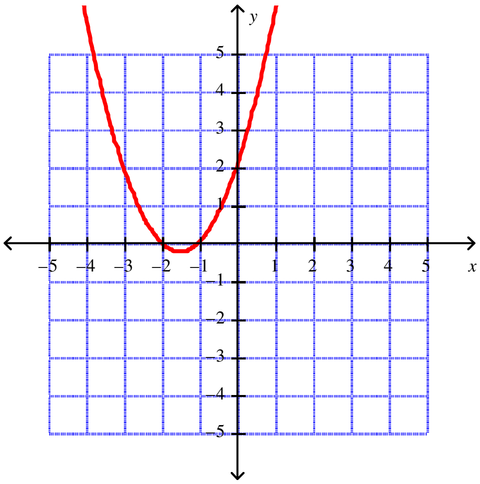


d. QUOTIENT linear ÷ exponential

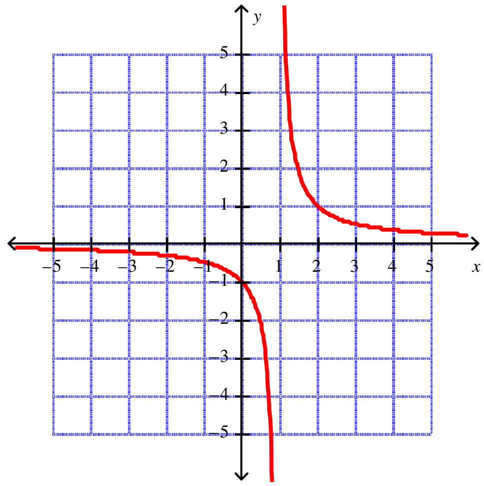


5. Come up with the equations for each of the following

$f(x) =$



$g(x) =$

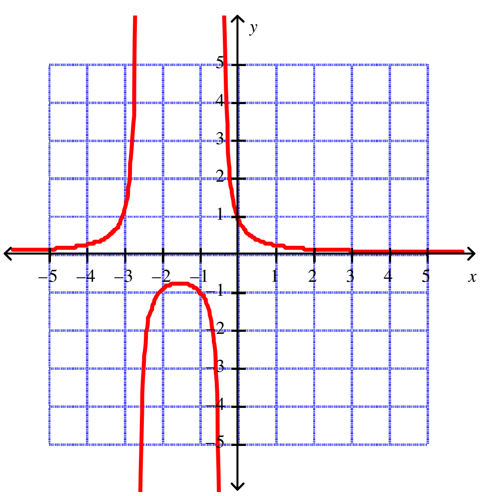


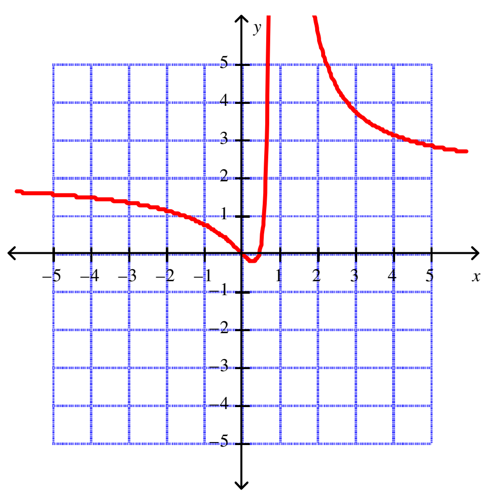
6. Write down and simplify the algebraic models of the different COMPOSTIONS

$h(x) = (f \circ g)(x) = f(g(x))$ and $i(x) = (g \circ f)(x) = g(f(x))$

7. Match these graphs to the composition equations above

(Use what you learned of rational functions to help you decide – zeros, y-int, VA, HA)





8. For the following DISCRETE (which means - not continuous, composed of separate points) functions
$f(x) = \{(0,2),(1,6),(2,4),(3,8),(4,10)\}$
$g(x) = \{(2,4),(4,2),(6,3),(8,0),(10,1)\}$
find

a. $(f - g)(x)$

b. $(f \circ g)(x)$

c. $(f \times g)(x)$

d. $(g \circ f)(x)$

9. Answer the following questions about inverses

   a. For $f(x) = \dfrac{1}{x+3}$, what is $f^{-1}(x)$?

   b. For $g(x) = (-x-5)^2 - 2$ what is $g^{-1}(x)$?

   c. Evaluate the following:

      i. $(f \circ g)(1)$

      ii. $(f^{-1} \circ g)(0)$

      iii. $(f \circ f^{-1})(-5)$

      iv. $(g^{-1} \circ g)(-4)$

      v. Why do iii. functions appear to cancel but not iv.?

Techniques of Solving

1. For the questions below sketch the two functions that are equal to each other on the same grid to see where the approximate solutions are.

a. $\sin \pi x = -2|x+5|$

b. $\sqrt{x+1} = x^2$

c. $4^x = 4x^4$

d. $6\cos x = 2x^2$

Real-Life Models

2. In an effort to boost fan support, the owners of a baseball team have agreed to gradually reduce ticket prices, P, in dollars, according to the function $P(g) = 25 - 0.1g$, where g is the number of games that have been played so far this season. The owners are also randomly giving away free baseball caps. The number, C, in hundreds, of caps given away per game can be modelled by the function $C(g) = 2 - 0.04g$. Since these marketing initiatives began, the number, N, in hundreds, of fans in attendance has been modelled by the function $N(g) = 10 + 0.2g$.
   a. Develop an algebraic and a graphical model (without technology) for $f(g) = P(g)N(g)$ and explain what it means.
   b. Develop an algebraic and a graphical model (without technology) for $h(g) = \dfrac{C(g)}{N(g)}$ and explain what it means.
   c. Will the owners increase or decrease their revenue from ticket sales under their current marketing plan?
   d. How likely are you to receive a free baseball cap if you attend game 5?

3. When the driver of a vehicle observes an obstacle in his path, the driver reacts to apply the brakes and bring the vehicle to a complete stop. The distance that the vehicle travels while coming to a stop is a combination of the reaction distance, r, in meters, given by $r(x)=0.15x$, and breaking distance, b, also in meters, given by $b(x)=0.005x^2$. The speed of the vehicle is x km/h. Determine the stopping distance as a function of speed, and calculate the speed if the stopping distance is 44 meters.

4. A music store has traditionally made a profit from sales of CDs and cassette tapes. The number, C, in thousands, of CDs sold yearly as a function of time, t, in years since the store opened, can be modelled by the function $C(t)=-0.03t^2+0.5t+3$. The number, T, in thousands, of cassette tapes sold as a function of time can be modelled by the function $T(t)=1.5-0.1t$. The store opened in 1990, at which point $t=0$.
   a. Graph both functions (without technology) up to the year 2008 and describe their trends.
   b. The total revenue function $R(t)=[3C(t)+2T(t)](1.04)^t$. Develop an algebraic and a graphical model (can use technology to help with the graph) for the store's revenue, and interpret the trend.

COMBINATION OF FUNCTIONS – review

1. What is the domain of $f - g$, where $f(x) = \sqrt{x+1}$ and $g(x) = 2\log[-(x+1)]$?

2.
a) If $f(x) = \frac{1}{3x+4}$ and $g(x) = \frac{1}{x-2}$, what is $f + g$?
b) What is the domain of $f + g$?
c) What is $(f + g)(8)$?

3. Describe or give an example of
a) two odd functions whose sum is an even function
b) two functions whose sum represents a vertical stretch applied to one of the functions
c) two rational functions whose difference is a constant function

4. Let $f(x) = x^2 - nx + 5$ and $g(x) = mx^2 + x - 3$. The functions are combined to form the new function $h(x) = f(x) + g(x)$. Points $(1, 3)$ and $(-2, 18)$ satisfy the new function. Determine the values of m and n.

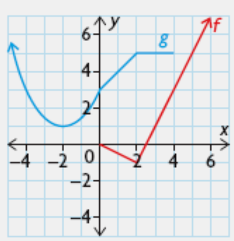
5. If $f(x) = \sqrt{1+x}$ and $g(x) = \sqrt{1-x}$, determine the domain of $y = (f \times g)(x)$.

6. Is the following statement true or false? "If $f(x) \times g(x)$ is an odd function, then both $f(x)$ and $g(x)$ are odd functions." Justify your answer.

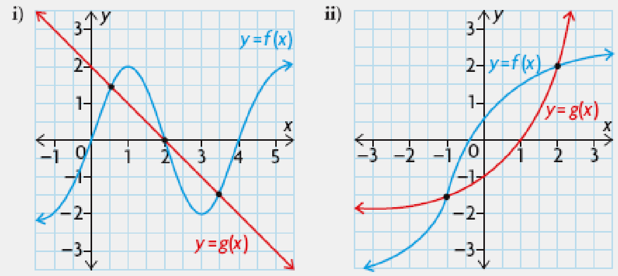
7. $f(x) = x^2$, $g(x) = \log(x)$. State the domain of $f \div g$.

8. Given $f = \{(0, 1), (1, 2), (2, 5), (3, 10)\}$ and $g = \{(2, 0), (3, 1), (4, 2), (5, 3), (6, 4)\}$, determine the following values.
a) $(g \circ f)(2)$ d) $(f \circ g)(0)$
b) $(f \circ f)(1)$ e) $(f \circ f^{-1})(2)$

9. Use the graphs of f and g to evaluate each expression.
a) $f(g(2))$ c) $(g \circ g)(-2)$



10. For each graph shown below, state the solution to each of the following:
a) $f(x) = g(x)$ c) $f(x) \le g(x)$



11. Find the number of solutions of the following by showing sketches
a) $\cos x = x$, when $x \in \left[0, \frac{\pi}{2}\right]$
b) $\sin(2\pi x) = -4x^2 + 16x - 12,\ 0 \le x \le 5$

12. Give an example of two functions, f and g, such that $f(x) \geq g(x)$ when $x \in [-4, -2]$ or $x \in [1, \infty)$.

13. Give an example of two functions, f and g, such that $f(x) \geq 0$ when $x \in [-5, 5]$ and $f(x) \geq g(x)$ when $x \in [-4, 5]$.

14. For $f(x) = -4x + 5$ on domain of $[-2, 12]$
$g(x) = -x^2 + 8x$ on domain of $[-5, 10]$

a. Find the ranges of $f(x)$ and $g(x)$ on the provided domains
b. Find domain of $(f - g)(x)$
c. Find domain of $(f \div g)(x)$
d. Find domain and range of $(f \circ g)(x)$