extraTIPSquestFULLans

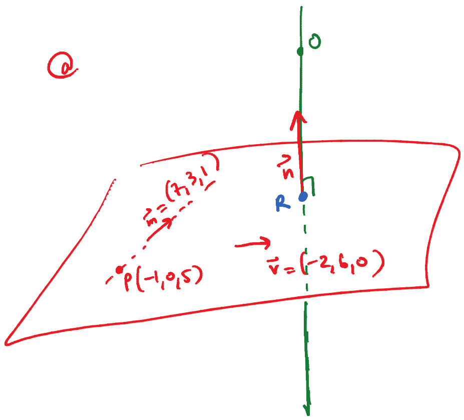
March-19-13
11:58 AM

1. A plane has a line $(x,y,z) = (-1,0,5) + t(7,3,1)$ on it and a vector $\vec{v} = (-2,6,0)$

a) Find the equation of the line (in all forms) that's perpendicular to this plane and goes through the origin.

b) State the equation of the plane and then find the point R where the line in a) meets the plane

a)



$\vec{n} = \vec{v} \times \vec{m}$

$$\begin{matrix} -2 & 6 & 0 & -2 & 6 & 0 \\ 7 & 3 & 1 & 7 & 3 & 1 \end{matrix}$$

$(6-0,\ 0+2,\ -6-42)$

$(6, 2, -48)$

or $(3,1,-24)$

$$\boxed{\therefore (x,y,z) = (0,0,0) + t(3,1,-24)}$$

OR $x = 3t$, $y = t$, $z = -24t$ OR $\dfrac{x}{3} = \dfrac{y}{1} = \dfrac{z}{-24}$

b) plane

$(x,y,z) = (-1,0,5) + r(7,3,1) + q(-2,6,0)$

meet line $(x,y,z) = t(3,1,-24)$

(1) $-1 + 7r - 2q = 3t$

(2) $3r + 6q = t$

(3) $5 + r = -24t$

eliminate q from (1) and (2)

$3\times$(1) $-3 + 21r - 6q = 9t$

(2) $3r + 6q = t$

add $-3 + 24r = 10t$ (4)

now eliminate r from (3) and (4)

$24\times$(3) $120 + 24r = -576t$

(4) $-3 + 24r = 10t$

subt $123 = -586t$

$\dfrac{-123}{586} = t$ don't need r and q to find pt. R just use line equation

"pt R" $\vec{OR} = (x,y,z) = (0,0,0) - \dfrac{123}{586}(3,1,-24)$

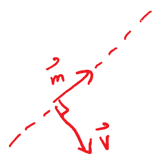
$$\therefore \boxed{R = \left(\frac{-369}{586}, \frac{-123}{586}, \frac{1476}{293}\right)}$$

2.) $\dfrac{-x+5}{7} = \dfrac{y-1}{3} = z$ a) Find the value of k so that vector $\vec{v} = (3,-6,k)$ is $\perp$ to the given line.

b) pt. $Q = (-5,1,0)$ is on the line, find two possible points R, 4 units away from Q in the direction of $\vec{v}$

Rewrite: $\dfrac{+(x-5)}{-7} = \dfrac{y-1}{3} = \dfrac{z-0}{1}$ $\quad \therefore \vec{m} = (-7,3,1)$

a)



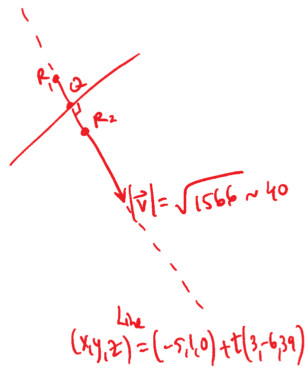
$\vec{m} \cdot \vec{v} = 0$

$(-7,3,1)\cdot(3,-6,k) = 0$

$-21 - 18 + k = 0$

$k = 39$

b)



"pt. R" = "pt Q" $\pm\ 4\hat{v}$

$\vec{OR} = \vec{OQ} \pm 4\dfrac{\vec{v}}{|\vec{v}|}$

$= (-5,1,0) \pm \dfrac{4}{\sqrt{1566}}(3,-6,39)$

$\vec{OR} \doteq (-5,1,0) \pm (0.3,-0.6,3.9)$

$\vec{OR} = (-5.3, 1.6, -3.9)$ or $(-4.7, 0.4, 3.9)$

$\therefore$ pt R can be approx. $(-5.3, 1.6, -3.9)$ or $(-4.7, 0.4, 3.9)$

OR another way

pt. R is on the line

$\therefore$ pt R $(-5+3t, 1-6t, 39t)$

$\therefore \vec{QR} = ((-5+3t)--5, (1-6t)-1, 39t-0)$

$= (3t, -6t, 39t)$

but $|\vec{QR}| = 4$

$\therefore \sqrt{(3t)^2 + (-6t)^2 + (39t)^2} = 4$

$1566t^2 = 16$

$t^2 = \dfrac{8}{783}$

$t = \pm\sqrt{\dfrac{8}{783}} \sim 0.1$

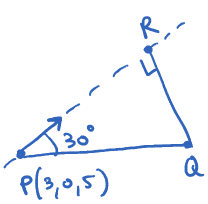
$\therefore$ pt. R $(-5+3(\pm 0.1), 1-6(\pm 0.1), 39(\pm 0.1))$

$(-4.7, 0.4, 3.9)$

or

$(-5.3, 1.6, -3.9)$

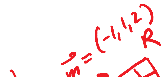
3.) Consider the given picture of a plane with line $\begin{cases} x = 3-t \\ y = t \\ z = 5+2t \end{cases}$ that has pts P and R



and point Q, such that $|\vec{PQ}| = 5$

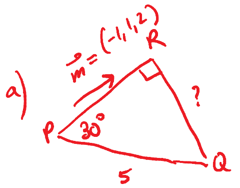
a) Find $|\vec{RQ}|$

b) Find point R



SOH CAH TOA

a)



SOH CAH TOA

$\sin 30^\circ = \frac{|\vec{RQ}|}{5}$

$5 \sin 30 = |\vec{RQ}|$

$\frac{5}{2} = |\vec{RQ}|$

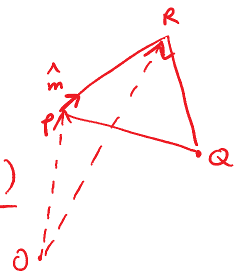
b) size $|\vec{PR}| = 5\cos 30^\circ$

$= \frac{5\sqrt{3}}{2} \doteq 4.3$

"pt. R" = "pt. P" + $\frac{5\sqrt{3}}{2}\hat{m}$ (ie. stretch the unit vector by the size needed.)

$\hat{m}$ ← unit vector in direction of the line.

$\vec{OR} = \vec{OP} + \frac{5\sqrt{3}}{2}\frac{\vec{m}}{|\vec{m}|}$



$(x,y,z) = (3,0,5) + \frac{5\sqrt{3}}{2}\frac{(-1,1,2)}{\sqrt{6}}$

$= (3,0,5) + \frac{5}{2\sqrt{2}}(-1,1,2)$

$\doteq (3,0,5) + (-1.8, 1.8, 3.5)$

$\vec{OR} \doteq (1.2, 1.8, 8.5)$

$\boxed{\therefore \text{pt. R is approx } (1.2, 1.8, 8.5)}$

or another way

pt R (x,y,z) is on line

$\therefore$ pt R $(3-t, t, 5+2t)$

$\therefore \vec{PR} = ((3-t)-3, t-0, (5+2t)-5)$

$= (-t, t, 2t)$

but $|\vec{PR}| = \frac{5}{2}\sqrt{3}$

$\sqrt{(-t)^2 + t^2 + (2t)^2} = \frac{5}{2}\sqrt{3}$

$\sqrt{6t^2} = \frac{5}{2}\sqrt{3}$

$6t^2 = \frac{25}{4}(3)$

$t = \pm\sqrt{\frac{25}{8}} \doteq 1.8$

approx. same

$\therefore$ pt. R $= (3 \pm 1.8, \pm 1.8, 5 + 2(\pm 1.8))$

$= (4.8, 1.8, 8.6)$

or

$= (1.2, -1.8, 1.4)$

Actually two possible R answers since in picture drawn don't know which way $\vec{m}$ is pointed

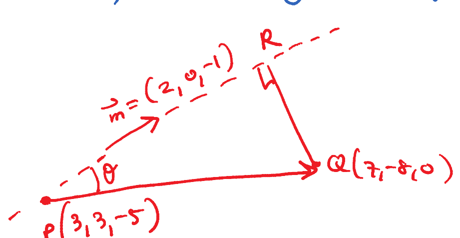
see that

4. Given line: $y = 3$, $\frac{x-3}{2} = \frac{z+5}{-1}$ and a pt. Q$(7,-8,0)$

a) Find the shortest distance from Q to the line

b) Find point R on the line where minimum distance occurs

c) Find the symmetric equation of the line with pts R and Q



a) distance $= |\vec{PQ}|\sin\theta$

$= \frac{|\vec{PQ} \times \vec{m}|}{|\vec{m}|} = \frac{|(11,14,22)|}{\sqrt{5}}$

$= \frac{\sqrt{801}}{\sqrt{5}}$

$\vec{PQ} = (7-3, -8-3, 0--5)$

4 / −11 5 4 −11 5

$P(3,3,-5)$

$\vec{PQ} = (7-3, -8-3, 0--5)$
$= (4,-11,5)$

$(11-0,\ 10+4,\ 0+22)$

$\sqrt{5}$

b) $\vec{OR} = \vec{OP} + \text{proj.}(\vec{PQ} \text{ on } \vec{m})$

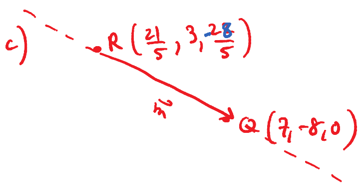
$= (3,3,-5) + \dfrac{(4,-11,5)\cdot(2,0,-1)}{\left(\sqrt{2^2+0^2+1^2}\right)^2}(2,0,-1)$

$= (3,3,-5) + \dfrac{8+0-5}{5}(2,0,-1)$

$= (3,3,-5) + \frac{3}{5}(2,0,-1)$

$\vec{OR} = \left(\frac{21}{5}, 3, -\frac{28}{5}\right)$ $\quad\therefore$ pt. $R = \left(\frac{21}{5}, 3, -\frac{28}{5}\right)$

c)



$(x,y,z) = (7,-8,0) + t(14,-55,+28)$

$\vec{RQ} = \left(7-\frac{21}{5}, -8-3, 0+\frac{28}{5}\right)$

$= \left(\frac{14}{5}, -11, +\frac{28}{5}\right)$

or

$\vec{m} = (14,-55,+28)$

$\boxed{\therefore \text{ Symm. equation is } \quad \dfrac{x-7}{14} = \dfrac{y+8}{-55} = \dfrac{z}{+28}}$

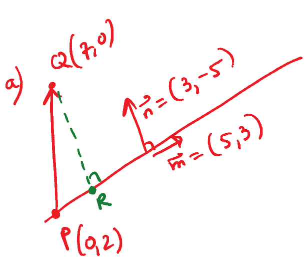
5.) Given line: $3x - 5y + 10 = 0$ and a pt. $Q(7,0)$

a) Find the shortest distance from Q to the line

b) Find point R on the line where minimum distance occurs

c) Find the symmetric equation of the line with pts R and Q

a)



find random pt. P on the line

distance $= \left|\text{proj}(\vec{PQ} \text{ on } \vec{n})\right|$

$= \dfrac{|\vec{PQ}\cdot n|}{|\vec{n}|}$

$= \left|\dfrac{(7,-2)\cdot(3,-5)}{\sqrt{3^2+5^2}}\right|$

$= 21+10$

$$\sqrt{3^2+5^2}$$

$$= \frac{21+10}{\sqrt{34}}$$

$$= \frac{31}{\sqrt{34}}$$

find random pt. P on the line
choose $x=0$
then $y=2$ $\therefore P(0,2)$

$$\therefore \vec{PQ} = (7-0,\ 0-2) = (7,-2)$$

b) Two ways:

option ① "pt. R" = "pt Q" − proj($\vec{PQ}$ on $\vec{n}$) OR "pt R" = "pt. P" + proj($\vec{PQ}$ on $\vec{m}$)

$$\vec{OR} = \vec{OQ} - \frac{\vec{PQ}\cdot\vec{n}}{|\vec{n}|^2}\vec{n}$$

why minus? since $\vec{PQ}$ points the wrong way
or can do $\vec{QP}$ projection on $\vec{n}$ without minus

$$= (7,0) - \frac{31}{34}(3,-5)$$

$$\vec{OR} = \left(\frac{145}{34}, \frac{155}{34}\right)$$

$$\boxed{\therefore \text{pt. } R = \left(\frac{145}{34}, \frac{155}{34}\right)}$$

$$\vec{OR} = \vec{OP} + \frac{\vec{PQ}\cdot\vec{m}}{|\vec{m}|^2}\vec{m}$$

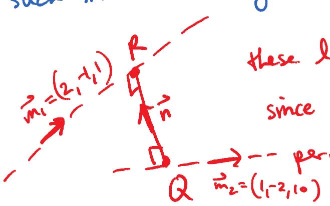
$$= (0,2) + \frac{35-6}{(\sqrt{34})^2}(5,3)$$

$$= (0,2) + \frac{29}{34}(5,3)$$

$$= \left(\frac{145}{34}, \frac{155}{34}\right)$$

c) $(x,y) = (7,0) + t(3,-5)$ $\qquad \boxed{\therefore \frac{x-7}{3} = \frac{y}{-5}}$

⑥ Skew lines (that do not intersect) are given
① $(x,y,z) = (0,2,1) + t(2,-1,1)$
and ② $(x,y,z) = (1,0,1) + r(1,-2,10)$
Find pt. R on line ① and pt. Q on line ②
such that $\vec{RQ}$ is of smallest size possible



these lines will not meet, hard to draw on 2-D paper ∵
since want shortest distance need a vector
perpendicular to both $\vec{m}_1$ and $\vec{m}_2$
do cross product

$$\begin{matrix} 2 & -1 & 1 & 2 & -1 & 1 \\ 1 & -2 & 10 & 1 & -2 & 10 \end{matrix}$$

$(-10+2,\ 1-20,\ -4+1)$
$(-8,-19,-3)$
or $(8,19,3) = \vec{n}$ can reduce direction vectors

pt. R is on line ① and pt. Q is on line ②
$\therefore R\,(2t,\ 2-t,\ 1+t)$ $\qquad \therefore Q\,(1+r,\ -2r,\ 1+10r)$

$$\therefore \vec{RQ} = \big((1+r)-2t,\ -2r-(2-t),\ (1+10r)-(1+t)\big)$$

$= (1+r-2t, -2-2r+t, 10r-t)$

Notice $\overrightarrow{RQ}$ is shorter/longer than $\vec{n}$
but collinear

$\therefore \overrightarrow{RQ} = k\vec{n}$

$(1+r-2t, -2-2r+t, 10r-t) = k(8, 19, 3)$

$\therefore$ ① $1+r-2t = 8k$
② $-2-2r+t = 19k$
③ $10r - t = 3k$

eliminate r from ① and ②

$2\times$① $2+2r-4t = 16k$
② $-2-2r+t = 19k$
add $-3t = 35k$ ④

eliminate r from ① and ③

$10\times$① $10+10r-20t = 80k$
③ $10r - t = 3k$
subt $10 - 19t = 77k$ ⑤

eliminate t from ④ and ⑤

$t = \frac{-35}{3}k$ into

$10 - 19\left(\frac{-35}{3}k\right) = 77k$

$10 = -\frac{434}{3}k$

$\frac{-15}{217} = k$

$\therefore t = \frac{25}{31}$ 0.81 $\therefore r = \frac{13}{217}$ 0.06

$\therefore$ pt. R $(2t, 2-t, 1+t)$
$\sim (1.62, 1.19, 1.81)$
$\left(\frac{50}{31}, \frac{37}{31}, \frac{56}{31}\right)$

$\therefore$ pt. Q $(1+r, -2r, 1+10r)$
$\sim (1.06, -0.12, 1.6)$
$\left(\frac{230}{217}, \frac{-26}{217}, \frac{347}{217}\right)$