ch5Review_22

June-12-13
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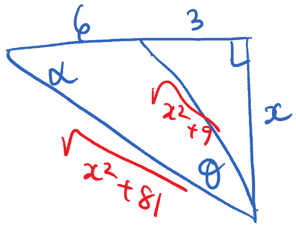

domain: $x \in (0, \infty)$

Use sine law

$$\frac{\sin\theta}{6} = \frac{\sin\alpha}{\sqrt{x^2+9}}$$

But $\sin\alpha = \dfrac{x}{\sqrt{x^2+81}}$

$$\therefore \sin\theta = \frac{6x}{\sqrt{x^2+9}\sqrt{x^2+81}}$$

Maximize: $\theta = \sin^{-1}\left[\dfrac{6x}{\sqrt{x^4+90x^2+729}}\right]$

don't know deriv. of sine inverse so ... do implicit (think $\theta(x)$!)

$$\frac{d}{dx}\left[\sqrt{x^4+90x^2+729}\,\sin\theta = 6x\right]$$

$$\frac{1}{\cancel{2}}(x^4+90x^2+729)^{-1/2}\overset{2x^3+90x}{(\cancel{4}x^3+180x)}\sin\theta + (x^4+90x^2+729)^{1/2}\cos\theta\frac{d\theta}{dx} = 6$$

$$\sqrt{x^4+90x^2+729}\,\cos\theta\frac{d\theta}{dx} = 6 - \frac{(2x^3+90x)\sin\theta}{\sqrt{x^4+90x^2+729}}$$

$$\sqrt{x^4+90x^2+729}\,\cos\theta\frac{d\theta}{dx} = \frac{6\sqrt{x^4+90x^2+729} - (2x^3+90x)\sin\theta}{\sqrt{x^4+90x^2+729}}$$

$$\frac{d\theta}{dx} = \frac{6\sqrt{x^4+90x^2+729} - (2x^3+90x)\sin\theta}{(x^4+90x^2+729)\cos\theta}$$

$$0 = 6\sqrt{x^4+90x^2+729} - (2x^3+90x)\sin\theta$$

$$0 = 6\sqrt{x^4+90x^2+729} - (2x^3+90x)\sin\theta$$

$$(2x^3+90x)\sin\theta = 6\sqrt{x^4+90x^2+729}$$

$$\frac{6x}{\sqrt{x^4+90x^2+729}}$$

$$\frac{(2x^3+90x)(\cancel{6}x)}{\sqrt{x^4+90x^2+729}} = \cancel{6}\sqrt{x^4+90x^2+729}$$

$$2x^4+90x^2 = x^4+90x^2+729$$

$$x^4-729 = 0$$

$$(x^2+27)(x^2-27)=0$$

$$(x^2+27)(x+\sqrt{27})(x-\sqrt{27})=0$$

$\therefore x=\sqrt{27}$ the only crit. pt.

$x \doteq 5.19$

show max:

all methods inconvenient

try #'s close by:

$\theta(5.3) = 29.995$

$\theta(5.19) = 30^\circ \leftarrow$ MAX

$\theta(5.1) = 29.9956$

answer question: the value of x is 5.19 to make θ max.