8. **a.** $x = -\frac{5}{7}t, y = 1 + \frac{2}{7}t, z = t, t \in \mathbf{R}$

b. $x = 3, y = \frac{1}{4}, z = -\frac{1}{2}$

c. $x = 3t - 3s + 7, y = t, z = s,$ $s, t \in \mathbf{R}$

9. **a.** $x = \frac{1}{2} + \frac{1}{36}t, y = -\frac{1}{2} + \frac{5}{12}t,$ $z = t, t \in \mathbf{R}$

b. $x = \frac{9}{8} - \frac{31}{24}t, y = \frac{1}{4} + \frac{1}{12}t, z = t,$ $t \in \mathbf{R}$

10. **a.** These three planes meet at the point $(-1, 5, 3)$.

b. The planes do not intersect. Geometrically, the planes form a triangular prism.

c. The planes meet in a line through the origin, with equation $x = t,$ $y = -7t, z = -5t, t \in \mathbf{R}$

11. 4.90

12. **a.** $x - 2y + z + 4 = 0$
$\vec{r} = (3, 1, -5) + s(2, 1, 0), s \in \mathbf{R}$
$\vec{m} \times \vec{n} = (2, 1, 0)(1, -2, 1) = 0$
Since the line's direction vector is perpendicular to the normal of the plane and the point $(3, 1, -5)$ lies on both the line and the plane, the line is in the plane.

b. $(-1, -1, -5)$

c. $x - 2y + z + 4 = 0$
$-1 - 2(-1) + (-5) + 4 = 0$
The point $(-1, -1, -5)$ is on the plane since it satisfies the equation of the plane.

d. $7x - 2y - 11z - 50 = 0$

13. **a.** 5.48

b. $(3, 0, -1)$

14. **a.** $(-2, -3, 0)$.

b. $\vec{r} = (-2, -3, 0) + t(1, -2, 1),$ $t \in \mathbf{R}$

15. **a.** $-10x + 9y + 8z + 16 = 0$

b. about 0.45

16. **a.** 1

b. $\vec{r} = (0, 0, -1) + t(4, 3, 7), t \in \mathbf{R}$

17. **a.** $x = 2, y = -1, z = 1$

b. $x = 7 - 3t, y = 3 - t, z = t, t \in \mathbf{R}$

18. $a = \frac{2}{3}, b = \frac{3}{4}, c = \frac{1}{2}$

19. $\left(4, -\frac{7}{4}, \frac{7}{2}\right)$

20. $\left(-\frac{5}{3}, \frac{8}{3}, \frac{4}{3}\right)$

21. **a.** $\vec{r} = \left(\frac{45}{4}, 0, -\frac{21}{4}\right) + t(11, 2, -5), t \in \mathbf{R};$
$\vec{r} = \left(-\frac{37}{2}, 0, \frac{15}{2}\right) + t(11, 2, -5), t \in \mathbf{R};$
$\vec{r} = (7, 0, -1) + t(11, 2, -5),$ $t \in \mathbf{R}; z = -1 - 5t, t \in \mathbf{R}$

b. All three lines of intersection found in part a. have direction vector $(11, 2, -5)$, and so they are all parallel. Since no pair of normal vectors for these three planes is parallel, no pair of these planes is coincident.

22. $\left(\frac{1}{2}, 1, \frac{1}{3}\right), \left(\frac{1}{2}, 1, -\frac{1}{3}\right), \left(\frac{1}{2}, -1, \frac{1}{3}\right),$ $\left(\frac{1}{2}, -1, -\frac{1}{3}\right), \left(-\frac{1}{2}, 1, \frac{1}{3}\right),$ $\left(\frac{1}{2}, -1, -\frac{1}{3}\right) \left(-\frac{1}{2}, 1, -\frac{1}{3}\right)$, and $\left(-\frac{1}{2}, -1, \frac{1}{3}\right)$

23. $y = \frac{7}{6}x^2 - \frac{3}{2}x - \frac{2}{3}$

24. $\left(\frac{29}{7}, \frac{4}{7}, -\frac{33}{7}\right)$

25. $A = 5, B = 2, C = -4$

26. **a.** $\vec{r} = (-1, -4, -6) + t(-5, -4, -3), t \in \mathbf{R}$

b. $\left(\frac{13}{2}, 2, -\frac{3}{2}\right)$

c. about 33.26 units^2

27. $6x - 8y + 9z - 115 = 0$

Chapter 9 Test, p. 556

1. **a.** $(3, -1, -5)$

b. $3 - (-1) + (-5) + 1 = 0$
$3 + 1 - 5 + 1 = 0$
$0 = 0$

2. **a.** $\frac{13}{12}$ or 1.08

b. $\frac{40}{3}$ or 13.33

3. **a.** $x = \frac{4t}{5}, y = 1 - \frac{t}{5}, z = t, t \in \mathbf{R}$

b. $(4, 0, 5)$

4. **a.** $(1, -5, 4)$

b. The three planes intersect at the point $(1, -5, 4)$.

5. **a.** $x = -\frac{1}{2} - \frac{t}{4}, y = \frac{3t}{4} + \frac{1}{2}, z = t,$ $t \in \mathbf{R}$

b. The three planes intersect at this line.

6. **a.** $m = -1, n = -3$

b. $x = -1, y = 1 - t, z = t, t \in \mathbf{R}$

7. 10.20

Cumulative Review of Vectors, pp. 557–560

1. **a.** about 111.0°

b. scalar projection: $-\frac{14}{13}$,
vector projection:
$\left(-\frac{52}{169}, \frac{56}{169}, -\frac{168}{169}\right)$

c. scalar projection: $-\frac{14}{3}$,
vector projection:
$\left(-\frac{28}{9}, \frac{14}{9}, \frac{28}{9}\right)$

2. **a.** $x = 8 + 4t, y = t, z = -3 - 3t,$ $t \in \mathbf{R}$

b. about 51.9°

3. **a.** $\frac{1}{2}$

b. 3

c. $\frac{3}{2}$

4. **a.** $-7\vec{i} - 19\vec{j} - 14\vec{k}$

b. 18

5. x-axis: about 42.0°, y-axis: about 111.8°, z-axis: about 123.9°

6. **a.** $(-7, -5, -1)$

b. $(-42, -30, -6)$

c. about 8.66 square units

d. 0

7. $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right)$ and $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$

8. **a.** vector equation: Answers may vary.
$\vec{r} = (2, -3, 1) + t(-1, 5, 2), t \in \mathbf{R};$
parametric equation:
$x = 2 - t, y = -3 + 5t,$
$z = 1 + 2t, t \in \mathbf{R}$

b. If the x-coordinate of a point on the line is 4, then $2 - t = 4$, or $t = -2$. At $t = -2$, the point on the line is $(2, -3, 1) - 2(-1, 5, 2) = (4, -13, -3)$. Hence, $C(4, -13, -3)$ is a point on the line.

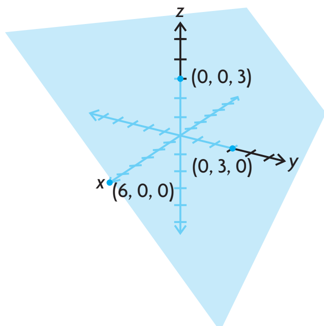
9. The direction vector of the first line is $(-1, 5, 2)$ and of the second line is $(1, -5, -2) = -(-1, 5, 2)$. So they are collinear and hence parallel. The lines coincide if and only if for any point on the first line and second line, the vector connecting the two points is a multiple of the direction vector for the lines. $(2, 0, 9)$ is a point on the first line and $(3, -5, 10)$ is a point on the second line. $(2, 0, 9) - (3, -5, 10) = (-1, 5, -1) \neq k(-1, 5, 2)$ for $k \in \mathbf{R}$. Hence, the lines are parallel and distinct.

10. vector equation:
$\vec{r} = (0, 0, 4) + t(0, 1, 1), t \in \mathbf{R}$;
parametric equation: $x = 0, y = t,$
$z = 4 + t, t \in \mathbf{R}$

11. -13

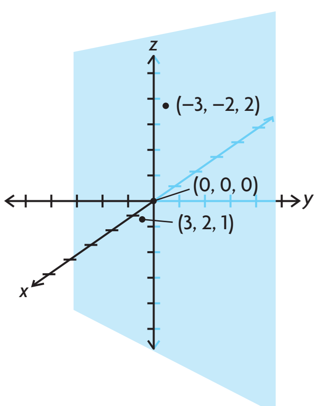
12. $\left(\frac{3}{2}, -\frac{31}{6}, \frac{13}{6}\right)$

13. **a.**



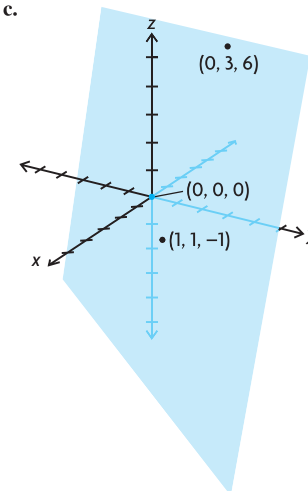
Answers may vary. For example, $(0, 3, -3)$ and $(6, 0, -3)$.

b.



Answers may vary. For example, $(-3, -2, 2)$ and $(3, 2, 1)$.

c.



Answers may vary. For example, $(0, 3, 6)$ and $(1, 1, -1)$.

14. $(-7, 10, 20)$

15. $\vec{q} = (1, 0, 2) + t(-11, 7, 2), t \in \mathbf{R}$

16. **a.** $12x - 9y - 6z + 24 = 0$
b. about 1.49 units

17. **a.** $3x - 5y + 4z - 7 = 0$
b. $x - y + 12z - 27 = 0$
c. $z - 3 = 0$
d. $x + 2z + 1 = 0$

18. 336.80 km/h, N 12.1° W

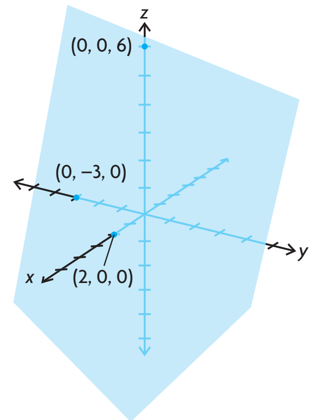
19. **a.** $\vec{r} = (0, 0, 6) + s(1, 0, -3) + t(0, 1, 2), s, t \in \mathbf{R}$. To verify, find the Cartesian equation corresponding to the above vector equation and see if it is equivalent to the Cartesian equation given in the problem. A normal vector to this plane is the cross product of the two directional vectors.

$\vec{n} = (1, 0, -3) \times (0, 1, 2)$

$= (0(2) - (-3)(1), -3(0) - 1(2), 1(1) - 0(0))$
$= (3, -2, 1)$

So the plane has the form $3x + 2y + z + D = 0$, for some constant D. To find D, we know that $(0, 0, 6)$ is a point on the plane, so $3(0) - 2(0) + (6) + D = 0$. So, $6 + D = 0$, or $D = -6$. So, the Cartesian equation for the plane is $3x - 2y + z - 6 = 0$. Since this is the same as the initial Cartesian equation, the vector equation for the plane is correct.

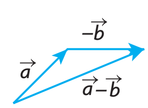
b.



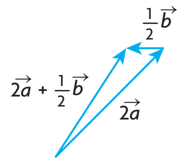
20. **a.** 16°
b. The two planes are perpendicular if and only if their normal vectors are also perpendicular. A normal vector for the first plane is $(2, -3, 1)$ and a normal vector for the second plane is $(4, -3, -17)$. The two vectors are perpendicular if and only if their dot product is zero.
$(2, -3, 1) \cdot (4, -3, -17)$
$= 2(4) - 3(-3) + 1(-17)$
$= 0$
Hence, the normal vectors are perpendicular. Thus, the planes are perpendicular.
c. The two planes are parallel if and only if their normal vectors are also parallel. A normal vector for the first plane is $(2, -3, 2)$ and a normal vector for the second plane is $(2, -3, 2)$. Since both normal vectors are the same, the planes are parallel. Since $2(0) - 3(-1) + 2(0) - 3 = 0$, the point $(0, -1, 0)$ is on the second plane. Yet since $2(0) - 3(-1) + 2(0) - 1 = 2 \neq 0$, $(0, -1, 0)$ is not on the first plane. Thus, the two planes are parallel but not coincident.

21. resultant: about 56.79 N, 37.6° from the 25 N force toward the 40 N force, equilibrant: about 56.79 N, 142.4° from the 25 N force away from the 40 N force

22. **a.**



b.



23. **a.** $\left(\frac{6}{7}, \frac{2}{7}, -\frac{3}{7}\right)$

b. $\left(-\frac{6}{7}, -\frac{2}{7}, \frac{3}{7}\right)$

24. **a.** $\overrightarrow{OC} = (8, 9)$,
$\overrightarrow{BD} = (10, -5)$

b. about 74.9°

c. about 85.6°

25. **a.** $x = t, y = -1 + t, z = 1, t \in \mathbf{R}$

b. $(1, 2, -3)$

c. $x = 1, y = t, z = -3 + t, t \in \mathbf{R}$

d. $x = 1 + 3s + t, y = t, z = s,$
$s, t \in \mathbf{R}$

26. **a.** yes; $x = 0, y = -1 + t, z = t, t \in \mathbf{R}$

b. no

c. yes;
$x = 2 - 2t, y = t, z = 3t, t \in \mathbf{R}$

27. 30°

28. **a.** $-\frac{3}{2}$

b. 84

29. $\vec{r} = t(-1, 3, 1), t \in \mathbf{R}$,
$-x + 3y + z - 11 = 0$

30. $(-1, 1, 0)$

31. **a.** 0.8 km

b. 12 min

32. **a.** Answers may vary.
$\vec{r} = (6, 3, 4) + t(4, 4, 1), t \in \mathbf{R}$

b. The line found in part a will lie in the plane $x - 2y + 4z - 16 = 0$ if and only if both points $A(2, -1, 3)$ and $B(6, 3, 4)$ lie in this plane. We verify this by substituting these points into the equation of the plane, and checking for consistency.
For A:
$2 - 2(-1) + 4(3) - 16 = 0$
For B:
$6 - 2(3) + 4(4) - 16 = 0$
Since both points lie on the plane, so does the line found in part a.

33. 20 km/h at N 53.1° E

34. parallel: 1960 N,
perpendicular: about 3394.82 N

35. **a.** True; all non-parallel pairs of lines intersect in exactly one point in $\mathbf{R}^2$. However, this is not the case for lines in $\mathbf{R}^3$ (skew lines provide a counterexample).

b. True; all non-parallel pairs of planes intersect in a line in $\mathbf{R}^3$.

c. True; the line $x = y = z$ has direction vector $(1, 1, 1)$, which is not perpendicular to the normal vector $(1, -2, 2)$ to the plane $x - 2y + 2z = k$, k is any constant. Since these vectors are not perpendicular, the line is not parallel to the plane, and so they will intersect in exactly one point.

d. False; a direction vector for the line $\frac{x}{2} = y - 1 = \frac{z + 1}{2}$ is $(2, 1, 2)$. A direction vector for the line $\frac{x - 1}{-4} = \frac{y - 1}{-2} = \frac{z + 1}{-2}$ is $(-4, -2, -2)$, or $(2, 1, 1)$ (which is parallel to $(-4, -2, -2)$). Since $(2, 1, 2)$ and $(2, 1, 1)$ are obviously not parallel, these two lines are not parallel.

36. **a.** A direction vector for
L_1: $x = 2, \frac{y - 2}{3} = z$ is $(0, 3, 1)$,
and a direction vector for
L_2: $x = y + k = \frac{z + 14}{k}$ is $(1, 1, k)$.
But $(0, 3, 1)$ is not a nonzero scalar multiple of $(1, 1, k)$ for any k, since the first component of $(0, 3, 1)$ is 0. This means that the direction vectors for L_1 and L_2 are never parallel, which means that these lines are never parallel for any k.

b. 6; $(2, -4, -2)$

Calculus Appendix

Implicit Differentiation, p. 564

1. The chain rule states that if y is a composite function, then $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$. To differentiate an equation implicitly, first differentiate both sides of the equation with respect to x, using the chain rule for terms involving y, then solve for $\frac{dy}{dx}$.

2. **a.** $-\frac{x}{y}$

b. $\frac{x^2}{5y}$

c. $\frac{-y^2}{2xy + y^2}$

d. $\frac{9x}{16y}$

e. $-\frac{13x}{48y}$

f. $-\frac{2x}{2y + 5}$

3. **a.** $y = \frac{2}{3}x - \frac{13}{3}$

b. $y = \frac{2}{3}(x + 8) + 3$

c. $y = -\frac{3\sqrt{3}}{5}x - 3$

d. $y = \frac{11}{10}(x + 11) - 4$

4. $(0, 1)$

5. **a.** 1

b. $\left(\frac{3}{\sqrt{5}}, \sqrt{5}\right)$ and $\left(-\frac{3}{\sqrt{5}}, -\sqrt{5}\right)$

6. -10

7. $7x - y - 11 = 0$

8. $y = \frac{1}{2}x - \frac{3}{2}$

9. **a.** $\frac{4}{(x + y)^2} - 1$

b. $4\sqrt{x + y} - 1$

10. **a.** $\frac{3x^2 - 8xy}{4x^2 - 3}$

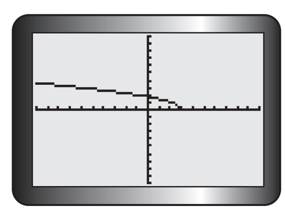
b. $y = \frac{x^3}{4x^2 - 3}; \frac{4x^4 - 9x^2}{(4x^2 - 3)^2}$

c. $\frac{dy}{dx} = \frac{3x^2 - 8xy}{4x^2 - 3}$

$y = \frac{x^3}{4x^2 - 3}$

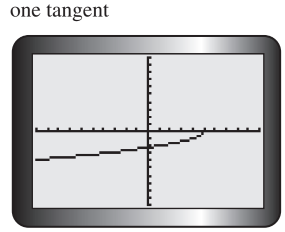
$$\frac{dy}{dx} = \frac{3x^2 - 8x\left(\frac{x^3}{4x^2 - 3}\right)}{4x^2 - 3}$$
$$= \frac{3x^2 - (4x^2 - 3) - 8x^4}{(4x^2 - 3)^2}$$
$$= \frac{12x^4 - 9x^2 - 8x^4}{(4x^2 - 3)^2}$$
$$= \frac{4x^4 - 9x^2}{(4x^2 - 3)^2}$$

11. **a.**



one tangent

b.



one tangent