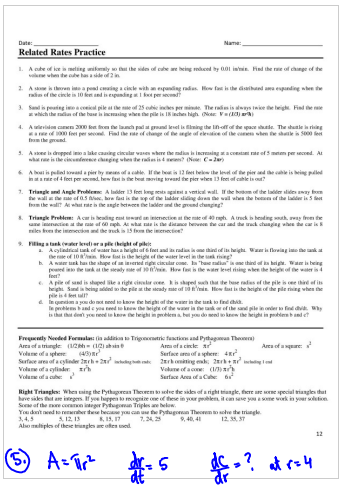
ansPRACTICERelatedRates



Date: _____ Name: _____

Related Rates Practice

1. A cube of ice is melting uniformly so that the sides of cube are being reduced by 0.01 in/min. Find the rate of change of the volume when the cube has a side of 2 in.
2. A stone is thrown into a pond creating a circle with an expanding radius. How fast is the distributed area expanding when the radius of the circle is 10 feet and is expanding at 1 foot per second?
3. Sand is pouring into a conical pile at the rate of 25 cubic inches per minute. The radius is always twice the height. Find the rate at which the radius of the base is increasing when the pile is 18 inches high. (Note: $V = \frac{1}{3}\pi r^2 h$)
4. A television camera 2000 feet from the launch pad at ground level is filming the lift-off of the space shuttle. The shuttle is rising at a rate of 1000 feet per second. Find the rate of change of the angle of elevation of the camera when the shuttle is 5000 feet from the ground.
5. A stone is dropped into a lake causing circular waves where the radius is increasing at a constant rate of 5 meters per second. At what rate is the circumference changing when the radius is 4 meters? (Note: $C = 2\pi r$)
6. A boat is pulled toward a pier by means of a cable. If the boat is 12 feet below the level of the pier and the cable is being pulled in at a rate of 4 feet per second, how fast is the boat moving toward the pier when 13 feet of cable is out?
7. **Triangle and Angle Problems:** A ladder 13 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at the rate of 0.5 ft/sec, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 5 feet from the wall? At what rate is the angle between the ladder and the ground changing?
8. **Triangle Problem:** A car is heading east toward an intersection at the rate of 40 mph. A truck is heading south, away from the same intersection at the rate of 60 mph. At what rate is the distance between the car and the truck changing when the car is 8 miles from the intersection and the truck is 15 from the intersection?
9. **Filling a tank (water level) or a pile (height of pile):**
   a. A cylindrical tank has a radius of 6 feet and a height of 10 feet and its radius is one third of its height. Water is flowing into the tank at the rate of 10 ft^3/min. How fast is the height of the water level in the tank rising?
   b. A water tank has the shape of an inverted right circular cone. Its "base radius" is one third of its height. Water is being poured into the tank at the steady rate of 10 ft^3/min. How fast is the water level rising when the height of the water is 4 feet?
   c. A pile of sand is shaped like a right circular cone. It is shaped such that the base radius of the pile is one third of its height. Sand is being added to the pile at the steady rate of 10 ft^3/min. How fast is the height of the pile rising when the pile is 6 feet tall?
   d. In question a you do not need to know the height of the water in the tank to find chith. In problems b and c you need to know the height of the water in the tank or of the sand pile in order to find chith. Why is that this don't you need to know the height in problem a, but you do need to know the height in problem b and c?

Frequently Needed Formulas: (in addition to Trigonometric functions and Pythagorean Theorem)

Area of a triangle: $(1/2)bh = (1/2)ab\sin\theta$	Area of a circle: πr^2	Area of a square: s^2
Volume of a sphere: $(4/3)\pi r^3$	Surface area of a sphere: $4\pi r^2$	
Surface area of a cylinder $2\pi rh + 2\pi r^2$ including both ends:	$2\pi rh$ omitting ends; $2\pi rh + \pi r^2$ including 1 end	
Volume of a cylinder: $\pi r^2 h$	Volume of a cone: $(1/3)\pi r^2 h$	
Volume of a cube: s^3	Surface Area of a Cube: $6s^2$	

Right Triangles: When using the Pythagorean Theorem to solve the sides of a right triangle, there are some special triangles that have sides that are integers. If you happen to recognize one of these in your problem, it can save you a some work in your solution. Some of the more common integer Pythagorean Triples are below.
You don't need to remember these because you can use the Pythagorean Theorem to solve the triangle.
3, 4, 5 5, 12, 13 8, 15, 17 7, 24, 25 9, 40, 41 12, 35, 37
Also multiples of these triangles are often used.

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(1.) $SA = 6s^2$, $V = s^3$; $\frac{ds}{dt} = 0.01$; $\frac{dV}{dt} = ?$ at $s = 2$

$\frac{dV}{dt} = 3s^2\frac{ds}{dt}$

$\frac{dV}{dt} = 3(2)^2(0.01) = 0.12$ in^2/min

(2.) $A = \pi r^2$; $\frac{dA}{dt} = ?$ at $r = 10$; $\frac{dr}{dt} = 1$

$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

$\frac{dA}{dt} = 2(\pi)(10)(1) = 20\pi$ ft^2/sec

(3.) $V = \frac{\pi}{3}r^2h$; $\frac{dV}{dt} = 25$; $\frac{dr}{dt} = ?$ at $h = 18$; $r = 2h$

$\frac{dr}{dt} = 2\frac{dh}{dt} \rightarrow \frac{dh}{dt} = \frac{1}{2}\frac{dr}{dt}$

sub

$\frac{dV}{dt} = \frac{\pi}{3}\left(2r\frac{dr}{dt}h\right) + \frac{\pi}{3}r^2\frac{dh}{dt}$

$25 = \frac{2\pi}{3}(36)(18)\frac{dr}{dt} + \frac{\pi}{3}(36)^2\frac{1}{2}\frac{dr}{dt}$

$25 = 432\pi\frac{dr}{dt} + 216\pi\frac{dr}{dt}$

$\frac{25}{648\pi} = \frac{dr}{dt}$ in/min

(4.) $\frac{dh}{dt} = 1000$; $\frac{d\theta}{dt} = ?$ at $h = 5000$; $\theta = \tan^{-1}\left(\frac{5}{2}\right)$; $\theta \approx 68°$

$\tan\theta = \frac{h}{2000}$

$\sec^2\theta\frac{d\theta}{dt} = \frac{1}{2000}\frac{dh}{dt}$

$\sec^2 68°\frac{d\theta}{dt} = \frac{1}{2000}(1000)$

$\frac{d\theta}{dt} = \frac{1}{2\sec^2 68} \approx \frac{\cos^2 68}{2}(0.0689°/\text{sec})$

(5.) $A = \pi r^2$; $\frac{dr}{dt} = 5$; $\frac{dC}{dr} = ?$ at $r = 4$

$C = 2\pi r$

$\frac{dC}{dt} = 2\pi\frac{dr}{dt} = 2\pi(5) = 10\pi$ m/s

(6.) $\frac{dc}{dt} = 4$; $\frac{dH}{dt} = ?$ at $C = 13$; $H = \sqrt{13^2 - 12^2}$; $H = 5$

$H^2 + 12^2 = C^2$

$2H\frac{dH}{dt} + 0 = 2C\frac{dC}{dt}$

$2(5)\frac{dH}{dt} = 2(13)(4)$

$\frac{dH}{dt} = 10.4$ ft/sec

(7.) $\cos\theta = \frac{H}{13}$; $\frac{dH}{dt} = 0.5$; $\frac{d\theta}{dt} = ?$ at $H = 5$; $\theta = \cos^{-1}\left(\frac{5}{13}\right)$; $\theta \approx 65.4°$

$-\sin\theta\frac{d\theta}{dt} = \frac{1}{13}\frac{dH}{dt}$

$-\sin(65.4°)\frac{d\theta}{dt} = \frac{0.5}{13}$

$\frac{d\theta}{dt} = \frac{-0.5}{13\sin 65.4°} \approx -0.0423°/\text{sec}$

(8.) $\frac{dE}{dt} = 40$; $\frac{dS}{dt} = 60$; $\frac{dD}{dt} = ?$ at $E = 8$, $S = 15$, $\therefore D = 17$

$D^2 = E^2 + S^2$

$2D\frac{dD}{dt} = 2E\frac{dE}{dt} + 2S\frac{dS}{dt}$

$2(17)\frac{dD}{dt} = 2(8)(40) + 2(15)(60)$

$\frac{dD}{dt} = \frac{2440}{34} \approx 71.76$ mph

(9.) @ $\frac{dV}{dt} = 10$; $\frac{dh}{dt} = ?$; $r = 2$, $h = 6$

radius will not change with time! consider constant.

$V = \pi r^2 h$

$\frac{dV}{dt} = \pi r^2\frac{dh}{dt}$

$10 = \pi(2)\frac{dh}{dt}$

$\frac{5}{\pi}$ ft/min $= \frac{dh}{dt}$

(10.) $\frac{dV}{dt} = 10$; $\frac{dh}{dt} = ?$ at $h = 4$, $r = \frac{4}{3}$

$V = \frac{\pi}{3}r^2h$; $r = \frac{1}{3}h$; $\frac{dr}{dt} = \frac{1}{3}\frac{dh}{dt}$

$\frac{dV}{dt} = \frac{\pi}{3}\left(2r\frac{dr}{dt}\right)h + \frac{\pi}{3}r^2\frac{dh}{dt}$

$10 = \frac{2\pi}{3}\left(\frac{4}{3}\right)(4)\frac{1}{3}\frac{dh}{dt} + \frac{\pi}{3}\left(\frac{4}{3}\right)^2\frac{dh}{dt}$

$10 = \left(\frac{32\pi}{27} + \frac{16\pi}{27}\right)\frac{dh}{dt}$

$\frac{270}{48\pi}$ ft/min $= \frac{dh}{dt}$

(c) Because radius stays constant in the cylinder but changes with time in a cone.