ansPRACTICE review

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Derivative Worksheet #1

Find the derivative of the following functions:

1. $f(t) = 7t - 12$ $f'(t) = 7$

2. $f(x) = 6$ $f'(x) = 0$

3. $f(x) = 12x^4 + 3x^2 + 7$ $f'(t) = 48x^3 + 6x$

4. $y = -6x^3 + 5x^2 - 8x + 2$ $y' = -18x^2 + 10x - 8$

5. $d(t) = 360 + 40t - 16t^2$ $d'(t) = 40 - 32t$

6. $g(t) = 7t^4 - 4t^3 + 6t^2 + 9t - 19$ $g'(t) = 28t^3 - 12t^2 + 12t + 9$

7. $y = 2 - 4x + 7x^2 - 9x^3$ $y' = -4 + 14x - 27x^2$

8. $f(x) = 0$ $f'(x) = 0$

9. $f(x) = e^x$ $f'(x) = e^x$

10. $f(x) = e^2$ $f'(x) = 0$

11. $f(t) = (t + 2)(t - 1)$ $f'(t) = (1)(t-1) + (1)(t+2) = 2t + 1$

12. $y = (2x + 1)(3x + 4)$

$$f'(t) = (2)(3x+4) + (2x+1)(3)$$
$$= 6x + 8 + 6x + 3$$
$$= 12x + 11$$

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Derivative Worksheet #2

Find the derivative of the following functions using product/quotient rules:

1. f(t) = (7t – 12) (4t³)

$f'(t) = (7)(4t^3) + (7t-12)(12t^2)$
$= 28t^3 + 84t^3 - 144t^2$
$= 112t^3 - 144t^2$

2. f(x) = 6(7x -3) (2x²)

$f'(x) = 6(7)(2x^2) + 6(7x-3)(4x) = 84x^2 + 168x^2 - 72x = 252x^2 - 72x$

3. f(x) = (2x⁴ + 3x² + 7) (9 - x³)

$f'(x) = (8x^3 + 6x)(9 - x^3) + (2x^4 + 3x^2 + 7)(-3x^2)$

4. y = -(6x³ + 5x² - 8x + 2) (4 – x)

$y' = -(18x^2 + 10x - 8)(4 - x) + -(6x^3 + 5x^2 - 8x + 2)(-1)$

5. d(t) = (4t) (10 – 4t)

$d'(t) = 4(10 - 4t) + 4t(-4)$

6. g(t) = (7t⁴ – 4t³) (6t² + 9t – 19)

$g'(t) = (28t^3 + 12t^2)(6t^2 + 9t - 19) + (7t^4 - 4t^3)(12t + 9)$

7. y = (2 – 4x) / (x² – 3x³)

$$y' = \frac{-4(x^2 - 3x^3) - (2 - 4x)(2x - 9x^2)}{(x^2 - 3x^3)^2}$$

8. f(x) = (2 – 3x + 5x² – 8x³) / 9

$f'(x) = \frac{1}{9}(-3 + 10x - 24x^2)$

9. f(x) = eˣ / x

$$f'(x) = \frac{e^x(x) - (1)e^x}{x^2}$$

10. f(x) = e² / x⁴

$$f'(x) = \frac{\cancel{0(x^4)} - e^2(4x^3)}{x^8}$$

11. f(t) = (6t + 2) / (7t - 1)

$$f'(t) = \frac{6(7t-1) + (6t+2)(7)}{(7t-1)^2}$$

12. y = (2x + 1) / (3x + 4)

$$y' = \frac{2(3x+4) + 3(2x+1)}{(3x+4)^2}$$

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Product & Quotient Rule Practice:

1. $\frac{d}{dx}(x^3-2x+1)(x^4+x-3)$ $=(3x^2-2)(x^4+x-3)+(x^3-2x+1)(4x^3+1)$

2. $\frac{d}{dx}\left(\frac{x^2+1}{x^2-1}\right)=\frac{(2x)(x^2-1)-(x^2+1)(2x)}{(x^2-1)^2}=\frac{2x\left[x^2-1-x^2-1\right]}{(x^2-1)^2}=\frac{-4x}{(x^2-1)^2}$

3. $\frac{d}{dx}\left(\frac{x^2}{\sin x}\right)=\frac{2x(\sin x)-x^3(\cos x)}{(\sin x)^2}$

4. $\frac{d}{dx}(\sin x\cos x)=\cos x(\cos x)+\sin x(-\sin x)=\cos^2 x-\sin^2 x$

5. $\frac{d}{dx}(\sin^2 x)=2(\sin x)^1(\cos x)$

6. $\frac{d}{dx}\left(\frac{x+1}{\sqrt{x}}\right)=\frac{(1)\sqrt{x}-(x+1)\frac{1}{2}(x)^{-1/2}}{x}=\left(\sqrt{x}-\frac{x+1}{2\sqrt{x}}\right)\div x=\frac{\sqrt{x}}{x}-\frac{x+1}{2x\sqrt{x}}$

7. $\frac{d}{dx}\left(\frac{1+\sin x}{1-\cos x}\right)=\frac{\cos x(1-\cos x)-(1+\sin x)(--\sin x)}{(1-\cos x)^2}=\frac{\cos x-\cos^2 x-\sin x-\sin^2 x}{(1-\cos x)^2}=\frac{\cos x-\sin x-1}{(1-\cos x)^2}$

8. $\frac{d}{dx}\left(\sqrt{x}e^x\right)$

$=\frac{1}{2}x^{-1/2}e^x+\sqrt{x}e^x=\frac{x}{2\sqrt{x}}+\sqrt{x}e^x=\frac{e^x\sqrt{x}}{2x}+\sqrt{x}e^x=\frac{e^x\sqrt{x}+2\sqrt{x}xe^x}{2x}=\frac{e^x\sqrt{x}}{2x}\left[1+2x\right]$

9. $\frac{d}{dx}\left(\frac{e^x}{1+x}\right)=\frac{e^x(1+x)-e^x(1)}{(1+x)^2}=\frac{e^x}{(1+x)^2}\left[1+x-1\right]=\frac{e^x x}{(1+x)^2}$

10. $\frac{d}{dx}(2^x e^x)=2^x\ln 2\, e^x+2^x e^x=e^x 2^x\left[\ln 2+1\right]$

11. $\frac{d}{dx}\left(\frac{3^x}{x+1}\right)=\frac{3^x\ln 3(x+1)-3^x(1)}{(x+1)^2}=\frac{3^x}{(x+1)^2}\left[\ln 3(\right.$

12. $\frac{d}{dx}\left(\frac{x^2-x-2}{x+1}\right)=\frac{(2x-1)(x+1)-(x^2-x-2)(1)}{(x+1)^2}=\frac{2x^2+x-1-x^2+x+2}{(x+1)^2}=\frac{x^2+2x+1}{(x+1)^2}=\frac{(x+1)^2}{(x+1)^2}=1$

13. Find the equation of the tangent line to the curve $y=\frac{\sqrt{x}}{x+1}$ at x=4

$y'=\frac{\frac{1}{2}x^{-1/2}(x+1)-\sqrt{x}(1)}{(x+1)^2}=\frac{x+1-2x}{2\sqrt{x}(x+1)^2}$

$=\frac{1-x}{2\sqrt{x}(x+1)^2}$

at $x=4$ $y=\frac{2}{5}$

$y'(4)=\frac{1-4}{2\sqrt{4}(4+1)^2}$

$=\frac{-3}{100}=m$

$\therefore y=mx+b$

$\frac{2}{5}=\frac{-3}{100}(4)+b$

$\frac{2}{5}=\frac{-3}{25}+b$

$b=\frac{2}{5}+\frac{3}{25}$

$b=\frac{13}{25}$

$\therefore y=\frac{-3}{100}x+\frac{13}{25}$

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Chain Rule Practice

1. $\frac{d}{dx}\cos\left(x^2\right) = -\sin(x^2)(2x)$

2. $\frac{d}{dx}\sqrt{1+\frac{1}{x}} = \frac{1}{2}\left(1+\frac{1}{x}\right)^{-1/2}\left(-x^{-2}\right) = \frac{-1}{x^2\sqrt{1+\frac{1}{x}}}$

3. $\frac{d}{dx}\left(3+\left(x^3-2x\right)^5\right)^8 = 8\left(3+(x^3-2x)^5\right)^7\left(5(x^3-2x)^4\right)(3x^2-2)$

4. $\frac{d}{dx}\tan^3\sqrt{x} = 3\tan^2\sqrt{x}\left(\sec^2\sqrt{x}\right)\left(\frac{1}{2}x^{-1/2}\right)$

5. $\frac{d}{dx}\sqrt{x^3+6x} = \frac{1}{2}\left(x^3+6x\right)^{-1/2}\left(3x^2+6\right)$

6. $\frac{d}{dx}\sec\left(x^2\right) = \sec x^2 \tan x^2 \,(2x)$

7. $\frac{d}{dx}\sec^2 x = 2\sec^1 x\left(\sec x \tan x\right)$

8. $\frac{d}{dx}\cos^3\left(x^2\right) = 3\cos^2(x^2)\left(-\sin(x^2)\right)(2x)$

9. $\frac{d}{dx}\sin 2x\cos 3x = \cos 2x(2)\cos 3x + \sin 2x(-\sin 3x)(3)$

10. $\frac{d}{dx}x^2\cos e^x = 2x\cos e^x + x^2(-\sin e^x)(e^x)$

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PRACTICE 1 - Implicit Differentiation

Find $\frac{dy}{dx}$:

1. $y^3 + 7y = x^3$

$3y^2y' + 7y' = 3x^2$

$y' = \frac{3x^2}{3y^2+7}$

2. $4x^2y - 3y = x^3 - 1$

$8xy + 4x^2y' - 3y' = 3x^2$

$y' = \frac{3x^2 - 8xy}{4x^2 - 3}$

3. $x^2 + 5y^3 = x + 9$

$2x + 15y^2y' = 1$

$y' = \frac{1-2x}{15y^2}$

4. Find $D_t y$ if $t^3 + t^2y - 10y^4 = 0$

$3t^2 + 2ty + t^2y' - 40y^3y' = 0$

$y' = \frac{-3t^2 - 2ty}{t^2 - 40y^3}$

5. Find the equation of the *tangent line* to the curve $y^3 - xy^2 + cos(xy) = 2$ at $x = 0$.

$y^3 - 0 + \cos(0) = 2$

$y^3 = 1$

$y = 1$

$3y^2y' - 1y^2 - x(2yy') - \sin(xy)(1y + xy') = 0$

$3y^2y' - 2xyy' - xy'\sin(xy) = y^2 + y\sin(xy)$

$y' = \frac{y^2 + y\sin(xy)}{3y^2 - 2xy - x\sin(xy)} = \frac{1^2 + \cancel{1(\sin 0)}}{3(1) - \cancel{2(0)(1)} - 0} = \frac{1}{3} = m$

$\therefore y = mx + b$

$1 = \frac{1}{3}(0) + b$

$1 = b$

$\therefore y = \frac{1}{3}x + 1$

6. Find $\frac{d^2y}{dx^2}$ at (2,1) if $2x^2y - 4y^3 = 4$.

$4xy + 2x^2y' - 12y^2y' = 0 \rightarrow$ sub pt.

$8 + 8y' - 12y' = 0 \quad \frac{-8}{8-12} = \frac{-8}{-4} = 2$

$4y + 4xy' + 4xy' + 2x^2y'' - 12(2y)y'y' - 12y^2y'' = 0$

sub

$4 + 8(2) + 8(2) + 8y'' - 24(2)^2 - 12y'' = 0$

$y'' = \frac{-60}{8-12} = \frac{-60}{-4} = 15$

7. Find the equation of the *normal line* (line perpendicular to the tangent line) to the curve $8(x^2 + y^2)^2 = 100(x^2 - y^2)$ at the point (3,1).

$16(x^2+y^2)(2x+2yy') = 100(2x - 2yy')$

$32x^3 + 32x^2yy' + 16xy^2 + 16y^3y' = 200x - 200yy'$

$y'(32x^2y + 16y^3 + 200y) = 200x - 32x^3 - 16xy^2$

$y' = \frac{200(3) - 32(27) - 48}{288 + 16 + 200} = \frac{600 - 864 - 48}{504} = -\frac{312}{504} = -\frac{5}{8}$

$m_{\perp} = \frac{8}{5}$

$y = mx + b$

$1 = \frac{8}{5}(3) + b$

$1 - \frac{24}{5} = b$

$\frac{-19}{5} = b$

$\therefore y = \frac{8}{5}x - \frac{19}{5}$

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PRACTICE 2 - Implicit Differentiation

Compute $\frac{dy}{dx}$ for the problems below:

1. $x^2 + y^2 = 1$ $\quad 2x + 2yy' = 0 \rightarrow y' = \frac{-2x}{2y}$

2. $x^2 - \frac{1}{2}y^2 = 1$ $\quad 2x - yy' = 0 \rightarrow y' = \frac{2x}{y}$

3. $2x^3y^2 + 2 = 4x$ $\quad 6x^2y^2 + 2x^3(2yy') = 4 \rightarrow y' = -\frac{6x^2y^2}{4x^3y} = \frac{-3y}{2x}$

4. $x^4y^4 + 3x^2y^2 = 1$ $\quad 4x^3y^4 + x^4(4y^3y') + 6xy^2 + 3x^2(2yy') = 0 \rightarrow y' = \frac{-4x^3y^4 - 6xy^2}{4x^4y^3 + 6x^2y} = \frac{-2xy^2(2x^2y^2+3)}{2x^2y(2x^2y^2+3)} = \frac{-y}{x}$

5. $5xy + 3x^2 = 4y^2$ $\quad y' = \frac{-5y-6x}{5x-8y}$
$5y + 5xy' + 6x = 8yy'$

6. $(x+y)^2 = 4xy$ $\quad 2x + 2xy' + 2y + 2yy' = 4y + 4xy'$ $\quad y' = \frac{4y-2x-2y}{2x+2y-4x} = \frac{y-x}{y-x} = 1$
$2(x+y)(1+y') = 4y + 4xy'$

7. $e^{xy} + xy = 3x^2$ $\quad e^{xy}y + e^{xy}xy' + y + xy' = 6x$ $\quad y' = \frac{6x - e^{xy}y - y}{e^{xy}x + x}$
$e^{xy}(1y + xy') + y + xy' = 6x$

8. $ln(x^2 + y^2) = y^3$ $\quad 2x + 2yy' = 3x^2y^2y' + 3y^4y'$ $\quad \frac{2x}{3x^2y^2 + 3y^4 - 2y} = y'$
$\frac{1}{x^2+y^2}(2x + 2yy') = 3y^2y'$

9. $\frac{x+y}{xy} + 2x = \frac{1}{5}y^5$ $\quad \frac{(1+y')(xy) - (x+y)(y+xy')}{x^2y^2} + 2 = y^4y'$

$\cancel{xy} + \cancel{xyy'} - \cancel{xy} - x^2y' - y^2 - \cancel{xyy'} + 2x^2y^2 = x^2y^6y'$

$\frac{-y^2 + 2x^2y^2}{x^2 + x^2y^6} = y'$

$\frac{y^2(2x^2-1)}{x^2(y^6+1)} = y'$

10. $\frac{x}{y}$ y^{-1} $+ \frac{y}{x}$ x^{-1} $= e^{x^2+2xy+y^2}$

$1y^{-1} + x(-1y^{-2}y') + y'x^{-1} + y(-x^{-2}) = e^{x^2+2xy+y^2}(2x + 2y + 2xy' + 2yy')$

$\frac{1}{y} - \frac{xy'}{y^2} + \frac{y}{x} - \frac{y}{x^2} = e^{\sim}(2x + 2y + 2yy' + 2xy')$

$yx^2 + x^3y' + xy^3 + y^3 = e^{\sim} x^2y^2(2x^2 + 2y + 2yy' + 2xy')$

$y' = \frac{e^{x^2+2xy+y^2}(2x^2+2y) - yx^2 - xy^3 - y^3}{x^3 - e^{x^2+2xy+y^2}(2y+2x)}$

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PRACTICE 3 - Implicit Differentiation

1. Differentiate each expression, treating y as a function of x. (If you cannot differentiate single expressions accurately, you certainly cannot use implicit differentiation on equations.)

(a) $\frac{d\,5y^3}{dx} = 15y^2y'$

(b) $\frac{d}{dx}(7x^5 + 2y^5) = 35x^4 + 10y^4y'$

(c) $\frac{d\,\sin(2y - e^x)}{dx} = \cos(2y-e^x)(2y'-e^x)$

(d) $\frac{d}{dx}(\ln(3x + 2y)) = \frac{1}{3x+2y}(3+2y')$

(e) $\frac{d\,x^3y^7}{dx} = 3x^2y^7 + x^3(7y^6y')$

(f) $\frac{d}{dx}e^{(3x + 5y)} = e^{3x+5y}(3+5y')$

(g) $\frac{d}{dx}(7y + e^{5y}) = 7y' + e^{5y}(5y')$

(h) $\frac{d}{dx}(\ln(3+y) + \cos(2y) + \ln(5)) =$

$\frac{1}{3+y}y' + -\sin(2y)(2y') + 0$

2. Use Implicit Differentiation on each equation to find $\frac{dy}{dx}$ (or y')

(a) $5x^3 + 2y^5 = 7 + y^2 + 3x$

(b) $x^3 + 2y^3 = 7y + 5x + 4$

(c) $\ln(3 + y) + \cos(5x) = e^y + x^2$

(d) $xy + \cos(2y) = \ln(7 - x^3) + 7^x + 3$

(e) $\tan(2 + 5y^2) + x^2y^3 = \ln(3 + y)$

(f) $e^{3x} + \sin(5y) = y^3 + x^5 + 4$

ⓐ $15x^2 + 10y^4y' = 2yy' + 3$

$y' = \frac{3 - 15x^2}{10y^4 - 2y}$

ⓑ $3x^2 + 6y^2y' = 7y' + 5$

$y' = \frac{5 - 3x^2}{6y^2 - 7}$

ⓔ $\sec^2(2+5y^2)(10yy') + 2xy^3 + x^2(3y^2y') = \frac{1}{(3+y)}y'$

$y' = \frac{-2xy^3}{10y\sec^2(2+5y^2) + 3x^2y^2 - \frac{1}{(3+y)}} = \frac{-2xy^3(3+y)}{(10y\sec^2(2+5y^2) + 3x^2y^2)(3+y) - 1}$

ⓒ $\frac{1}{(3+y)}y' + -\sin(5x)(5) = e^y y' + 2x$

$y' = \frac{2x + 5\sin 5x}{\frac{1}{3+y} - e^y}$

$= \frac{(2x + 5\sin 5x)(3+y)}{1 - e^y(3+y)}$

ⓓ $1y + xy' - \sin(2y)(2y') = \frac{1}{7-x^3}(-3x^2) + 7^x\ln 7$

$y' = \frac{-3x^2 + 7^x\ln 7(7-x^3) - y}{(7-x^3)(x - 2\sin 2y)}$

ⓕ $e^{3x}(3) + \cos(5y)(5y') = 3y^2y' + 5x^4$

$y' = \frac{5x^4 - 3e^{3x}}{5\cos 5y - 3y^2}$