Review Ques

April-15-13
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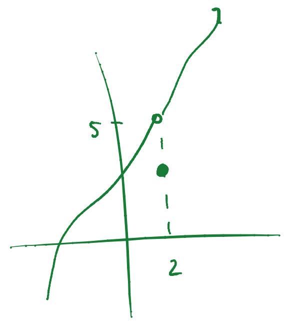
1. Explain in your own words what is meant by the equation

$$\lim_{x \to 2} f(x) = 5$$

Is it possible for this statement to be true and yet $f(2) = 3$? Explain.

$\lim_{x \to 2} f(x) = 5$ means the output of the graph approaches 5 from both sides ($x \to 2^-$ and $x \to 2^+$)

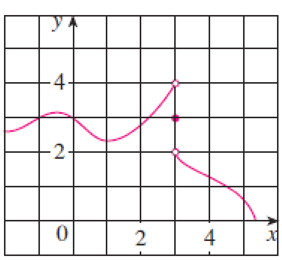
Yes. Since this graph has limit approaching 5 from both sides yet $f(2) = 3$



4. For the function f whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.

(a) $\lim_{x \to 0} f(x)$ (b) $\lim_{x \to 3^-} f(x)$ (c) $\lim_{x \to 3^+} f(x)$

(d) $\lim_{x \to 3} f(x)$ (e) $f(3)$



(a) 3 (b) 4 (c) 2

(d) DNE (e) 3

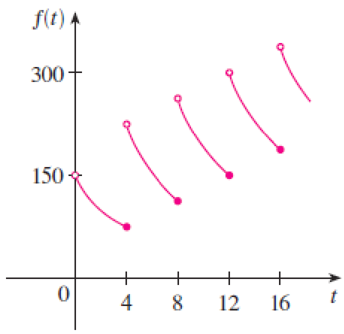
since

$\lim_{x \to 3^+} f(x) \neq \lim_{x \to 3^-} f(x)$

10. A patient receives a 150-mg injection of a drug every 4 hours. The graph shows the amount $f(t)$ of the drug in the bloodstream after t hours. Find

$$\lim_{t \to 12^-} f(t) \qquad \text{and} \qquad \lim_{t \to 12^+} f(t)$$

and explain the significance of these one-sided limits.



$\lim_{t \to 12^-} f(t) = 150$

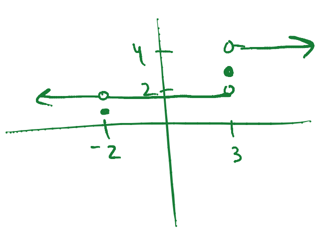
drug in blood just before injection

$\lim_{t \to 12^+} f(t) = 300$

drug in blood just after injection

Sketch

$\boxed{15.}$ $\lim_{x \to 3^+} f(x) = 4, \quad \lim_{x \to 3^-} f(x) = 2, \quad \lim_{x \to -2} f(x) = 2,$
$f(3) = 3, \quad f(-2) = 1$



Evaluate limit or justify why it doesn't exist

4. $\lim_{x \to 2} \dfrac{2x^2 + 1}{x^2 + 6x - 4}$

$\boxed{15.}$ $\lim_{t \to -3} \dfrac{t^2 - 9}{2t^2 + 7t + 3}$

40. $\lim_{x \to -6} \frac{2x + 12}{|x + 6|}$

30. $\lim_{x \to -4} \frac{\sqrt{x^2 + 9} - 5}{x + 4}$

(4) $\frac{2(2)^2+1}{2^2+6(2)-4} = \frac{8+1}{4+12-4} = \frac{9}{12} = 3/4$

(15) $\lim_{t \to -3} \frac{(\cancel{t+3})(t-3)}{(2t+1)(\cancel{t+3})} = \lim_{t \to -3} \frac{t-3}{2t+1} = \frac{-3-3}{2(-3)+1} = \frac{-6}{-5} = \frac{6}{5}$

(40) $\lim_{x \to -6^-} \frac{2(\cancel{x+6})}{-(\cancel{x+6})} = -2$

$\lim_{x \to -6^+} \frac{2(x+6)}{+(x+6)} = 2$

$\therefore \lim_{x \to -6} f(x) = \text{D.N.E}$

(30) $\lim_{x \to -4} \frac{(\sqrt{x^2+9} - 5)}{(x+4)} \frac{(\sqrt{x^2+9} + 5)}{(\sqrt{x^2+9} + 5)} = \lim_{x \to -4} \frac{(x^2+9) - 25}{(x+4)(\sqrt{x^2+9} + 5)}$

$= \lim_{x \to -4} \frac{x^2 - 16}{(x+4)(\sqrt{x^2+9}+5)} = \lim_{x \to -4} \frac{(\cancel{x+4})(x-4)}{(\cancel{x+4})(\sqrt{x^2+9} + 5)} = \frac{-4-4}{\sqrt{16+9} + 5} = \frac{-8}{10} = -\frac{4}{5}$

15–20 Explain why the function is discontinuous at the given number a. Sketch the graph of the function.

explain w/o using graph. Sketch after to verify

19. $f(x) = \begin{cases} \cos x & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 - x^2 & \text{if } x > 0 \end{cases}$ $\quad a = 0$

Continuity needs 3 conditions

(1) $\lim_{x \to 0^-} \cos x \stackrel{?}{=} \lim_{x \to 0^+} 1 - x^2$

$1 = 1$

condition 1 is ok.

(2) $f(0)$ is defined

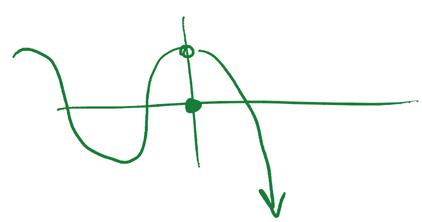
$f(0) = 0$

(3) But $\lim_{x \to 0} f(x) \neq f(0)$

$1 \neq 0$

$\therefore$ discontinuous at $x = 0$

Sketch



40. The gravitational force exerted by the earth on a unit mass at a distance r from the center of the planet is

$$F(r) = \begin{cases} \dfrac{GMr}{R^3} & \text{if } r < R \\ \dfrac{GM}{r^2} & \text{if } r \geqslant R \end{cases}$$

where M is the mass of the earth, R is its radius, and G is the gravitational constant. Is F a continuous function of r?

Check each piece: 1st one is linear
2nd one is rational $r \neq 0$, but that's not in domain $r \geqslant R$

(1) $\lim_{r \to R^-} \dfrac{GMr}{R^3} = \dfrac{GM\,R}{R^3} = \dfrac{GM}{R^2}$

$\lim_{r \to R^+} \dfrac{GM}{r^2} = \dfrac{GM}{R^2}$

$\therefore \lim_{r \to R}$ exists and equals $\dfrac{GM}{R^2}$

(2) $F(R)$ is defined and equals $\dfrac{GM}{R^2}$

(3) $\lim_{r \to R} F(r) = \dfrac{GM}{R^2} = F(R)$

$\therefore$ $F(r)$ is continuous

41. For what value of the constant c is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } x \geqslant 2 \end{cases}$$

each piece is continuous (quadratic + cubic)

need $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$

$\lim_{x \to 2^-} cx^2 + 2x = \lim_{x \to 2^+} x^3 - cx$

$4c + 4 = 8 - 2c$

$6c = 4$

$c = \dfrac{2}{3}$

Find m_{tan} of $y = x^3 - 2$ at $x = 1$
using the alternate definition of the limit.

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to 1} \frac{(x^3 - 2) - (1^3 - 2)}{x - 1}$$

$$= \lim_{x \to 1} \frac{x^3 - 1}{x - 1}$$

$$= \lim_{x \to 1} \frac{\cancel{(x-1)}(x^2 + x + 1)}{\cancel{(x-1)}}$$

$$= 1^2 + 1 + 1$$

$$= 3$$