Review game

February-12-13
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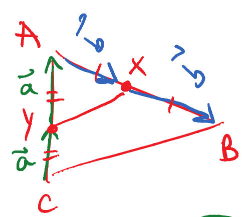
1. Draw any ΔABC with midpoint of AB labelled as X and midpoint of AC labelled as Y.
Using vectors Prove that XYCB is a trapezoid with a base twice as long as the other parallel side.



① $\overrightarrow{YX} = \overrightarrow{YA} + \overrightarrow{AX}$

same as $\overrightarrow{CY}$ — same as $\overrightarrow{XB}$

② $\overrightarrow{YX} = \overrightarrow{YC} + \overrightarrow{CB} + \overrightarrow{BX}$

same as $-\overrightarrow{CY}$ — same as $-\overrightarrow{XB}$

$\therefore$ ① rewritten $\overrightarrow{YX} = \overrightarrow{CY} + \overrightarrow{XB}$

② rewritten $\overrightarrow{YX} = -\overrightarrow{CY} + \overrightarrow{CB} + -\overrightarrow{XB}$

①+② $2\overrightarrow{YX} = \overrightarrow{CB}$

$\therefore \overrightarrow{YX}$ is parallel to $\overrightarrow{CB}$ and $\frac{1}{2}$ as long

$\therefore$ XYCB is a trapezoid with required measurments

Arshan's short version :)

$\overrightarrow{YX} = \vec{a} + \vec{b}$

$\overrightarrow{CB} = 2\vec{a} + 2\vec{b}$

$\overrightarrow{CB} = 2(\vec{a} + \vec{b})$

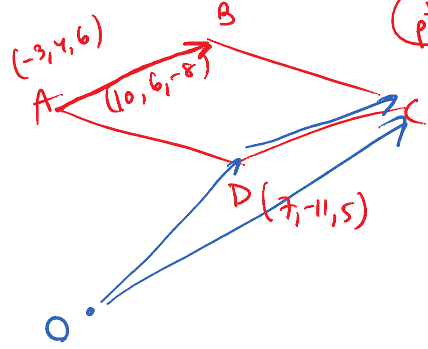
$\overrightarrow{CB} = 2\overrightarrow{YX}$

$\therefore \overrightarrow{CB}$ is parallel to $\overrightarrow{YX}$ and twice the size.

Good Job Arshan!

2. Find points C and B if ABCD is a parallelogram and $A(-3,4,6)$, $\overrightarrow{AB} = (10,6,-8)$, $D(7,-11,5)$

2.



pt. B

$\vec{AB} = \vec{OB} - \vec{OA}$

$(10,6,-8) = (x,y,z) - (-3,4,6)$

$10 = x - -3$ $\quad$ $6 = y - 4$ $\quad$ $-8 = z - 6$

$7 = x$ $\quad$ $10 = y$ $\quad$ $-2 = z$

$\therefore$ pt. B $(7,10,-2)$

pt. C like $\vec{OC}$

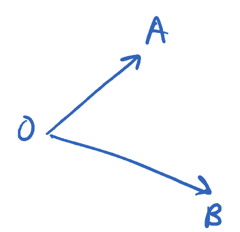
$\vec{OC} = \vec{OD} + \vec{AB}$

$\vec{OC} = (7,-11,5) + (10,6,-8)$

$\vec{OC} = (17,-5,-3)$

$\therefore$ pt. C $= (17,-5,-3)$

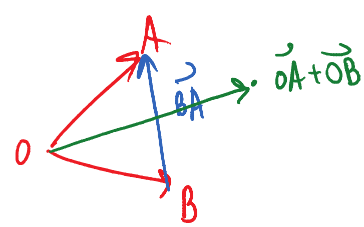
3. $A(1,-3,7)$, $B(-2,-5,0)$



a) Find $\vec{BA}$ and draw it on the diagram at the side

b) Find $\vec{OA} + \vec{OB}$ and draw it

c) Convert vector $\vec{OA}$ into geometric form



3. a) $\vec{BA} = (3,2,7)$

b) $\vec{OA} + \vec{OB} = (-1,-8,7)$

c) $|\vec{OA}| = \sqrt{59}$ $[\alpha = 83°, \beta = 113°, \gamma = 24°]$

4. $\vec{A} = 5\ [\theta = 45^\circ]$

$\vec{B} = 8\ [\theta = 150^\circ]$

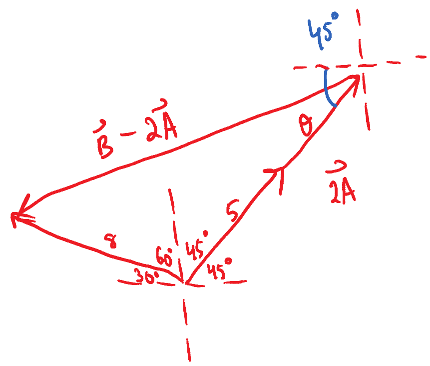
a) draw vector $\vec{B} - 2\vec{A}$

b) find its magnitude + direction

c) convert vector $\vec{B}$ into algebraic form

4.

ⓐ



ⓑ $|\vec{B} - 2\vec{A}|^2 = 8^2 + 10^2 - 2(8)(10)\cos 105^\circ$

$|\vec{B} - 2\vec{A}| \doteq 14$

$\cos\theta = \dfrac{14^2 + 10^2 - 8^2}{2(14)(10)}$

$\theta = 33^\circ$ $\quad\therefore\ \vec{B} - 2\vec{A} = 14\ [\theta = 192^\circ]$

ⓒ $\vec{B} = 8(\cos 150^\circ, \sin 150^\circ)$

$\vec{B} = 8\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\vec{B} = (-4\sqrt{3}, 4)$

5. What do vectors span ($\overset{1D}{\text{line}}$, $\overset{2D}{\text{plane}}$, $\overset{3D}{\text{space}}$) in $\mathbb{R}^3$?

a) $\vec{u} = (6, -5, -8)$

$\vec{v} = (-10, 12, -6)$

$\vec{w} = (15, -18, 9)$

$\vec{x} = (12, -10, 7)$

b) $\vec{u} = (3, -2, 0)$

$\vec{v} = (1, 1, -5)$

$\vec{w} = (2, -1, -1)$

5. a) $\vec{u} = (6, -5, -8)$

$\vec{v} = (-10, 12, -6)$ parallel

$\vec{w} = (15, -18, 9)$ $\therefore$ ignore one of them

$\vec{x} = (12, -10, 7)$

Assume

$\boxed{\vec{u} = a\vec{v} + b\vec{x}}$

+ get a contradiction

$\therefore$ not coplanar

$\therefore$ span a 3D space in $\mathbb{R}^3$

$(6, -5, -8) = a(-10, 12, -6) + b(12, -10, 7)$

$6 = -10a + 12b$ ①

$-5 = 12a - 10b$ ② $\rightarrow$ $-5 = 12a - 10b$

$-8 = -6a + 7b$ ③ $\times 2$ $\rightarrow$ $-16 = -12a + 14b$

$\times 2 \quad \times 2 \quad \times 2$

$-21 = \cancel{0a} + 4b$ ② + ③×2

$-\frac{21}{4} = b$

sub in ③ $-8 = -6a + 7\left(-\frac{21}{4}\right)$

$-8 + \frac{147}{4} = -6a$

$\frac{115}{4} = -6a$

$\frac{115}{24} = a$

check in ①

$$6 = -10\left(\frac{115}{24}\right) + 12\left(-\frac{21}{4}\right)$$

$$6 \neq \frac{-1331}{12}$$

contradiction
the assumption $\vec{u} = a\vec{v} + b\vec{x}$
is wrong $\therefore \vec{u}, \vec{v}, \vec{x}$ are not
coplanar + span a space

b) $\vec{u} = (3, -2, 0)$
$\vec{v} = (1, 1, -5)$
$\vec{w} = (2, -1, -1)$

Assume
$\vec{u} = a\vec{v} + b\vec{w}$

no contradictions $\nearrow$ do same as above

$\therefore$ coplanar or
span a plane in $\mathbb{R}^3$

$\left(a = -\frac{1}{3}, b = \frac{5}{3}\right)$

6. $A(1, -2, 3)$ $B(0, -7, 4)$

a) determine the magnitude and direction of $\vec{BA}$

b) give a vector that is collinear to $\vec{BA}$

ⓐ $\vec{BA} = (1, 5, -1)$

$|\vec{BA}| = \sqrt{27}$

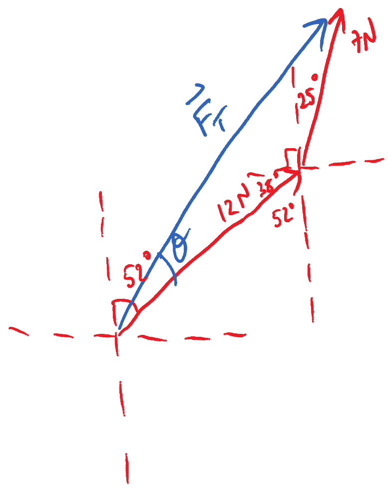
dir $\alpha = \cos^{-1}\left(\frac{1}{\sqrt{27}}\right) = 79°$

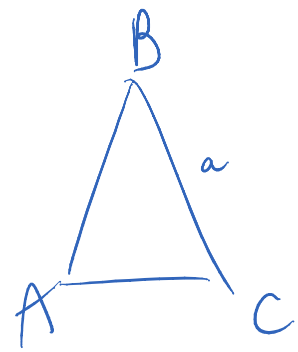
$\beta = \cos^{-1}\left(\frac{5}{\sqrt{27}}\right) = 16°$

$\gamma = \cos^{-1}\left(\frac{-1}{\sqrt{27}}\right) = 101°$

b) many answers - any scalar multiple of $(1, -5, 1)$
ex. $(2, -10, 2)$

7. $\vec{F_1} = 12\,N$ [N52°E]
$\vec{F_2} = 7\,N$ [N25°E]
find the net (or total) force





$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Total force = $\vec{F_T}$

$$|\vec{F_T}|^2 = 12^2 + 7^2 - 2(12)(7)\cos 153°$$

$$|\vec{F_T}| \doteq 18.5$$

$$\cos\theta = \frac{12^2 + 18.5^2 - 7^2}{2(12)(18.5)}$$

$$\theta \doteq 10°$$

$$\therefore \vec{F_T} = 18.5 \text{ [N42°E]}$$

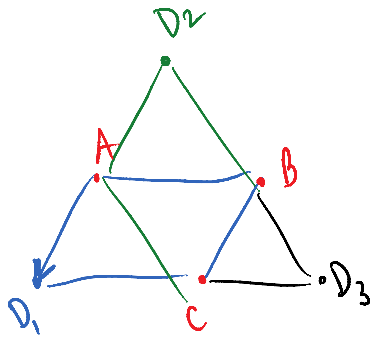
8. there is a missing point

of the parallelogram

Find all possibilities for it.

$A(1,1,-3)$ $B(-2,5,0)$ $C(6,-5,4)$

Note: not drawn to scale at all



$\vec{AC} = (5,-6,7)$

$\vec{CB} = (-8,10,-4)$

$\vec{BC} = (8,-10,4)$

D_1: $\vec{OD_1} = \vec{OA} + \vec{AD_1}$

$= \vec{OA} + \vec{BC}$ } same

$= (1,1,-3) + (8,-10,4)$

$= (9,-9,1)$ $\therefore$ pt. $D_1(9,-9,1)$

D_2: $\vec{OD_2} = \vec{OA} + \vec{AD_2}$

$= \vec{OA} + \vec{CB}$

$= (1,1,-3) + (-8,10,-4)$

$= (-7,11,-7)$ $\therefore$ pt $D_2(-7,11,-7)$

D_3: $\vec{OD_3} = \vec{OB} + \vec{BD_3}$

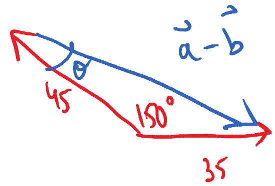
$= \vec{OB} + \vec{AC}$

$= (-2,5,0) + (5,-6,7)$

$= (3,-1,7)$ $\therefore$ pt $D_3(3,-1,7)$

9. if $|\vec{a}|$ and $|\vec{b}|$ are 35 km and 45 km respectively

with angle 150° between them
find $\vec{a}-\vec{b}$.



angles go between tail to tail

$$|\vec{a}-\vec{b}|^2 = 45^2 + 35^2 - 2(45)(35)\cos 150^\circ$$

$$|\vec{a}-\vec{b}| \doteq 77.3$$

$$\cos\theta = \frac{45^2 + 77.3^2 - 35^2}{2(45)(77.3)}$$

$\theta = 13^\circ$ $\quad\therefore \vec{a}-\vec{b} = 77.3$ Km [13° off the head of $\vec{b}$]