reviewDerivANS

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Product + Chain AND practice simplifying

17. $g(x) = (1+4x)^5(3+x-x^2)^8$

18. $h(t) = (t^4-1)^3(t^3+1)^4$

19. $y = (2x-5)^4(8x^2-5)^{-3}$ **20.** $y = (x^2+1)\sqrt[3]{x^2+2}$

17.) $g'(x) = 5(1+4x)^4(4)(3+x-x^2)^8 + (1+4x)^5(8)(3+x-x^2)^7(1-2x)$

$= 4(1+4x)^4(3+x-x^2)^7\left[5(3+x-x^2) + 2(1+4x)(1-2x)\right]$

$1-2x+4x-8x^2$

$= 4(1+4x)^4(3+x-x^2)^7\left[15+5x-5x^2+2+4x-16x^2\right]$

$= 4(1+4x)^4(3+x-x^2)^7\left[-21x^2+9x+17\right]$

18.) $h'(x) = 3(t^4-1)^2(4t^3)(t^3+1)^4 + (t^4-1)^3(4)(t^3+1)^3(3t^2)$

$= 12t^2(t^4-1)^2(t^3+1)^3\left[t(t^3+1) + (t^4-1)\right]$

$= 12t^2(t^4-1)(t^3+1)^3\left[2t^4+t-1\right]$

19) $y' = 4(2x-5)^3(2)(8x^2-5)^{-3} + (2x-5)^4(-3)(8x^2-5)^{-4}(16x)$

$= 8(2x-5)^3(8x^2-5)^{-4}\left[(8x^2-5) - 6x(2x-5)\right]$

$-12x^2+30x$

$= 8\dfrac{(2x-5)^3(-4x^2+30x-5)}{(8x^2-5)^4}$

20) $y' = 2x(x^2+2)^{1/3} + (x^2+1)\left(\frac{1}{3}\right)(x^2+2)^{-2/3}(2x)$

$y' = \frac{2x}{3}(x^2+2)^{-2/3}\left[3(x^2+2) + x^2+1\right]$

$= \dfrac{2x(4x^2+7)}{3(x^2+2)^{2/3}}$

Quotient (don't simplify, unless asked)

13. $y = \dfrac{x^3}{1-x^2}$ **14.** $y = \dfrac{x+1}{x^3+x-2}$

15. $y = \dfrac{t^2+2}{t^4-3t^2+1}$ **16.** $y = \dfrac{t}{(t-1)^2}$ simplify this one.

13.) $y' = \dfrac{(1-x^2)(3x^2) - (x^3)(-2x)}{(1-x^2)^2}$

14) $y' = \dfrac{(x^3+x-2)(1) - (x+1)(3x^2+1)}{(x^3+x-2)^2}$

15) $y' = \dfrac{(t^4-3t^2+1)(2t) - (t^2+2)(4t^3-6t)}{(t^4-3t^2+1)^2}$

16) $y' = \dfrac{(t-1)^2(1) - t(2)(t-1)(1)}{(t-1)^4}$

Implicit find $\frac{dy}{dx}$

1. $xy + 2x + 3x^2 = 4$

3. $\frac{1}{x} + \frac{1}{y} = 1$

5. $x^3 + y^3 = 1$

7. $x^2 + xy - y^2 = 4$

6. $2\sqrt{x} + \sqrt{y} = 3$

8. $2x^3 + x^2y - xy^3 = 2$

① $x\frac{dy}{dx} + 1y + 2 + 6x = 0$

$\frac{dy}{dx} = \frac{-2-y-6x}{x}$

⑤ $3x^2 + 3y^2\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{3x^2}{3y^2}$

$= -\left(\frac{x}{y}\right)^2$

⑥ $2\left(\frac{1}{2}\right)x^{-1/2} + \frac{1}{2}(y)^{-1/2}\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-x^{-1/2}}{\frac{1}{2}y^{-1/2}}$

$= \frac{-2y^{1/2}}{x^{1/2}} = -2\sqrt{\frac{y}{x}}$

③ $-1x^{-2} - 1y^{-2}\frac{dy}{dx} = 0$

$\frac{-1x^{-2}}{y^{-2}} = \frac{dy}{dx}$

$-\frac{y^2}{x^2} = \frac{dy}{dx}$

⑦ $2x + x\frac{dy}{dx} + 1y - 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx}\left[x - 2y\right] = -y - 2x$

$\frac{dy}{dx} = \frac{-y-2x}{x-2y}$

⑧ $6x^2 + 2xy + x^2\frac{dy}{dx} - 1y^3 - x3y^2\frac{dy}{dx} = 0$

$\frac{dy}{dx}\left[x^2 - 3xy^2\right] = y^3 - 6x^2 - 2xy$

$\frac{dy}{dx} = \frac{y^3 - 6x^2 - 2xy}{x^2 - 3xy^2}$

Chain + Tips

69. Let $r(x) = f(g(h(x)))$, where $h(1) = 2$, $g(2) = 3$, $h'(1) = 4$, $g'(2) = 5$, and $f'(3) = 6$. Find $r'(1)$.

70. If g is a twice differentiable function and $f(x) = xg(x^2)$, find f'' in terms of g, g', and g''.

71. If $F(x) = f(3f(4f(x)))$, where $f(0) = 0$ and $f'(0) = 2$, find $F'(0)$.

72. If $F(x) = f(xf(xf(x)))$, where $f(1) = 2$, $f(2) = 3$, $f'(1) = 4$, $f'(2) = 5$, and $f'(3) = 6$, find $F'(1)$.

69. $r'(x) = f'(g(h(x)))\,g'(h(x))\,h'(x)$

$r'(1) = f'(g(h(1)))\,g'(h(1))\,h'(1)$

$= f'(g(2))\,g'(2)\,(4)$

$= f'(3)\,(5)(4)$

$= 6(5)(4)$

$= 120$

70. $f'(x) = xg'(x^2)(2x) + 1g(x^2)$

$= 2x^2g'(x^2) + g(x^2)$

$f''(x) = 2x^2g''(x^2)(2x) + 4xg'(x^2) + g'(x^2)(2x)$

$= 4x^3g''(x^2) + 6xg'(x^2)$

71. $F'(x) = f'(3f(4f(x)))\,3f'(4f(x))\,4f'(x)$

$F'(0) = f'(3f(4f(0)))\,3f'(4f(0))\,4f'(0)$

$= f'(3f(0))\,3f'(0)\,4(2)$

$= f'(0)\,3(2)(4)(2)$

$= 2(3)(2)(4)(2)$

$= 96$

72. $F'(x) = f'(xf(xf(x)))\left[1f(xf(x)) + xf'(xf(x))\left[1f(x) + xf'(x)\right]\right]$

$F'(1) = f'(f(f(1)))\left[f(f(1)) + f'(f(1))\left[f(1) + f'(1)\right]\right]$

$= f'(f(2))\left[f(2) + f'(2)\left[2 + 4\right]\right]$

$= f'(3)\left[3 + 5(6)\right]$

$= 6(33)$

$= 198$

More Tips

46. If $h(2) = 4$ and $h'(2) = -3$, find

$$\frac{d}{dx}\left(\frac{h(x)}{x}\right)\Bigg|_{x=2}$$

$$\frac{d}{dx}\left[\frac{h(x)}{x}\right] = \frac{x\,h'(x) - 1\,h(x)}{x^2}$$

$$\frac{d}{dx}\left[\frac{h(x)}{x}\right]\Bigg|_{x=2} = \frac{2h'(2) - h(2)}{2^2} = \frac{2(-3) - 4}{4} = \frac{-10}{4} = -\frac{5}{2}$$

50. If f is a differentiable function, find an expression for the derivative of each of the following functions.

(a) $y = x^2 f(x)$ (b) $y = \dfrac{f(x)}{x^2}$

(c) $y = \dfrac{x^2}{f(x)}$ (d) $y = \dfrac{1 + xf(x)}{\sqrt{x}}$

(a) $y' = x^2 f'(x) + 2x f(x)$

(b) $y' = \dfrac{x^2 f'(x) - 2x f(x)}{x^4}$

(c) $y' = \dfrac{f(x)2x - x^2 f'(x)}{[f(x)]^2}$

(d) $y = x^{-1/2} + x^{1/2} f(x)$

$y' = -\frac{1}{2}x^{-3/2} + \frac{1}{2}x^{-1/2} f(x) + x^{1/2} f'(x)$

52. Find equations of the tangent lines to the curve

$$y = \frac{x-1}{x+1}$$

that are parallel to the line $x - 2y = 2$. $\longrightarrow$ $x - 2 = 2y$

$\frac{1}{2}x - 1 = y$ $\therefore m = \frac{1}{2}$

$$y' = \frac{(x+1)(1) - (x-1)(1)}{(x+1)^2}$$

$$\frac{1}{2} = \frac{\cancel{x}+1 - \cancel{x}+1}{(x+1)^2}$$

$(x+1)^2 = 4$

$x^2 + 2x + 1 = 4$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x = -3$ and $x = 1$

$y = \frac{-4}{-2} = 2$ $y = 0$

pt. $(-3, 2)$ and $(1, 0)$

$y = mx + b$ and $y = mx + b$

$2 = \frac{1}{2}(-3) + b$ $0 = \frac{1}{2}(1) + b$

$\frac{7}{2} = b$ $-\frac{1}{2} = b$

$\therefore$ equations are

$y = \frac{1}{2}x + \frac{7}{2}$ and $y = \frac{1}{2}x - \frac{1}{2}$

56. (a) The curve $y = |x|/\sqrt{2 - x^2}$ is called a *bullet-nose curve*. Find an equation of the tangent line to this curve at the point $(1, 1)$.

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$y = \frac{|x|}{\sqrt{2 - x^2}} = |x|(2 - x^2)^{-1/2}$$

$$y = \begin{cases} x(2 - x^2)^{-1/2} & \text{if } x \geq 0 \\ -x(2 - x^2)^{-1/2} & \text{if } x < 0 \end{cases}$$

$$y' = \begin{cases} 1(2 - x^2)^{-1/2} + x\left(-\cancel{\tfrac{1}{2}}\right)(2 - x^2)^{-3/2}(-\cancel{2}x) & \text{if } x \geq 0 \\ -1(2 - x^2)^{-1/2} - x\left(-\tfrac{1}{2}\right)(2 - x^2)^{-3/2}(-2x) & \text{if } x < 0 \end{cases}$$

at pt. $(1, 1)$ can use 1st piece so

$y' = \begin{cases} \\ -1(2-x^2)^{-1/2} - x\left(-\frac{1}{2}\right)(2-x^2)^{-3/2}(-2x) & \text{if } x<0 \end{cases}$

at pt. (1,1)
can use 1st piece x

$y'(1) = (2-1)^{-1/2} + (2-1)^{-3/2}$
$= 1+1$
$= 2$ ∴ slope is $m=2$ pt. (1,1) ∴ $y = mx+b$
$1 = 2(1)+b$
$-1 = b$ ∴ equation is $y = 2x-1$

(a) Write $|x| = \sqrt{x^2}$ and use the Chain Rule to show that

$$\frac{d}{dx}|x| = \frac{x}{|x|}$$

$\frac{d}{dx}\left[\sqrt{x^2}\right] = \frac{1}{\cancel{2}}(x^2)^{-1/2}(\cancel{2}x)$
$= \frac{x}{\sqrt{x^2}}$
$= \frac{x}{|x|}$

21. If $f(x) + x^2[f(x)]^3 = 10$ and $f(1) = 2$, find $f'(1)$.

$f'(x) + 2x[f(x)]^3 + x^2 3[f(x)]^2 f'(x) = 0$
$f'(x)\left[1 + 3x^2[f(x)]^2\right] = -2x[f(x)]^3$
$f'(x) = \frac{-2x[f(x)]^3}{1+3x^2[f(x)]^2}$

$f'(1) = \frac{-2(2)^3}{1+3(2)^2} = \frac{-16}{13}$

33–36 Find y'' by implicit differentiation.

33. $9x^2 + y^2 = 9$
34. $\sqrt{x} + \sqrt{y} = 1$
35. $x^3 + y^3 = 1$
36. $x^4 + y^4 = a^4$

33.) $18x + 2yy' = 0$
$18 + 2yy'' + 2y'y' = 0$
$y'' = \frac{-18 - 2(y')^2}{2y}$

34.) $\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}y' = 0$
$-\frac{1}{4}x^{-3/2} \quad -\frac{1}{4}y^{-3/2}y'y' + \frac{1}{2}y^{-1/2}y'' = 0$
$y'' = \frac{\frac{1}{4}x^{-3/2} + \frac{1}{4}y^{-3/2}[y']^2}{\frac{1}{2}y^{-1/2}}$

35.) $3x^2 + 3y^2y' = 0$
$6x + 6yy'y' + 3y^2y'' = 0$
$y'' = \frac{-6x - 6y[y']^2}{3y^2}$

36.) $4x^3 + 4y^3y' = 0$
$12x^2 + 12y^2y'y' + 4y^3y'' = 0$
$y'' = \frac{-12x^2 - 12y^2[y']^2}{4y^3}$

ⓐ $y(y^2-1)(y-2) = x(x-1)(x-2)$

At how many points does this curve have horizontal tangents? Estimate the x-coordinates of these points.

(b) Find equations of the tangent lines at the points (0, 1) and (0, 2).

(c) Find the exact x-coordinates of the points in part (a).

ⓐ $(y^3-y)(y-2) = (x^2-x)(x-2)$

$y^4-2y^3-y^2+2y = x^3-2x^2-x^2+2x$

$y^4-2y^3-y^2+2y = x^3-3x^2+2x$

now take deriv.

$4y^3y' - 6y^2y' - 2yy' + 2y' = 3x^2-6x+2$

$y'(4y^3-6y^2-2y+2) = 3x^2-6x+2$

$y' = \dfrac{3x^2-6x+2}{4y^3-6y^2-2y+2}$

$m=0$ for horiz. tangents

$\dfrac{0}{1} = \dfrac{3x^2-6x+2}{\cancel{1}\,(4y^3-6y^2-2y+2)}$

$0 \doteq 3(x-1.577)(x-0.423)$

$\therefore$ at $x \doteq 1.577$ or $x \doteq 0.423$ there are horiz. tangents

ⓑ $y'\Big|_{\substack{x=0\\y=1}} = \dfrac{2}{-2} = -1$ $\quad\therefore m=-1$ pt $(0,1)$

$\therefore y=mx+b$

$1=-1(0)+b$

$1=b$

$\boxed{\therefore y=-1x+1}$

$y'\Big|_{\substack{x=0\\y=2}} = \dfrac{2}{6} = \dfrac{1}{3}$ $\quad\therefore m=\dfrac{1}{3}$ pt. $(0,2)$

$\therefore y=mx+b$

$2=\frac{1}{3}(0)+b$

$2=b$

$\boxed{\therefore y=\frac{1}{3}x+2}$

ⓒ $x = \dfrac{+6 \pm \sqrt{36-4(3)(2)}}{2(3)}$

$x = \dfrac{6 \pm \sqrt{12}}{6}$ $\qquad \sqrt{4}\sqrt{3}$ $\quad 2\sqrt{3}$

$x = 1 \pm \frac{1}{3}\sqrt{3}$