Common Errors **Review for Calculus**

Error	Reason/Correct/Justification/Example
$\frac{2}{0} \neq 0$ and $\frac{2}{0} \neq 2$	Division by zero is undefined!
$-3^2 \neq 9$	$-3^2 = -9$, $(-3)^2 = 9$ Watch parenthesis!
$(x^2)^3 \neq x^5$	$(x^2)^3 = x^2x^2x^2 = x^6$
$\frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c}$	$\frac{1}{2} = \frac{1}{1+1} \neq \frac{1}{1} + \frac{1}{1} = 2$
$\frac{1}{x^2+x^3} \neq x^{-2} + x^{-3}$	A more complex version of the previous error.
$\frac{\not{a}+bx}{\not{a}} \neq 1+bx$	$\frac{a+bx}{a} = \frac{a}{a} + \frac{bx}{a} = 1 + \frac{bx}{a}$ Beware of incorrect canceling!
$-a(x-1) \neq -ax - a$	$-a(x-1) = -ax + a$ Make sure you distribute the "-"!
$(x+a)^2 \neq x^2 + a^2$	$(x+a)^2 = (x+a)(x+a) = x^2 + 2ax + a^2$
$\sqrt{x^2+a^2} \neq x + a$	$5 = \sqrt{25} = \sqrt{3^2+4^2} \neq \sqrt{3^2} + \sqrt{4^2} = 3 + 4 = 7$
$\sqrt{x+a} \neq \sqrt{x} + \sqrt{a}$	See previous error.
$(x+a)^n \neq x^n + a^n$ and $\sqrt[n]{x+a} \neq \sqrt[n]{x} + \sqrt[n]{a}$	More general versions of previous three errors.
$2(x+1)^2 \neq (2x+2)^2$	$2(x+1)^2 = 2(x^2+2x+1) = 2x^2+4x+2$ $(2x+2)^2 = 4x^2+8x+4$ Square first then distribute!
$(2x+2)^2 \neq 2(x+1)^2$	See the previous example. You can not factor out a constant if there is a power on the parethesis!
$\sqrt{-x^2+a^2} \neq -\sqrt{x^2+a^2}$	$\sqrt{-x^2+a^2} = (-x^2+a^2)^{\frac{1}{2}}$ Now see the previous error.
$\frac{a}{\left(\frac{b}{c}\right)} \neq \frac{ab}{c}$	$\frac{a}{\left(\frac{b}{c}\right)} = \frac{\left(\frac{a}{1}\right)}{\left(\frac{b}{c}\right)} = \left(\frac{a}{1}\right)\left(\frac{c}{b}\right) = \frac{ac}{b}$
$\frac{\left(\frac{a}{b}\right)}{c} \neq \frac{ac}{b}$	$\frac{\left(\frac{a}{b}\right)}{c} = \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{1}\right)} = \left(\frac{a}{b}\right)\left(\frac{1}{c}\right) = \frac{a}{bc}$

Elements of Proof

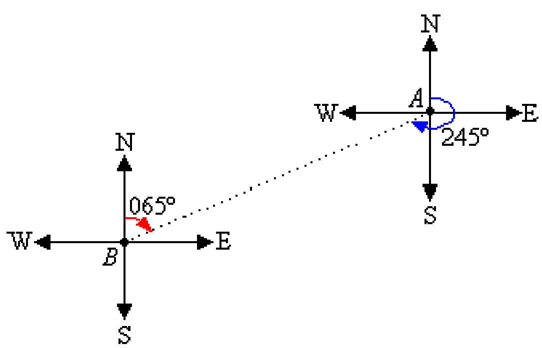
Special words and what they mean:

	AND	OR	IF	ONLY IF	IF AND ONLY IF
Symbol					
Ex.					

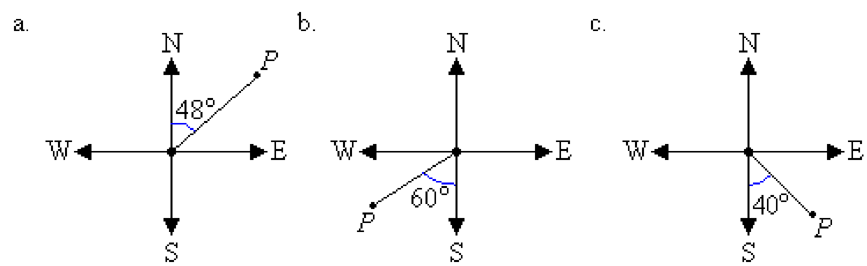
Bearings

The **true bearing** of a point is the number of degrees in the angle measured in a ____________________ direction from the ________________ line to the line joining the centre of the compass with the point.

A bearing is used to represent the direction of one point relative to another point. For example, the bearing of *A* from *B* is 065°. The bearing of *B* from *A* is 245°.



1. State the bearing of the point *P* in each of the following diagrams:



The **conventional bearing** of a point is stated as the number of degrees ____________________________ of the ____________________________ line. We will refer to the conventional bearing simply as the **direction**.

Quadrant bearings are given in the format of

N 40°E (northeast)
S 26°W (southwest)
N30ºE means the direction is 30º east of north.

Whenever you measure a quadrant bearing, it should always be recorded with _________________________ listed first, followed by _________________________ away from north or south, and the _________________________ away from north or south.

In other words, you would never give a quadrant bearing as E 40°N

2. Describe each of the following bearings as directions.

a. 076°

b. 150°

c. 225°

d. 290°

Formulas

Linear/Geometry

$y = mx + b$ $\quad m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad Ax + By + C = 0$ $\quad y - y_0 = m(x - x_0)$

$Midpt = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$$dist = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Quadratic

$y = a(x-h)^2 + k$ $\quad y = a(x-r)(x-t)$ $\quad y = ax^2 + bx + c$ $\quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometry

Cosine Laws

$c^2 = a^2 + b^2 - 2ab\cos C$

$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

Sine Law

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Pythagorean

$a^2 + b^2 = c^2$

Circles

Arc length: $s = r\theta$

Area: $A = \frac{1}{2}r^2\theta$

$x^2 + y^2 = r^2$

$\sin\theta = \frac{opp}{hyp} = \frac{y}{r}$

$\cos\theta = \frac{adj}{hyp} = \frac{x}{r}$

$\tan\theta = \frac{opp}{adj} = \frac{y}{x}$

Quotient Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

Reciprocal Identities

$\csc\theta = \frac{1}{\sin\theta}$

$\sec\theta = \frac{1}{\cos\theta}$

$\cot\theta = \frac{1}{\tan\theta}$

Double Angle

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$

Sum/Difference

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

Polynomial/Factoring

- $a^2 - b^2 = (a-b)(a+b)$
- $a^2 + b^2$ is prime
- $a^2 + 2ab + b^2 = (a+b)^2$
- $a^2 - 2ab + b^2 = (a-b)^2$
- $a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)$
- $a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)$

Sequences & Series

Arithmetic Series

- $a_n = a + (n-1)d$
- $S_n = \sum_{k=0}^{n-1}(a+kd) = \frac{n}{2}[2a+(n-1)d]$
- $S_n = \sum_{k=0}^{n-1}(a+kd) = n\left(\frac{a+a_n}{2}\right)$

Geometric Series

- $a_n = ar^{n-1}$
- $S_n = \sum_{k=0}^{n-1} ar^k = a\left[\frac{1-r^n}{1-r}\right]$ if $r \neq 1$
- $S = \sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}$ if $|r| < 1$

Finance

$A = P + Prt$ $\quad I = Prt$

$A = P(1+i)^n$ $\quad i = \frac{r}{C}$ $\quad n = Ct$

$FV = \frac{R\left[(1+i)^n - 1\right]}{i}$ $\quad PV = \frac{R\left[1-(1+i)^{-n}\right]}{i}$

Exponential/Logarithmic

$y = ab^{\frac{x}{p}}$ for word problems $\qquad y = ab^x + c$ for sketching

$y = a\log_b[k(x-d)] + c$

Exponent Rules

$a^n a^m = a^{n+m}$ $\qquad \frac{a^n}{a^m} = a^{n-m} = \frac{1}{a^{m-n}}$

$\left(a^n\right)^m = a^{nm}$ $\qquad a^0 = 1, \quad a \neq 0$

$(ab)^n = a^n b^n$ $\qquad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

$a^{-n} = \frac{1}{a^n}$ $\qquad \frac{1}{a^{-n}} = a^n$

$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$ $\qquad a^{\frac{n}{m}} = \left(a^{\frac{1}{m}}\right)^n = \left(a^n\right)^{\frac{1}{m}}$

$\sqrt[n]{a} = a^{\frac{1}{n}}$ $\qquad \sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$

$\sqrt[m]{\sqrt[n]{a}} = \sqrt[nm]{a}$ $\qquad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

$\sqrt[n]{a^n} = a$, if n is odd

$\sqrt[n]{a^n} = |a|$, if n is even

Log Rules

The domain of $\log_b x$ is $x > 0$

$\log_b x = y \iff x = b^y$

$b^{\log_b x} = x$

$\log_b b^x = x$

$\log_b 1 = 0$

$\log_b b = 1$

$\log_b xy = \log_b x + \log_b y$

$\log_b \frac{x}{y} = \log_b x - \log_b y$

$\log_b x^y = y \log_b x$

$\log_b x = \frac{\log_a x}{\log_a b}$

Special Logarithms

$\ln x = \log_e x$ natural log

$\log x = \log_{10} x$ common log

where $e = 2.718281828\ldots$

If $a \neq 1, a^x = a^y \Leftrightarrow x = y$

If $b \neq 1, \log_b x = \log_b y \Leftrightarrow x = y$

If $x \neq 0, a^x = b^x \Leftrightarrow a = b$

If $x \neq 1, \log_a x = \log_b x \Leftrightarrow a = b$

ESSENTIAL Skills PRACTICE

Radicals

1. Simplify

a. $\sqrt{384x^4y^3}$ b. $5\sqrt{27}+4\sqrt{48}$

c. $3\sqrt{2}\left(2\sqrt{6}+\sqrt{10}\right)$ d. $4\sqrt{3}+3\sqrt{20}-2\sqrt{12}+\sqrt{45}$

2. Rationalize the denominators

a. $\frac{2}{\sqrt{6}}$ b. $\frac{6\sqrt{5}}{7\sqrt{2}}$

c. $\frac{4}{\sqrt{2}-5\sqrt{3}}$ d. $\frac{5-\sqrt{3}}{2\sqrt{3}+\sqrt{5}}$

Pascal's Triangle

3. Expand

a. $(x+y)^4$ b. $(y-x^2)^3$ c. $(2x^3+1)^5$ d. $(x-3y)^5$

Factoring

4. Factor fully

a. $4x^2-7x+3$ b. $21n^2+8n-4$

c. $6a^2-13a-5$ d. $49a^2-56ab+16b^2$

e. $12t^2-15t-18$ f. $196x^2-25z^2$

g. $56x^3y^2+18x^2y^2-8xy^2$ h. $9x^2+6x+1-4y^2$

i. $24x^2y-16x^2y^2+8x^3y^2$ j. $27(x+y)^3-12(x+y)$

k. $(x+y)^2-4(x+y)-45$ l. $(a+b)^2-81$

m. $256a^4-625b^4$ n. $121-(a+b)^2$

o. $p^4+21p^2q^2-100q^4$ p. $3\sin^2A-14\sin A+8$

5. Perform synthetic division

a. $2y^3+y^2-27y-36$ by $y+3$

b. $-6x^3+29x^2+7x-13$ by $2x-1$

6. Perform long division

a. $2x^3+3x^2-17x-30$ by $2x^2+6x-8$

b. $3x^3+7x^2-2x-11$ by x^2-1

7. Factor the polynomials using the factor theorem

a. $f(x)=4x^3-9x^2+6x-1$

b. $f(x)=2x^3+9x^2+19x+15$

c. $f(x)=5x^3+29x^2+19x-5$

Solving no rounding!

8. Solve radical equations

a. $26=-1+(27x)^{\frac{3}{4}}$ b. $\sqrt{-10+7p}=p$

c. $5=3+4a^{-\frac{1}{6}}$ d. $2=\sqrt{3b-2}-\sqrt{10-b}$

9. Solve polynomial equations

a. $3x^3-24=-x^2+22x$ b. $2x^3-5x^2=2x-5$

Solving continued no rounding!

10. Solve rational equations

a. $\frac{b+6}{4b^2}+\frac{3}{2b^2}=\frac{b+4}{2b^2}$ b. $\frac{1}{r-2}+\frac{1}{r^2-7r+10}=\frac{6}{r-2}$

c. $\frac{5}{n^3+5n^2}=\frac{4}{n+5}+\frac{1}{n^2}$ d. $\frac{1}{x^2-5x}=\frac{x+7}{x}-1$

11. Solve inequalities

a. $39x+20>2x^3\cdot 3x^2$ b. $x^4+4x+12\le 9x^2$

c. $\frac{1}{(x+2)(x-3)^2}>0$ d. $\frac{2}{p-1}\ge\frac{3}{4}$

e. $\frac{5}{x-2}+\frac{3}{2-x}\ge 1$ f. $\frac{x}{2x-4}-\frac{3}{x-6}+1\le\frac{3}{2}$

12. Solve trig equations

a. $\cos\theta=-\frac{\sqrt{2}}{2}$ b. $\sin\theta=-\frac{1}{2}$ c. $\tan\theta=\frac{\sqrt{3}}{3}$

d. $2\cos^2 x=1+\sin x$

e. $2\sin x\sec x-2\sqrt{3}\sin x=0$

f. $2\sin x\tan x+\tan x-2\sin x-1=0$

13. Solve exponential or log equations

a. $3^x = 9^{x+3}$ b. $(5)(5)^{x+2} = 25^{2x}$

c. $4\left(3^x\right)+3^{2x}=-4$ d. $3^{x+2} - 3^x = 216$

e. $2^x = 3^{x+1}$ f. $\log_5(3x + 8) = 2$

g. $\log_x\frac{16}{81}=4$ h. $19^{\frac{x}{2}-4}=81$

i. $\log_5(x+2\sqrt{6})+\log_5(x-2\sqrt{6})=2$

Graphing

14. Lines

a. $y=-6$ b. $x=2$

c. $y=-\frac{1}{2}x-1$ d. $3x-2y=-6$

15. Quadratics

a. $y=1.5(x+2)^2+1$ b. $y=(4-2x)(x+8)$

16. Square Root

$y=-2\sqrt{4-x}+3$

17. Parent Polynomials

a. $y=x^2, y=x^4, y=x^6\ldots$ b. $y=x^3, y=x^5, y=x^7$

18. Cube Root

$y=-\sqrt[3]{2x+10}-4$

19. Tangent

$y=\tan\frac{\pi}{4}x+1$

20. Sinusoidals

a. $y=2\sin\left(\frac{1}{2}x+\frac{\pi}{4}\right)$ b. $y=-3\cos\left(\pi x-6\pi\right)+5$

21. Reciprocal parent trig

a. $y=\csc x$ b. $y=\sec x$ c. $y=\cot x$

22. Inverse parent trig

a. $y=\sin^{-1}x$ b. $y=\cos^{-1}x$ c. $y=\tan^{-1}x$

23. Exponential

a. $y=-4(0.8)^{2x-4}$ b. $y=0.5(2.5)^{\frac{2-x}{3}}-4$

24. Logarithmic

a. $y=2\log_{0.5}(15-3x)+1$ b. $y=2+\log_3(4x+1)$

25. Polynomial in standard form

$y=x^3-3x+7x^4-9x^5+1$

26. Polynomial in transformed form

$y=5(1-3x)^6+8$

27. Polynomial in factored form

a. $y=x(x+3)^2(2-2x)$ b. $y=-x^2(x+2)^2(1-x)^5$

28. Rational (transformation of parent)

$f(x)=\frac{-2}{3x+6}+4$

Graphing continued

29. Rational (transformation of parent after long division)

$$f(x)=\frac{x+4}{-2x-6}$$

30. Rational (reciprocal of polynomial)

a. $y=\dfrac{1}{(x+3)(x-5)}$ b. $y=\dfrac{1}{x^2+0.5}$

31. Rational with OA

a. $f(x)=\dfrac{2x^2-5x+2}{x+1}$ b. $y=\dfrac{x^3-8}{x^2-3x-4}$

32. Complicated rational

a. $f(x)=\dfrac{2x+1}{x^2-4x-5}$ b. $f(x)=\dfrac{-5x^3(x-3)^2(x+3)}{(x-1)^3(x+1)^2(x-4)}$

c. $f(x)=\dfrac{x^3(x+4)^2}{(x-1)^2(x+3)^2}$ d. $f(x)=\dfrac{(x+1)^2(x-3)(x-1)^3}{x^3(x-2)(x+2)^2}$

Word Problems

33. Linda has a garden. The width of her garden is 8 feet longer than the length of it. Around the garden she has a 4 foot wide sidewalk. The area of the sidewalk is 320 feet squared. What are the dimensions of the garden?

34. You have made a quilt that is 4 feet by 5 feet. You want to use the remaining 10 square feet as a decorative border of uniform width. What should the width of the border be?

35. A balloon is rising from the ground at 4 feet per second. Another balloon is at 756 feet and descending at 3 feet per second. In how many seconds will they be at the same altitude?

36. A motor boat travels 400km. If the boat went 18km/h faster, it could have traveled 600 km in the same amount of time. What was the original speed of the boat?

37. Rachel and Ken are knitting scarves to sell at the craft show. The wool for each scarf costs $6. They were planning to sell the scarves for $10 each, the same as last year when they sold 40 scarves. However, they know that if they raise the price, they will be able to make more profit even if they end up selling fewer scarves. They have been told that for every 50¢ increase in the price, they can expect to sell four fewer scarves. What selling price will maximize their profit and what will the profit be?

38. From an airplane, Janara looked down to see a bit. If she looked down at an angle of 9 degrees and the airplane was half a mile above the ground, what was the horizontal distance to the city?

39. The dimensions of a rectangular box are consecutive integers. If the box has volume of 13800 cubic centimetres, what are its dimensions?

40. Charice is painting the lines for her own basketball court. The free throw section will be a rectangle with a semi-circle on top. The length of the rectangle will be 2.25 metres greater than the width. Using 3.14 for π, the area of the court is 31.28 m^2. Determine the dimensions of the free throw section.

41. When you board a Ferris wheel your feet are 1 foot off the ground. At the highest point of the ride, your feet are 99 feet above the ground. It takes 30 seconds for the ride to complete one full revolution. Write a trigonometric equation for your height above the ground at t seconds after the ride starts. Find at what two times within one cycle you are exactly at 90 feet off the ground.

42. The average depth of water at the end of a dock is 6 feet. This varies 2 feet in both directions with the tide. Suppose there is a high tide at 4 AM. If the tide goes from low to high every 6 hours, write a cosine function d(t) describing the depth of the water as a function of time with t = 4 corresponding to 4 AM. At what two times within one cycle is the tide at a depth of 5 feet?

Word Problems continued

43. Members of a hiking club raise a total of \$60. When two members drop out of the hike their money is refunded. The remaining members going on the hike then had to pitch in \$1 each to make up the difference. How many members went on the hike?

44. A motorboat travels 30 miles up a river and returned a distance of 27 miles. The entire trip takes 5 hours. If the rate of the motorboat in still water is 12 mph, find the rate of the current of the river.

45. A store is clearing merchandise by reducing the price of all items by 8% at the start of each week. If a MP3 player was \$207 before any of the discounts were applied, find the cost after 8 weeks of discount.

46. Victor wants to buy a new car that costs \$90,000. He has saved \$20,000. Determine how many years it will take his \$20,000 to grow to \$90,000 at 6.25% interest compounded continuously. (Use $A = Pe^{rt}$ for continual compounding where A is final amount, P is original amount, e is the natural number 2.718…, r is the interest rate and t is time in years.)

47. Biologists use the logarithmic model $n = k\log A$ to estimate the number of species (*n*) that live in a region of area (*A*). In the model, *k* represents a constant. If 2800 species live in a rain forest of 500 square kilometers, then how many species will be left when half of this rain forest is destroyed by logging?

48. One example of a logarithmic modeling problem involves finding the magnitude of an earthquake. The Richter scale is a common method used to measure the intensity of an earthquake. The scale converts seismographic readings into numbers that offer an easy reference for measuring the magnitude (M) of an earthquake. All earthquakes are compared to a zero-level earthquake (x_0) whose seismographic reading measures 0.001 millimeter at a distance of 100 kilometers from the epicenter. The formula used to find the measure of the magnitude of an earthquake is: $M(x) = \log\left(\frac{x}{x_0}\right)$. In the formula, *x* stands for the *intensity* of the earthquake and *x* represents the seismographic reading in millimeters. The x_0 represents a zero-level earthquake the same distance from the epicenter.

Mexico City had an earthquake in 1985 that had a seismographic reading of 125,892 millimeters 100 kilometers from the center. Find the magnitude of the earthquake.

49. The surface area S of the right circular cone in the figure above is given by $S = \pi r\sqrt{r^2 + h^2}$. What radius should be used to produce a cone of height 5 inches and surface area of 100 square inches?

50. A rope is to be stretched at uniform height from a tree to a 35 foot long fence, which is 20 feet from the tree, and then to the side of a building at a point 30 feet from the fence, as shown. If 63 feet of rope is to be used, how far from the building wall should the rope meet the fence?