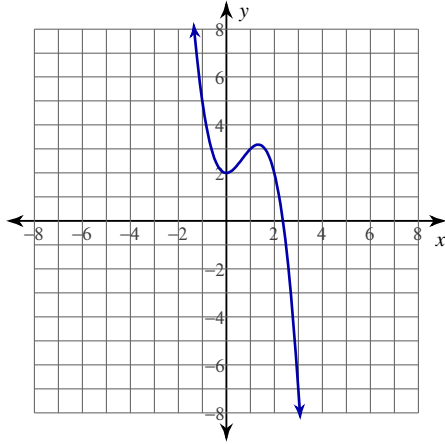


## Intervals of Increase and Decrease

Date \_\_\_\_\_ Period \_\_\_\_\_

For each problem, find the x-coordinates of all critical points, find all discontinuities, and find the open intervals where the function is increasing and decreasing.

1)  $y = -x^3 + 2x^2 + 2$



2)  $y = x^3 - 11x^2 + 39x - 47$

3)  $y = -x^4 + 3x^2 - 3$

4)  $y = \frac{x^2}{4x + 4}$

$$5) y = \frac{3x^2 - 3}{x^3}$$

$$6) y = (2x - 8)^{\frac{2}{3}}$$

$$7) y = -\frac{1}{5}(x - 4)^{\frac{5}{3}} - 2(x - 4)^{\frac{2}{3}} - 1$$

**Critical thinking question:**

8) If functions  $f$  and  $g$  are increasing on an interval, show that  $f + g$  is increasing on the same interval.

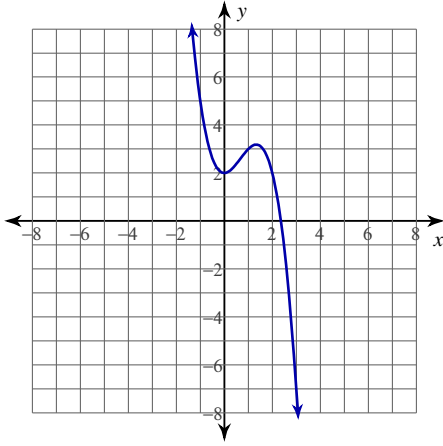
9) Give an example where functions  $f$  and  $g$  are increasing on the interval  $(-\infty, \infty)$ , but where  $f - g$  is decreasing.

## Intervals of Increase and Decrease

Date \_\_\_\_\_ Period \_\_\_\_\_

For each problem, find the x-coordinates of all critical points, find all discontinuities, and find the open intervals where the function is increasing and decreasing.

1)  $y = -x^3 + 2x^2 + 2$



Critical points at:  $x = 0, \frac{4}{3}$  No discontinuities exist.

Increasing:  $\left(0, \frac{4}{3}\right)$  Decreasing:  $(-\infty, 0), \left(\frac{4}{3}, \infty\right)$

2)  $y = x^3 - 11x^2 + 39x - 47$

Critical points at:  $x = 3, \frac{13}{3}$  No discontinuities exist.

Increasing:  $(-\infty, 3), \left(\frac{13}{3}, \infty\right)$  Decreasing:  $\left(3, \frac{13}{3}\right)$

3)  $y = -x^4 + 3x^2 - 3$

Critical points at:  $x = -\frac{\sqrt{6}}{2}, 0, \frac{\sqrt{6}}{2}$  No discontinuities exist.

Increasing:  $\left(-\infty, -\frac{\sqrt{6}}{2}\right), \left(0, \frac{\sqrt{6}}{2}\right)$  Decreasing:  $\left(-\frac{\sqrt{6}}{2}, 0\right), \left(\frac{\sqrt{6}}{2}, \infty\right)$

4)  $y = \frac{x^2}{4x + 4}$

Critical points at:  $x = -2, 0$  Discontinuity at:  $x = -1$

Increasing:  $(-\infty, -2), (0, \infty)$  Decreasing:  $(-2, -1), (-1, 0)$

$$5) y = \frac{3x^2 - 3}{x^3}$$

Critical points at:  $x = -\sqrt{3}, \sqrt{3}$  Discontinuity at:  $x = 0$   
 Increasing:  $(-\sqrt{3}, 0), (0, \sqrt{3})$  Decreasing:  $(-\infty, -\sqrt{3}), (\sqrt{3}, \infty)$

$$6) y = (2x - 8)^{\frac{2}{3}}$$

Critical point at:  $x = 4$  No discontinuities exist.  
 Increasing:  $(4, \infty)$  Decreasing:  $(-\infty, 4)$

$$7) y = -\frac{1}{5}(x - 4)^{\frac{5}{3}} - 2(x - 4)^{\frac{2}{3}} - 1$$

Critical points at:  $x = 0, 4$  No discontinuities exist.  
 Increasing:  $(0, 4)$  Decreasing:  $(-\infty, 0), (4, \infty)$

### Critical thinking question:

8) If functions  $f$  and  $g$  are increasing on an interval, show that  $f + g$  is increasing on the same interval.

We know that if  $x_1 < x_2$ , then  $f(x_1) < f(x_2)$  and  $g(x_1) < g(x_2)$ . Therefore,  
 $f(x_1) + g(x_1) < f(x_2) + g(x_2)$ .

9) Give an example where functions  $f$  and  $g$  are increasing on the interval  $(-\infty, \infty)$ , but where  $f - g$  is decreasing.

Many answers. Ex:  $f = x$  and  $g = 2x$