Date: ____________________ Name: ________________________

PRACTICE 1: Find $\frac{dy}{dx}$ using implicit differentiation.

Do on separate paper & hand in.

Part A

1) $y^3 + y^2 - 5y - x^2 = -4$
2) $y^3 + x^3 = 8$
3) $x^2 + 2xy + y^2 = 4$
4) $4x^2y^2 + x = y^4$
5) $x^3 - 7x^2y^3 + 4y^2 = -16$

(1) $3y^2y' + 2yy' - 5y' - 2x = 0$

$y' = \frac{2x}{3y^2+2y-5}$

(2) $3y^2y' + 3x^2 = 0$

$y' = -\frac{3x^2}{3y^2} = -\frac{x^2}{y^2}$

(3) $2x + 2y + 2xy' + 2yy' = 0$

$y' = \frac{-2(x+y)}{2(x+y)} = -1$

(4) $8xy^2 + 4x^2(2yy') + 1 = 4y^3y'$

$\frac{8xy^2+1}{4y^3-8x^2y} = y'$

(5) $3x^2 - 14xy^3 - 7x^2(3y^2y') + 8yy' = 0$

$y' = \frac{14xy^3 - 3x^2}{8y - 21x^2y^2}$

Part B Find $\frac{dy}{dx}$ by implicit differentiation for each of the following:

(1) $-3xy - 4y^2 = 2$

$-3y - 3xy' - 8yy' = 0$

$y' = \frac{3y}{-3x-8y}$

(2) $8x^2 = 2y^3 + 3xy^2$

$16x = 6y^2y' + 3y^2 + 3x(2yy')$

$y' = \frac{16x - 3y^2}{6y^2 + 6xy}$

(3) $\frac{3}{2x} + \frac{1}{y} = y$

$\frac{-3}{2x^2} - \frac{1}{y^2}y' = y'$

$y' = \frac{-3}{2x^2} \div \left(1 + \frac{1}{y^2}\right)$

$= \frac{-3y^2}{2x^2(y^2+1)}$

(4) $3x^2 = \frac{2-y}{2+y}$

$3x^2(2+y) = 2-y$

$6x(2+y) + 3x^2(y') = -y'$

$y' = -\frac{6x(2+y)}{3x^2+1}$

(5) $x = \tan y$

$1 = \sec^2 y\, y'$

$y' = \frac{1}{\sec^2 y}$

(6) $y = \cos(x - y)$

$y' = -\sin(x-y)(1-y')$

$y' = \frac{-\sin(x-y)}{1-\sin(x-y)}$

(7) $x\sin y + y\sin x = 1$

$1\sin y + x\cos y\, y' + y'\sin x + y\cos x = 0$

$y' = \frac{-\sin y - y\cos x}{x\cos y + \sin x}$

(8) $x = \sec^3(y^2 - 1)$

$1 = 3\sec^2(y^2-1)\sec(y^2-1)\tan(y^2-1)(2yy')$

$y' = \frac{1}{\text{all factors from}}$

Part C Find the slope of the tangent line at the given point on each curve defined by the given equation:

(1) $x^2 + 3y^2 = 21$; $(3, -2)$

(2) $x^3 + \sqrt[3]{y} = 3$; $(1, 8)$

(3) $\sqrt{xy} - y = -2$; $(1, 4)$

(4) $3xy - 2x^4 = y^3 - 23$; $(2, -3)$

(5) $x = \cos y$; $\left(\frac{1}{2}, \frac{-\pi}{3}\right)$

(6) $\sin(xy) = x$; $\left(1, \frac{\pi}{2}\right)$

(1) $2x + 6yy' = 0$

$y' = \frac{-2x}{6y}$

$y'\Big|_{(3,-2)} = \frac{-1(3)}{3(-2)} = \frac{1}{2}$

(2) $3x^2 + \frac{1}{3}y^{-2/3}y' = 0$

$y' = -3x^2 \div \left(\frac{1}{3y^{2/3}}\right)$

$y'\Big|_{(1,8)} = -3 \times 3(8^{2/3}) = -36$

(3) $\frac{1}{2}(xy)^{-1/2}(1y + xy') - y' = 0$

$\frac{y}{2\sqrt{xy}} + \frac{xy'}{2\sqrt{xy}} - y' = 0$

$y' = -\frac{y}{2\sqrt{xy}} \div \left(\frac{x}{2\sqrt{xy}} - 1\right)$

$y'\Big|_{(1,4)} = \frac{-4}{2\sqrt{4}} \div \left(\frac{1}{2\sqrt{4}} - 1\right)$

$= -1 \div \left(\frac{-3}{4}\right) = \frac{4}{3}$

(4) $3y + 3xy' - 8x^3 = 3y^2y'$

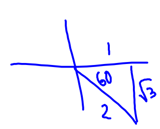
$y' = \frac{8x^3 - 3y}{3x - 3y^2}$

$y'\Big|_{(1,4)} = \frac{8-12}{3-48} = \frac{-4}{-45} = \frac{4}{45}$

(5) $1 = -\sin y\, y'$

$y' = \frac{1}{-\sin y}$

$y'\Big|_{(\frac{1}{2}, -\frac{\pi}{3})} = \frac{1}{-\sin(-\frac{\pi}{3})} = \frac{1}{+\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$



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(6) $\cos(xy)(1y + xy') = 1$

$\cos\left(\frac{\pi}{2}\right)\left(\frac{\pi}{2} + y'\right) = 1$

$0 = 1$

never true!

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PRACTICE 2 - Implicit Diff. Application (equations given)

1. A company determines that when x hundred units of a particular commodity are produced, the total cost of production is c thousand dollars, where $2c^3 - x^3 = 7000$. When 10,000 units are being produced, the level of production is increasing at the rate of 600 units per week. What is the total cost of production at this time and at what rate is it changing?

2. When the price of a certain commodity is p dollars per unit, customers demand x hundred units of the commodity, where
$$x^3 + 10px^2 + p^2x = 92x$$
How fast is the demand x changing with respect to time when the price is 6 dollars per unit and is decreasing at the rate of 24 cents per month?

3. At a certain factory, output is given by
$$Q = 40K^{\frac{1}{2}}L^{\frac{1}{2}} \text{ units,}$$
where K is the capital investment (in thousands of dollars) and L is the size of the labor force, measured in worker-hours. If output is kept constant, at what rate is capital investment changing at a time when $K = 10$, $L = 100$, and L is increasing at the rate of 40 worker-hours per week?

4. When the price of a certain commodity is p dollars per unit, the manufacturer is willing to supply x hundred units of the commodity, where
$$2p^2 + x^3 = 45$$
How fast is the supply x changing with respect to time when the price is 3 dollars per unit and is increasing at the rate of 90 cents per month.

(1) at $x = 10$ thousand, $c = 3000$, $\frac{dx}{dt} = 600$

$2c^2\frac{dc}{dt} - 3x^2\frac{dx}{dt} = 0$

$2(3000)^2\frac{dc}{dt} - 3(10)^2(600) = 0$

$\therefore \frac{dc}{dt} = 0.01$ \$/week

(2) at $p = 6$, $\frac{dp}{dt} = -0.24$

$x^3 + 60x^2 + 36x - 96x = 0$

$x(x^2 + 60x - 56) = 0$

$x = 0$ or $x = \frac{-60 \pm 61.84}{2}$ → 0.919, → −60.919

$3x^2\left(\frac{dx}{dt}\right) + 10\frac{dp}{dt} + 10p\left(2x\frac{dx}{dt}\right) + 2p\frac{dp}{dt}x + p^2\frac{dx}{dt} = 92\frac{dx}{dt}$

if demand is 0.919:

$\frac{dx}{dt} \doteq \frac{-10(-0.24) - 2(6)(0.919)(-0.24)}{3(0.919)^2 + 10(6)(2)(0.919) + 6^2 - 92}$

$\frac{dx}{dt} \sim \frac{5.04672}{56.813683} \sim 0.0888$ units/month

if demand is 0:

$\frac{dx}{dt} \doteq \frac{-10(-0.24) - 0}{0 + 0 + 6^2 - 92} \sim \frac{2.4}{-56} \sim -0.043$ units/month

if demand is −60.9: $\frac{dx}{dt} \doteq \frac{-173.04672}{-18499.65...} \sim 0.00935$ units/month

(3) $0 = 40\frac{1}{2}K^{-1/2}\frac{dK}{dt}L^{1/2} + 40K^{1/2}\frac{1}{2}L^{-1/2}\frac{dL}{dt}$

$0 = \frac{20\sqrt{L}}{\sqrt{K}}\frac{dK}{dt} + \frac{20\sqrt{K}}{\sqrt{L}}\frac{dL}{dt}$

$0 = \frac{20\sqrt{100}}{\sqrt{10}}\frac{dK}{dt} + \frac{20\sqrt{10}}{\sqrt{100}}(40)$

$\frac{-80\sqrt{10}}{20\sqrt{10}} = \frac{dK}{dt}$

$-4 = \frac{dK}{dt}$

\$$-4000$/week $= \frac{dK}{dt}$

(4) $4p\frac{dp}{dt} + 3x^2\frac{dx}{dt} = 0$

$2(3)^2 + x^3 = 45$

$x^3 = 27$

$x = 3$

$4(3)(0.90) + 3(3)^2\frac{dx}{dt} = 0$

$\frac{dx}{dt} = \frac{-10.8}{27} \sim -0.4$ hundred units/month