Intro to Vectors - Unit 1

Tentative TEST date_____

Big idea/Learning Goals

Did you know that bees use vectors? A honey bee that has found a beautiful meadow full of ripe flowers must come back to the hive and communicate that information. A bee must tell its fellows in what direction and how far to travel to get to the meadow, they can even compensate for any wind direction in their communications! (Show a video from youtube.)

You will be introduced to the idea of a directed line segment, called a **vector**. You will explore vectors in their geometric and algebraic form, and will learn the notation used to describe vectors. You will then study vector addition and properties and how to understand vectors in 2D and 3D spaces. Vectors will enable you to define a line in 2D or 3D space (unit 3) after which you can then solve where two such lines meet, if ever (unit 4).

Corrections for the textbook answers:

Success Criteria

☐ I understand the new topics for this unit if I can do the practice questions in the textbook/handouts
Specific questions will not be assigned, since it will depend on your knowledge and skill (everyone is at a different level). The goal is to do all types of questions quickly and without reference to notes or back of textbook or another individual. BUT you may not have time to do every single question available… so… If you are a strong student you may just concentrate on harder TIPS or APP questions, while if you are a weak student you may want to use all your time practicing the basic KU or COMM questions. The number of questions done should also be proportional to your mark so far. If you have very low scores, more practice is required.

Date	pg	Topics	# of quest. done? You may be asked to show them
	2-4	Introduction – Geometric Vectors 6.1	
	5-7	Vector Addition/Subtraction & Properties 6.2 & 6.3	
	8-10	Vectors in R^2 and R^3 – Algebraic Vectors 6.5	
	11-12	Operations with Vectors in R^2 and R^3 6.6 & 6.7	
	13-14	Linear Combinations and Spanning Sets 6.8	
		Review	

Reflect – DIAGNOSTIC TEST mark _____

Introduction – GEOMETRIC vectors

Vector

Scalar

1. Which of these physical quantities is a vector and which is a scalar?

a. the mass of the moon

b. the acceleration of a drag racer

c. the velocity of a wave at the beach

d. the speed of light

e. the force of gravity

f. the magnetic field of the earth

g. the area of a rectangle

h. the temperature of a swimming pool

Diagrams

Notation

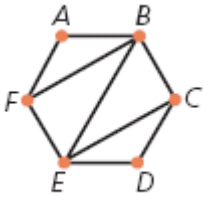
Equal vectors

Opposite Vectors RULE

2. ABCDEF is a regular hexagon. Give an example of vectors which are

a. equal

b. parallel but having different magnitudes



c. equal in magnitude but opposite in direction

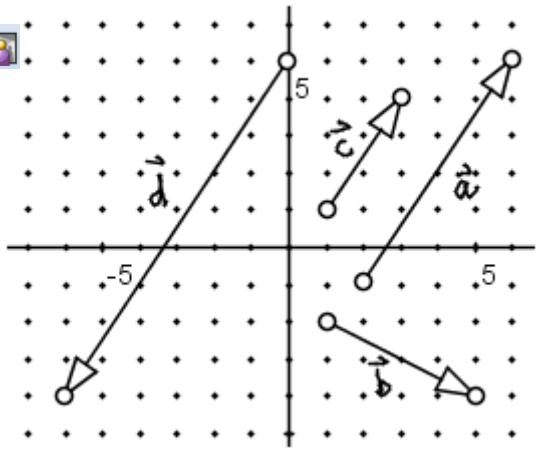
d. equal in magnitude but not equal vectors

e. different in both magnitude and direction

3. Angles can be quoted in different ways. Use the first two ways that follow for word problems, and the last one for non-word problem type question.

BEARING angles DIRECTION angles MATH angles

4. A student travels to school by bus, first riding 2 km west, then changing buses and riding a further 3 km north. Find the resultant vector.



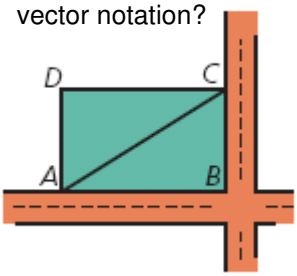
5. Determine the magnitude and direction of each of the vectors in the given diagram.

6. Examine the vectors in the diagram. Express $\vec{d}$ and $\vec{c}$ each as a scalar multiple of $\vec{a}$

7. A search and rescue aircraft, travelling at a speed of 240km/h, starts out at a heading of $N20^oW$. After travelling for one hour and fifteen minutes, it turns to a heading of $N80^oE$ and continues for another 2 hours before returning to base.
    a. Determine the displacement vector for each leg of the trip.
    b. Find the total distance the aircraft travelled and how long it took.

Vector Addition/Subtraction & Properties

1. Suppose rectangle ABCD is a park at a corner of an intersection. What are two ways to get from A to C written in vector notation?

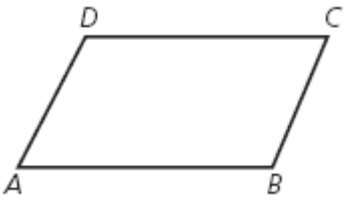


Adding Vectors RULE

Triangle Law of Vector Addition.

Parallelogram Law of Vector Addition.

2. In parallelogram ABCD, find the difference $\overrightarrow{AB} - \overrightarrow{AD}$
   a. geometrically
   b. algebraically



Vector Subtraction.

3. Show a geometric proof of the associative law.

Commutative Law of Addition.

Associative Law of Addition.

Distributive properties

Another Associative property

4. Show an informal proof of the triangle inequality: $\left|\vec{u}+\vec{v}\right| \le \left|\vec{u}\right| + \left|\vec{v}\right|$. When does equality hold?

5. Show a formal proof that $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are perpendicular when $|\vec{a}|=|\vec{b}|$.

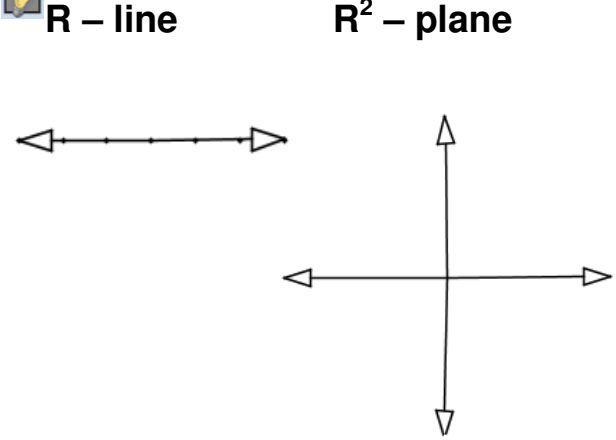
Unit Vector

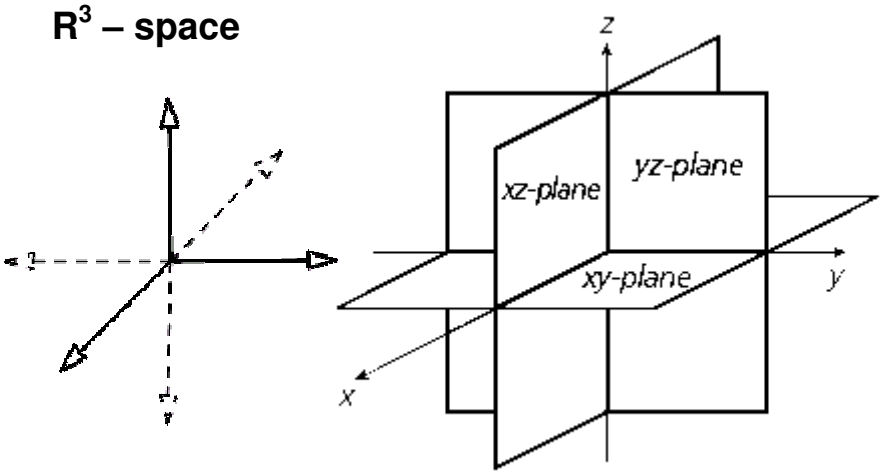
Collinear Vectors

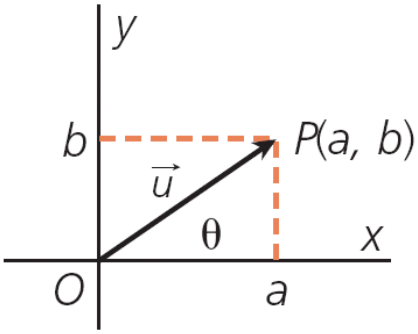
6. If $|\vec{a}|=5$, $|\vec{b}|=8$ and the angle between the two vectors is 120°.
   a. Calculate the vector $2\vec{a}-3\vec{b}$
   b. Determine the unit vector in the same direction as $2\vec{a}-3\vec{b}$

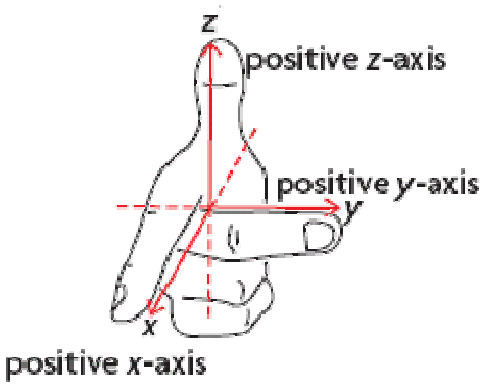
Vectors in R^2 and R^3 - ALGEBRAIC vectors

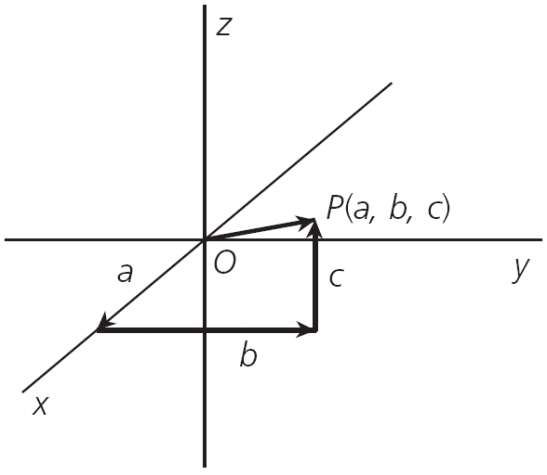
R – line **R^2 – plane** **R^3 – space**

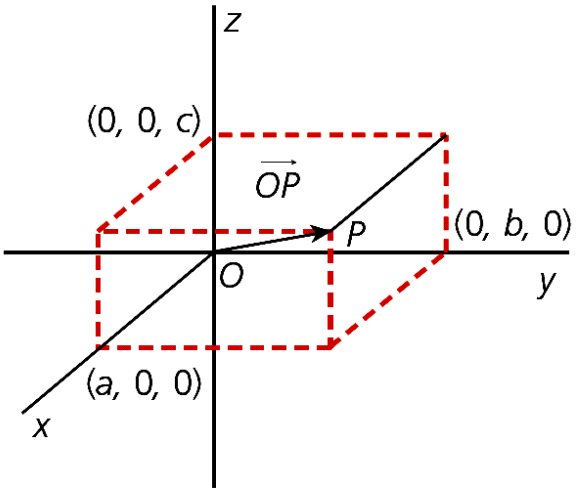






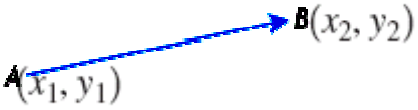




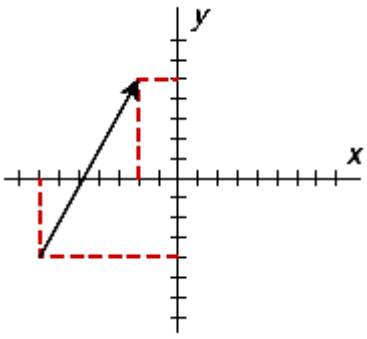


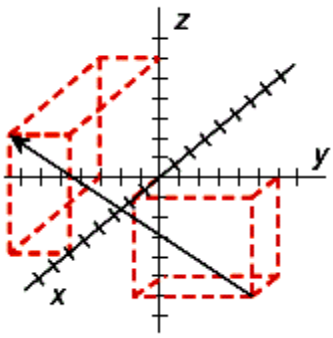
1. Draw a position vector of the point P(– 3, 7) then
   a. express it in both algebraic vector notations AND in geometric notation
   b. find the unit vector, how does it tie to unit circles you've learned in grade 11/12?

2. How do you find the related position vector of any vector between points?



3. Reposition each of the following vectors so that its initial point is at the origin, and determine its components.

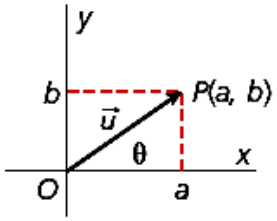




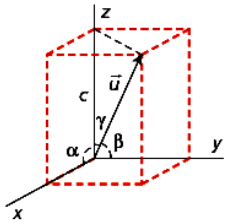
Formulas can be rewritten to use any vector between two point coordinates

$A(x_1, y_1)B(x_2, y_2)$ $A(x_1, y_1, z_1)B(x_2, y_2, z_2)$

Vectors in R²



Vectors in R³



4. Express as a vector in component form

a. $|\vec{a}| = 12$, $\theta = 330^o$ in R^2

b. $|\vec{u}| = 8$, $\alpha = 60^o$, $\beta = 150^o$ in R^3

Operations with Vectors in R^2 and R^3

1. Find a single vector equivalent to each of the following

   a. $-\frac{1}{2}(4,-6,8)+\frac{3}{2}(4,-6,8)$

   b. $5(9\hat{i}-7\hat{j})-5(-9\hat{i}+7\hat{k})$

   c. If $\vec{a}=(2,-1,4)$ and $\vec{b}=3\hat{i}+8\hat{j}-6\hat{k}$ find $2\vec{a}-\vec{b}$ and its magnitude.

2. If $A(1,-5,2)$ and $B(-3,4,4)$ are opposite vertices of parallelogram OAPB and O is the origin, find the coordinates of P. Show calculations in both component form and unit vector form.

3. Using vectors, demonstrate that the three points $A(5,-1), B(-3,4)$ and $C(13,-6)$ are collinear.

4. Find the components of the unit vector with the direction opposite to $\overrightarrow{XY}$ where $X(7,4,-2)$ and $Y(1,2,1)$.

5. Draw a position vector of the point T(– 5, – 7, 2) then

   a. find a unit vector in the same direction as $\overrightarrow{OT}$
   b. express it in two algebraic vector notations AND geometric notation

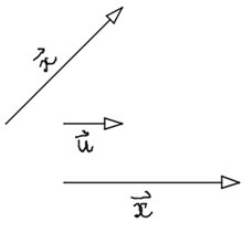
5. Find the point on the y-axis that is equidistant from the points (2, – 1, 1) and (0, 1, 3)

Linear Combinations and Spanning Sets

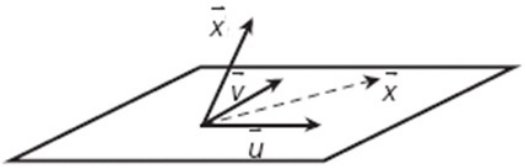
Linear combinations

CONSIDER: Which $\vec{x}$ cannot be written in terms of $\vec{u}$ and/or $\vec{v}$?

Collinear Vectors



Coplanar Vectors



Spanning sets

Spanning set for R

Spanning set for R^2

Spanning set for R^3

1. Explain what two vectors can span then determine if the following $\vec{x}$ and $\vec{u}$ are collinear.

a. $\vec{x} = 4\hat{i} - 8\hat{j}$
$\vec{u} = 6\hat{i} - 12\hat{j}$

b. $\vec{x} = (10, -8, 3)$
$\vec{u} = (5, -4, 6)$

2. Explain what three vectors can span then determine if the following three vectors are coplanar.

a. $\vec{u} = (3,-1,4)$, $\vec{v} = (6,-4,-8)$, $\vec{w} = (7,-3,4)$

b. $\vec{u} = (1,3,2)$, $\vec{v} = (1,-1,1)$, $\vec{w} = (5,1,-4)$