p1NOTES

January-22-13
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IntroVectors
NotesNEW
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see below

Intro to Vectors - Unit 1

Tentative TEST date Tue. Feb.12

Big idea/Learning Goals

Did you know that bees use vectors? A honey bee that has found a beautiful meadow full of ripe flowers must come back to the hive and communicate that information. A bee must tell its fellows in what direction and how far to travel to get to the meadow, they can even compensate for any wind direction in their communications! (Show a video from youtube.)

You will be introduced to the idea of a directed line segment, called a **vector**. You will explore vectors in their geometric and algebraic form, and will learn the notation used to describe vectors. You will then study vector addition and properties and how to understand vectors in 2D and 3D spaces. Vectors will enable you to define a line in 2D or 3D space (unit 3) after which you can then solve where two such lines meet, if ever (unit 4).

Corrections for the textbook answers:

Success Criteria

- ☐ I understand the new topics for this unit if I can do the practice questions in the textbook/handouts
Specific questions will not be assigned, since it will depend on your knowledge and skill (everyone is at a different level). The goal is to do all types of questions quickly and without reference to notes or back of textbook or another individual. BUT you may not have time to do every single question available… so… If you are a strong student you may just concentrate on harder TIPS or APP questions, while if you are a weak student you may want to use all your time practicing the basic KU or COMM questions. The number of questions done should also be proportional to your mark so far. If you have very low scores, more practice is required.

Date	pg	Topics	# of quest. done? You may be asked to show them
Feb.4	2-4	Introduction – Geometric Vectors 6.1	
	5-7	Vector Addition/Subtraction & Properties 6.2 & 6.3	
	8-10	Vectors in R^2 and R^3 – Algebraic Vectors 6.5	
	11-12	Operations with Vectors in R^2 and R^3 6.6 & 6.7	
	13-14	Linear Combinations and Spanning Sets 6.8	
		Review	

Reflect – DIAGNOSTIC TEST mark ______________

Introduction – GEOMETRIC vectors

Vector a quantity with magnitude (length/size) AND direction.

Scalar a quantity with magnitude only.

1. Which of these physical quantities is a vector and which is a scalar?

a. the mass of the moon S

b. the acceleration of a drag racer V

c. the velocity of a wave at the beach V

d. the speed of light S

e. the force of gravity V

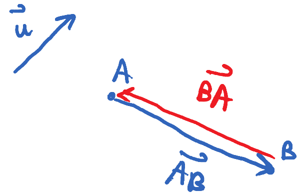
f. the magnetic field of the earth V

g. the area of a rectangle S

h. the temperature of a swimming pool S

Diagrams

- arrow to show direction
- length of line to show magnitude



"point to point notation"

Notation

vectors – use letters with an arrow on top

ex. $\vec{u}$, $\vec{x}$, $\overrightarrow{AB}$

scalars – use just letters (no arrow) or numbers

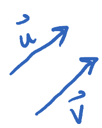
ex. x , 5 ...

magnitude – use absolute value around the vector name.

ex. $|\vec{x}|$ $|\overrightarrow{AB}|$ $\cancel{|5|}$ $|5| = 5$

Equal vectors

$\vec{u} = \vec{v}$ $\Longleftrightarrow$ if and only if magnitudes and directions are the same.

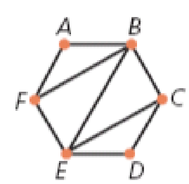


Opposite Vectors RULE

$\overrightarrow{AB}$ is opposite of $\overrightarrow{BA}$

$|\overrightarrow{AB}| = |\overrightarrow{BA}|$ $\overrightarrow{AB} = -\overrightarrow{BA}$

2. ABCDEF is a regular hexagon. Give an example of vectors which are



a. equal

$\overrightarrow{AB} = \overrightarrow{ED}$ many more answers.

b. parallel but having different magnitudes

$\overrightarrow{FA}$, $\overrightarrow{EB}$

c. equal in magnitude but opposite in direction

$\overrightarrow{AF}$, $\overrightarrow{DC}$

d. equal in magnitude but not equal vectors

$\overrightarrow{AF}$, $\overrightarrow{FE}$

e. different in both magnitude and direction

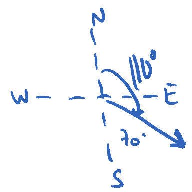
$\overrightarrow{AB}$, $\overrightarrow{BE}$

3. Angles can be quoted in different ways. Use the first two ways that follow for word problems, and the last one for non-word problem type question.

BEARING angles

Rotation clockwise from North Line.

ex. Bearing of 110°



DIRECTION angles

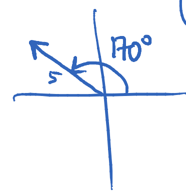
↳ tell you how much away from North/South line.

ex. S 70° E

Read as "70° east of south"

MATH angles Polar Form (r, θ)

counter clockwise from pos. x-axis

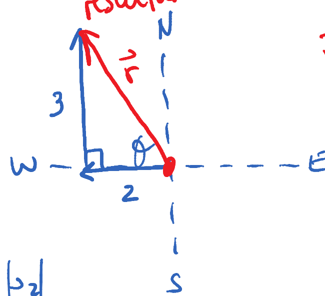


4. A student travels to school by bus, first riding 2 km west, then changing buses and riding a further 3 km north. Find the resultant vector.

vector! mag dir

two vectors added.

→ vector with "tail" at starting pt. and "head" at ending pt.



$\vec{r} = \sqrt{13}$ km [N 34° W]

mag direction

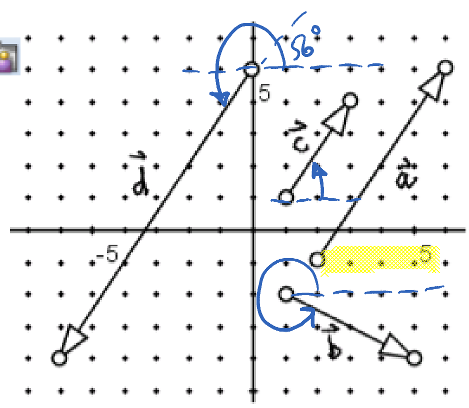
$|\vec{r}| =$ mag. only.

$\tan\theta = \frac{3}{2}$

$\therefore \theta = \tan^{-1}\left(\frac{3}{2}\right) = 56°$

$2^2 + 3^2 = |\vec{r}|^2$

$\sqrt{13} = |\vec{r}|$



5. Determine the magnitude and direction of each of the vectors in the given diagram.

$|\vec{a}| = \sqrt{4^2 + 6^2} = \sqrt{52} = 2\sqrt{13}$ $\theta_a = \tan^{-1}\left(\frac{6}{4}\right) = 56°$

polar : $(2\sqrt{13}, 56°)$

$|\vec{b}| = \sqrt{2^2 + 4^2} = \sqrt{20} = 2\sqrt{5}$ $\theta_b = \tan^{-1}\left(\frac{-2}{4}\right) = -27° = 333°$

$|\vec{c}| = \sqrt{13}$ $\theta_c = 56°$

+ 180° since opposite direction to $\vec{c}$

$|\vec{d}| = 3\sqrt{13}$ $\theta_d = 236°$

6. Examine the vectors in the diagram. Express $\vec{d}$ and $\vec{c}$ each as a scalar multiple of $\vec{a}$

stretch/compress.

$\vec{d} = -\frac{3}{2}\vec{a}$

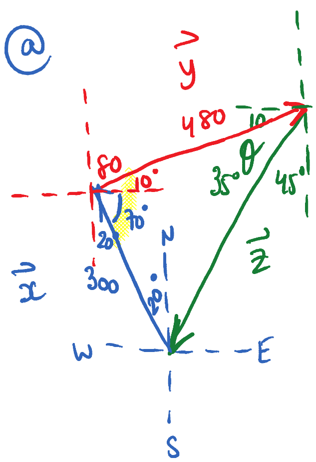
$\vec{c} = \frac{1}{2}\vec{a}$

☆ this is possible only on collinear/parallel vectors.

7. A search and rescue aircraft, travelling at a speed of 240km/h, starts out at a heading of $N20°W$. After travelling for one hour and fifteen minutes, it turns to a heading of $N80°E$ and continues for another 2 hours before returning to base.

   a. Determine the displacement vector for each leg of the trip.
   b. Find the total distance the aircraft travelled and how long it took.

ⓐ



$\therefore \vec{x} = 300\,km\,[N\,20°\,W]$

$\vec{y} = 480\,km\,[N\,80°\,E]$

$\vec{z} = 520\,km\,[S\,45°\,W]$

D / V T

$D = VT$

$= (240\,\frac{km}{h})(1.25\,h)$

$= 300\,km$

$D = (240)(2)$

$= 480\,km$

$|\vec{z}|^2 = 300^2 + 480^2 - 2(300)(480)\cos 80°$

$|\vec{z}| \doteq 520\,km$

$\frac{\sin\theta}{300} = \frac{\sin 80}{520}$

$\theta = \sin^{-1}\left(\frac{300 \sin 80}{520}\right)$

$\theta = 35°$

ⓑ Total distance = 1300 km (scalar)

$\therefore$ Total Time = 1 hr 15 min + 2 hr + 2 hr 10 min

= 5 hrs 25 min

$T = \frac{D}{V}$

$= \frac{520}{240}$

$= 2.1\bar{6}$

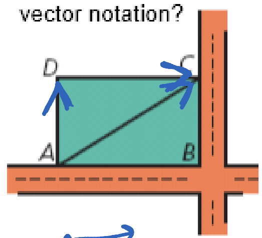
$0.1\bar{6} \times 60\,min$

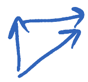
$\sim 10\,min$

Note: if you're told angle between vectors → draw it between tail-to-tail version!

Vector Addition/Subtraction & Properties

1. Suppose rectangle ABCD is a park at a corner of an intersection. What are two ways to get from A to C written in vector notation?





$\vec{AC} = \vec{AB} + \vec{BC}$

or

$\vec{AC} = \vec{AD} + \vec{DC}$

Adding Vectors RULE

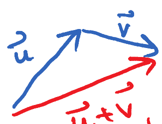
if you have addition and the inside letter matches then you can "collapse". $\vec{AC} = \vec{AX} + \vec{XC}$

(only 1st pt and last pt matter)

collapses

Triangle Law of Vector Addition.

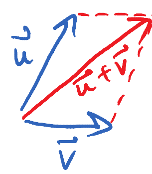
Place vectors head to tail



Resultant vector

Parallelogram Law of Vector Addition.

Place vectors Tail to tail

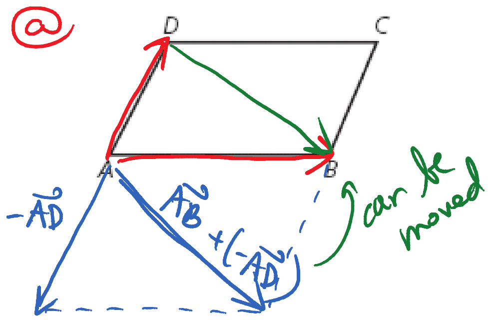


Resultant vector from the corner with tails.

2. In parallelogram ABCD, find the difference $\overrightarrow{AB} - \overrightarrow{AD}$
   a. geometrically
   b. algebraically

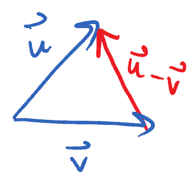
$\vec{AB} + (-\vec{AD})$

ⓐ



Vector Subtraction.

Place vectors Tail to tail



$\vec{u}-\vec{v}$

Resultant will point at the head of the "positive" one

ⓑ

$\vec{AB} - \vec{AD}$) opposite vector rule

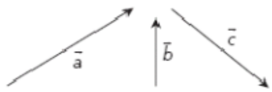
$= -\vec{BA} - \vec{AD}$) common factor neg

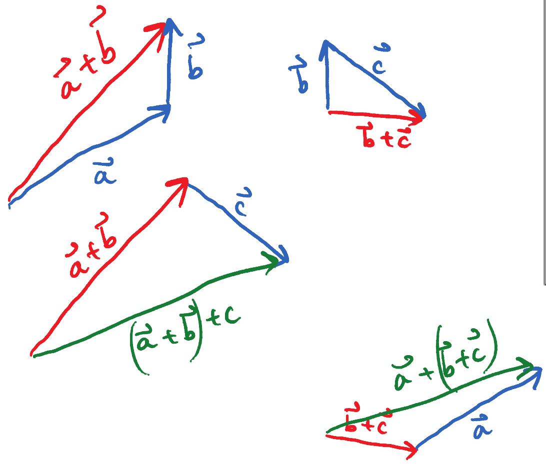
$= -(\vec{BA} + \vec{AD})$) "collapse" Adding Vectors Rule

$= -(\vec{BD})$) opposite vector rule.

$= \vec{DB}$

3. Show a geometric proof of the associative law.





Commutative Law of Addition.

$\vec{u}+\vec{v}=\vec{v}+\vec{u}$

Associative Law of Addition.

$(\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c})$

Distributive properties

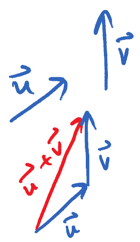
$k(\vec{a}+\vec{b})=k\vec{a}+k\vec{b}$

$(m+n)\vec{a}=m\vec{a}+n\vec{a}$

Another Associative property

$(mn)\vec{a}=m(n\vec{a})=mn\vec{a}$

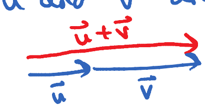
4. Show an informal proof of the triangle inequality: $|\vec{u}+\vec{v}|\leq|\vec{u}|+|\vec{v}|$. When does equality hold?



Resultant smaller than or equal to two vectors' magnitudes added separately.

magnitude of resultant is smaller since it can be thought of as the "shortcut".

equal to if $\vec{u}$ and $\vec{v}$ are parallel

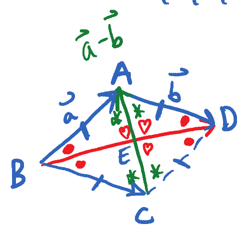


6

then if

5. Show a formal proof that $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are perpendicular when $|\vec{a}| = |\vec{b}|$

Assume $|\vec{a}|=|\vec{b}|$



$\angle ABD = \angle ADB$ • since ΔABD is isosceles

Similarly $\angle CBD = \angle CDB$ •

$\angle DAC = \angle DCA$ * since ΔDAC is isosceles

similarly $\angle BAC = \angle BCA$ *

Notice that the 4 Δ (small Δ of the same shape as ΔABE) are congruent / identical $\therefore \angle AEB = \angle AED = \angle DEC = \angle CEB$ ♡

$\therefore$ Full revolution at pt. E is $360°$ $4♡ = 360°$

$\therefore ♡ = 90°$ $\therefore \vec{a}+\vec{b} \perp \vec{a}-\vec{b}$

$(\hat{u})$ Unit Vector

has a magnitude of ONE

$$\hat{u} = \frac{\vec{u}}{|\vec{u}|}$$

Collinear Vectors (PARALLEL)

vectors that can be moved and placed on ONE line.

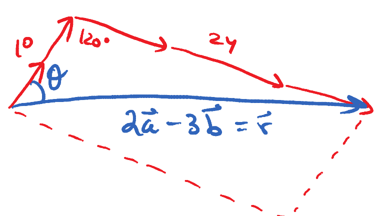
$\vec{a}$ and $\vec{b}$ are collinear $\Leftrightarrow$ iff $\vec{a} = k\vec{b}$

6. If $|\vec{a}| = 5$, $|\vec{b}| = 8$ and the angle between the two vectors is 120°.

a. Calculate the vector $2\vec{a} - 3\vec{b}$

b. Determine the unit vector in the same direction as $2\vec{a} - 3\vec{b}$

@



$|2\vec{a}-3b|^2$

$|\vec{r}|^2 = 10^2 + 24^2 - 2(10)(24)\cos 120°$

$|\vec{r}| \doteq 30.3 = \sqrt{916} = 2\sqrt{229}$

$$\frac{\sin\theta}{24} = \frac{\sin 120°}{30.3}$$

$\theta = 43°$

$\therefore 2\vec{a}-3\vec{b} = 30.3$ [43° off of $\vec{a}$]

b) unit vector $= \dfrac{2\vec{a}-3\vec{b}}{|2\vec{a}-3\vec{b}|} = \dfrac{1}{|2\vec{a}-3\vec{b}|}(2\vec{a}-3\vec{b})$

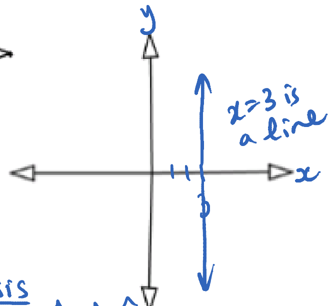
~~$2a-3b$~~

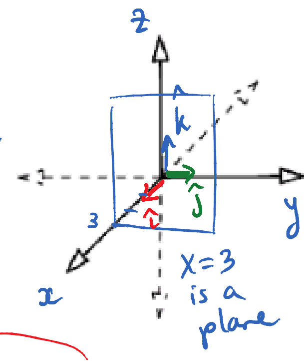
$= \dfrac{1}{2\sqrt{229}}(2\vec{a}-3\vec{b})$

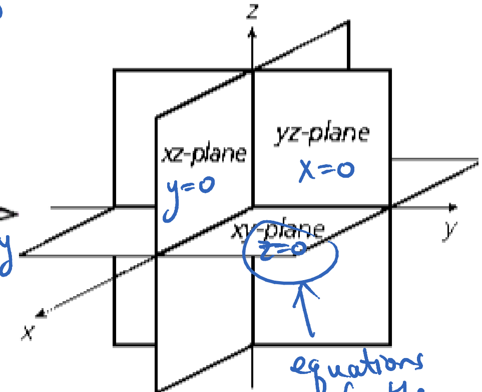
Vectors in R^2 and R^3 - ALGEBRAIC vectors

R – line 1D **R^2 – plane** 2D **R^3 – space** 3D

X=3 is a point

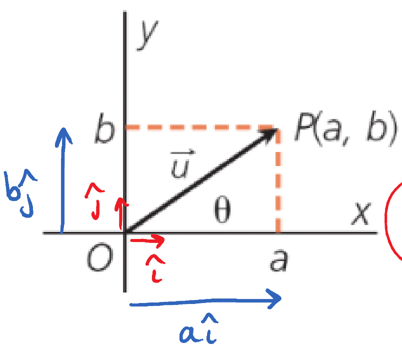






equations for the planes

Standard Basis
Unit vectors: $\hat{i}, \hat{j}, \hat{k}$



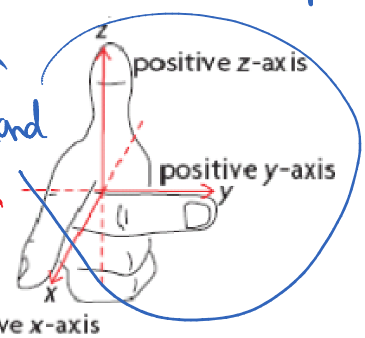
Position vector
- is a vector's tail moved to the origin of a coordinate system.

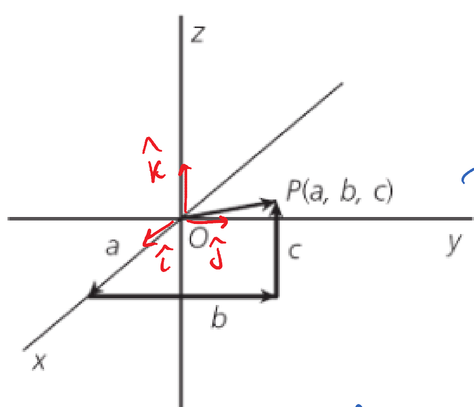
$\overrightarrow{OP} = \vec{u} = (a, b)$ ← component form

$\vec{u} = a\hat{i} + b\hat{j}$ ← linear combination of standard basis vectors

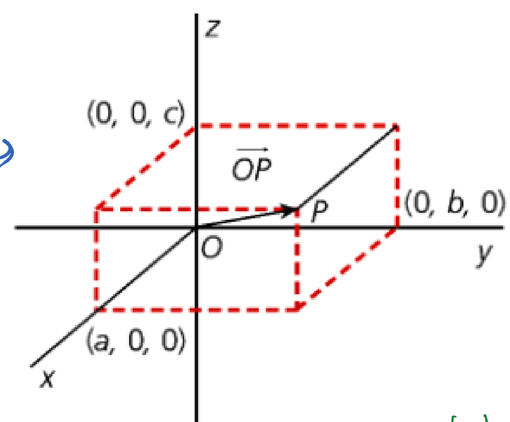
$|\vec{u}| = \sqrt{a^2 + b^2}$

Use your Right Hand





easier to visualize with a "Box"



$\overrightarrow{OP} = a\hat{i} + b\hat{j} + c\hat{k}$

$\overrightarrow{OP} = (a, b, c)$

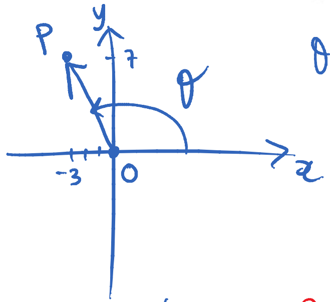
$|\overrightarrow{OP}| = \sqrt{a^2 + b^2 + c^2}$

2D Geometric form $\vec{u} = |\vec{u}|\,[\theta \text{ dir}]$

3D Geometric form $\vec{u} = |\vec{u}|\,[\alpha, \beta, \gamma]$

rotation counterclockwise from x-axis
y-axis
z-axis

1. Draw a position vector of the point P(– 3, 7) then
   a. express it in both algebraic vector notations AND in geometric notation
   b. find the unit vector, how does it tie to unit circles you've learned in grade 11/12?



$\theta = \tan^{-1}\left(\frac{7}{-3}\right) = -67^\circ$ not in II

$\therefore \theta = 113^\circ$

$|\vec{OP}| = \sqrt{3^2 + 7^2} = \sqrt{58}$

(b) $\hat{OP} = \frac{\vec{OP}}{|\vec{OP}|}$

$= \frac{1}{|\vec{OP}|}\vec{OP}$

$= \frac{1}{\sqrt{58}}(-3, 7)$

$= \left(\frac{-3}{\sqrt{58}}, \frac{7}{\sqrt{58}}\right)$

$\left(\frac{x}{r}, \frac{y}{r}\right)$

$\cos\theta$ $\sin\theta$

(a) $\left.\begin{array}{l}\vec{OP} = (-3, 7) \\ \vec{OP} = -3\hat{i} + 7\hat{j}\end{array}\right\}$ alg. Forms

$\vec{OP} = \sqrt{58}\ [\theta = 113^\circ]$

2. How do you find the related position vector of any vector between points?

if the vector doesn't have the tail at the origin - it can always be moved.

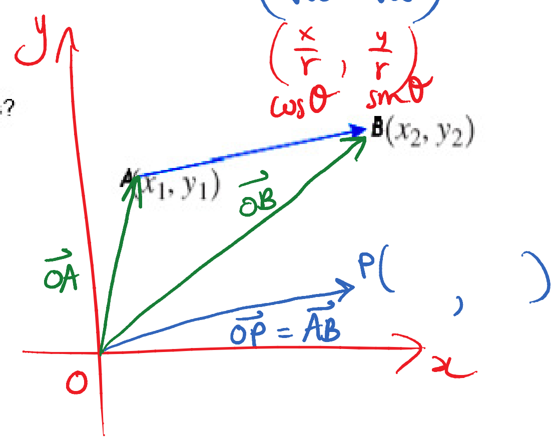
$\vec{OP} = \vec{AB} = \vec{AO} + \vec{OB}$ "uncollapse"

$= -\vec{OA} + \vec{OB}$

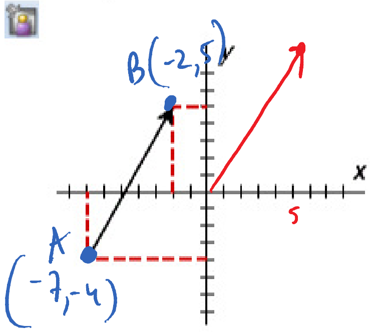
$= \vec{OB} - \vec{OA}$

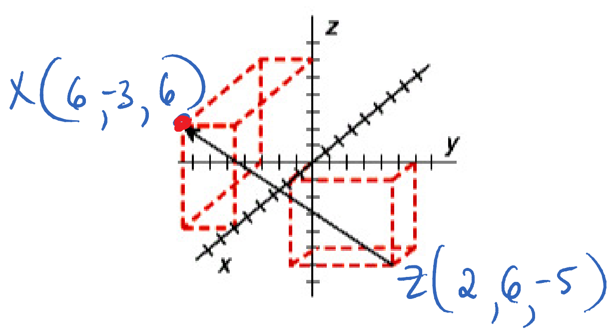
$= (x_2, y_2) - (x_1, y_1)$

$= (x_2 - x_1, y_2 - y_1)$



3. Reposition each of the following vectors so that its initial point is at the origin, and determine its components.





$\vec{AB} = (-2 - -7, 5 - -4)$

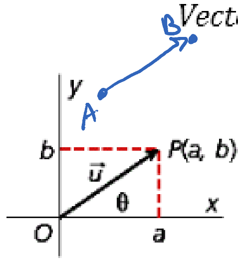
$\vec{AB} = (5, 9)$

$\therefore \vec{ZX} = (4, -9, 11)$

Formulas can be rewritten to use any vector between two point coordinates

$A(x_1, y_1)B(x_2, y_2)$ $A(x_1, y_1, z_1)B(x_2, y_2, z_2)$

Vectors in R^2



$\vec{AB} = (x_2 - x_1, y_2 - y_1)$

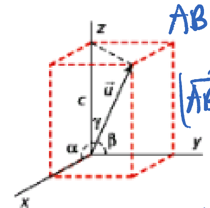
$|\vec{AB}| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$ (distance formula gr. 10)

geo. vector : $\vec{u} = |\vec{u}|\,[\theta]$

unit vector : $\hat{u} = \frac{\vec{u}}{|\vec{u}|} = (\cos\theta, \sin\theta)$

rearranged

Vectors in R^3



$\vec{AB} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$

$|\vec{AB}| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

geo. vect. $\vec{u} = |\vec{u}|\,[\alpha, \beta, \gamma]$

unit vector : $\hat{u} = \frac{\vec{u}}{|\vec{u}|} = (\cos\alpha, \cos\beta, \cos\gamma)$

4. Express as a vector in component form

a. $|\vec{a}| = 12$, $\theta = 330^\circ$ in R^2 $\vec{a} = 12\,[\theta = 330]$

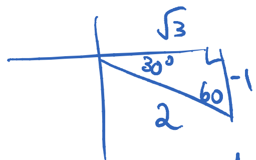
b. $|\vec{u}| = 8$, $\alpha = 60^\circ$, $\beta = 150^\circ$ in R^3

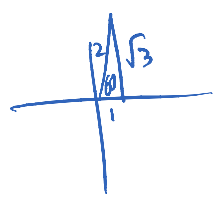
ⓐ $\vec{a} = |\vec{a}|\,\hat{a}$

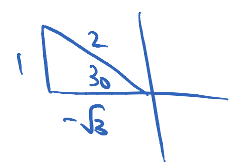
$= 12(\cos 330, \sin 330)$

$= 12\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$= (6\sqrt{3}, -6)$







ⓑ $\vec{u} = |\vec{u}|\,\hat{u}$

$= 8(\underbrace{\cos 60^\circ, \cos 150^\circ, \cos\gamma}_{\text{unit vector}})$ $|\hat{u}| = 1$

$= 8\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}, 0\right)$

$\vec{u} = (4, -4\sqrt{3}, 0)$ ← component form

$\vec{u} = 4\hat{i} - 4\sqrt{3}\hat{j}$ ← standard basis form

$1 = \sqrt{\cos^2 60^\circ + \cos^2 150^\circ + \cos^2\gamma}$

$1 = \cos^2 60 + \cos^2 150 + \cos^2\gamma$

$1 = \left(\frac{1}{2}\right)^2 + \left(\frac{-\sqrt{3}}{2}\right)^2 + \cos^2\gamma$

$1 = \frac{1}{4} + \frac{3}{4} + \cos^2\gamma$

$\sqrt{\ }$ $1 = 1 + \cos^2\gamma$

$0 = \cos\gamma$

$\gamma = 90^\circ$

Operations with Vectors in R^2 and R^3

6.6 + 6.7

1. Find a single vector equivalent to each of the following

   a. $-\frac{1}{2}(4,-6,8)+\frac{3}{2}(4,-6,8)$

   b. $5(9\hat{i}-7\hat{j})-5(-9\hat{i}+7\hat{k})$

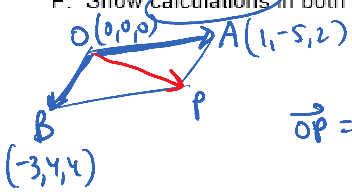
   c. If $\vec{a}=(2,-1,4)$ and $\vec{b}=3\hat{i}+8\hat{j}-6\hat{k}$ find $2\vec{a}-\vec{b}$ and its magnitude.

ⓐ $(-2,3,-4)+(6,-9,12)$
$=(4,-6,8)$

ⓑ $-90\hat{i}-35\hat{j}-35\hat{k}$

ⓒ $(1,-10,14)$
$=\hat{i}-10\hat{j}+14\hat{k}$
$3\sqrt{33}$

2. If $A(1,-5,2)$ and $B(-3,4,4)$ are opposite vertices of parallelogram OAPB and O is the origin, find the coordinates of P. Show calculations in both component form and unit vector form.

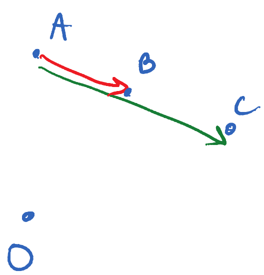


vector $\vec{OP}$ will "look like" pt. P

$\vec{OP}=\vec{OB}+\vec{OA}$
$=(-3,4,4)+(1,-5,2)$
$=(-2,-1,6)$ ∴ pt. $P=(-2,-1,6)$

$-3\hat{i}+4\hat{j}+4\hat{k}+\hat{i}-5\hat{j}+2\hat{k}$
$=-2\hat{i}-1\hat{j}+6\hat{k}$

3. Using vectors, demonstrate that the three points $A(5,-1)$, $B(-3,4)$ and $C(13,-6)$ are collinear. fall in one line



$\vec{AB}=(-3-5,\ 4--1)$
$=(-8,5)$

$\vec{AC}=(13-5,\ -6--1)$
$=(8,-5)$

since $\underbrace{\vec{AB}=k\vec{AC}}_{\text{scalar multiple}}$ then vectors $\vec{AB}$ and $\vec{AC}$ are parallel which means the pts. A, B, C are in one line.

4. Find the components of the unit vector with the direction opposite to $\overrightarrow{XY}$ where $X(7,4,-2)$ and $Y(1,2,1)$.

$\hat{XY}=\frac{1}{|\vec{XY}|}(\vec{XY})$

$\vec{XY}=(1-7,2-4,1--2)$
$=(-6,-2,3)$

$|\vec{XY}|=\sqrt{6^2+2^2+3^2}$
$=\sqrt{49}=7$

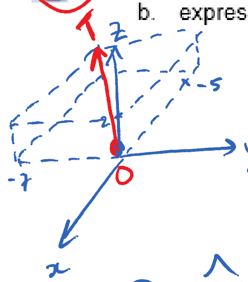
∴ $\hat{XY}$ in the opposite direction is swap signs
$\frac{1}{7}(6,2,-3)$

$\cos\alpha = \frac{-5}{\sqrt{78}}$

5. Draw a position vector $\vec{OT}$ of the point T(– 5, – 7, 2) then
   a. find a unit vector in the same direction as $\vec{OT}$ $\hat{OT}$
   b. express it in two algebraic vector notations AND geometric notation

$\vec{OT} = |\vec{OT}| \, [\alpha = \quad \beta = \quad \gamma = \quad]$

$= \sqrt{78} \, [\alpha = 124^\circ \; \beta = 142^\circ, \gamma = 77^\circ]$



(b) $\vec{OT} = (-5, -7, 2) = -5\hat{i} - 7\hat{j} + 2\hat{k}$

(a) $\hat{OT} = \frac{1}{\sqrt{78}}(-5, -7, 2)$

$\hat{u} = \frac{\vec{u}}{|\vec{u}|} = \frac{1}{|\vec{u}|}\vec{u}$

6. Find the point on the y-axis that is equidistant from the points A(2, – 1, 1) and B(0, 1, 3)

same distance

$P(0, y, 0)$

$\vec{PA} = (2-0, -1-y, 1-0)$

$\vec{PB} = (0, 1-y, 3)$

$|\vec{PA}| = |\vec{PB}|$

$\left(\sqrt{2^2 + (-1-y)^2 + 1^2}\right)^2 = \left(\sqrt{0^2 + (1-y)^2 + 3^2}\right)^2$

⋮

solve for y

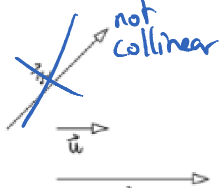
$y = 1$

Linear Combinations and Spanning Sets

Linear combinations are expressions $a\vec{u} + b\vec{v}$ where a, b are scalars and $\vec{u}, \vec{v}$ are vectors ex. $2\hat{i} - 3\hat{j}$ is a linear combination of $\hat{i} = (1,0)$ $\hat{j} = (0,1)$

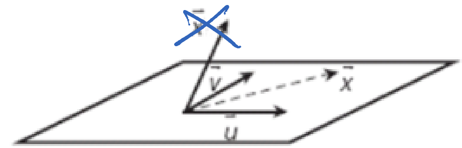
CONSIDER: Which $\vec{x}$ cannot be written in terms of $\vec{u}$ and/or $\vec{v}$?

Collinear Vectors



$\vec{x}$ and $\vec{u}$ are collinear $\overset{\text{iff}}{\Longleftrightarrow}$ there is a scalar k such that $\vec{x} = k\vec{u}$

Coplanar Vectors



$\vec{u}, \vec{v}$ and $\vec{x}$ are coplanar $\overset{\text{iff}}{\Longleftrightarrow}$ there are scalars a and b such that $\vec{x} = a\vec{u} + b\vec{v}$

Spanning sets smallest # of vectors that can generate any other vector in the given space.

Spanning set for 1D: 1D - any vector will span a LINE

Spanning set for 2D: 2D - any two noncollinear vectors will span a plane

Spanning set for 3D: 3D - any three non-coplanar vectors will span a space

1. Explain what two vectors can span then determine if the following $\vec{x}$ and $\vec{u}$ are collinear.

a. $\vec{x} = 4\hat{i} - 8\hat{j} = (4, -8)$ ×1.5
$\vec{u} = 6\hat{i} - 12\hat{j} = (6, -12)$
collinear
$\therefore$ spans a line in $\mathbb{R}^2$

b. $\vec{x} = (10, -8, 3)$ 3 components
$\vec{u} = (5, -4, 6)$
non collinear
$\therefore$ span a plane in $\mathbb{R}^3$

with algebra:
Assume $\vec{x} = k\vec{u}$
$(10, -8, 3) = k(5, -4, 6)$
$10 = 5k$, $-8 = -4k$, $3 = 6k$
$2 = k$ $\quad$ $2 = k$ $\quad$ $\frac{1}{2} = k$
contradiction
$\therefore \vec{x} \neq k\vec{u}$
$\vec{x}, \vec{u}$ are non collinear

2. Explain what three vectors can span then determine if the following three vectors are coplanar.

a. $\vec{u} = (3,-1,4)$, $\vec{v} = (6,-4,-8)$, $\vec{w} = (7,-3,4)$

$\vec{u} = a\vec{v} + b\vec{w}$

$(3,-1,4) = a(6,-4,-8) + b(7,-3,4)$

$= (6a,-4a,-8a) + (7b,-3b,4b)$

$(3,-1,4) = (6a+7b, -4a-3b, -8a+4b)$

b. $\vec{u} = (1,3,2)$, $\vec{v} = (1,-1,1)$, $\vec{w} = (5,1,-4)$

Assume $\vec{u} = a\vec{v} + b\vec{w}$

$(1,3,2) = a(1,-1,1) + b(5,1,-4)$

① $1 = 1a + 5b$

② $3 = -1a + 1b$

③ $2 = 1a - 4b$

① + ② $4 = 6b$

$\frac{2}{3} = b$

Sub in ①

$1 = a + 5(\frac{2}{3})$

$-\frac{7}{3} = a$

check in ③

$2 \stackrel{?}{=} 1(-\frac{7}{3}) - 4(\frac{2}{3})$

$\frac{-7-8}{3}$

$\frac{-15}{3}$

-5

$2 \neq -5$ contradiction

$\therefore \vec{u} \neq a\vec{v} + b\vec{w}$

$\therefore \vec{u}, \vec{v}, \vec{w}$ are non-coplanar