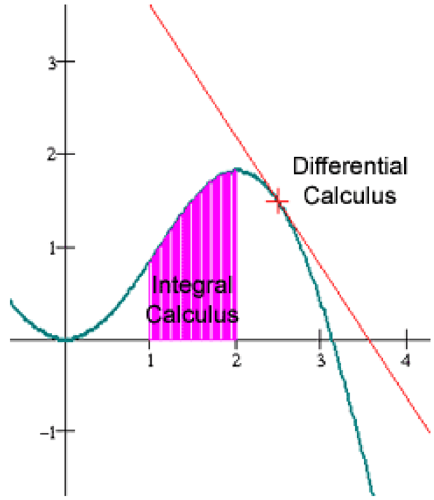
Introduction to Calculus Unit - Notes

Tentative TEST date_______________

Big idea/Learning Goals

Calculus is an entire branch of mathematics. Calculus is built on two major complementary ideas.



The first is differential calculus, which is concerned with the instantaneous rate of change. This can be illustrated by the slope of a tangent to a function's graph. The second is integral calculus, which studies the areas under a curve. These two processes act inversely to each other. The development of the mathematical methods of calculus has been credited to two great mathematicians; Sir Isaac Newton (1642 –1727) and Gottfried Wilhelm von Leibniz (1646 –1716). Although others as far back as 200B.C. had been working on solutions to these types of problems, Newton and Leibniz developed the process of differentiation and integration. Calculus allows you to find optimal solutions to mathematical expressions and is used in medicine, engineering, economics, computer science, business, physical sciences, statistics, and many more areas.

Corrections for the textbook answers:
1.2 #20 500

Success Criteria

☐ I understand the new topics for this unit if I can do the practice questions in the textbook/handouts

Date	pg	Topics	# of quest. done? You may be asked to show them
	2-4	Limit of a Function 1.4	
	5-7	Properties of Limits 1.5	
	8-10	Continuity 1.6	
	11-13	Slopes of Secants and Tangents with Limits 1.2 & 1.3	
		Review	

Reflect – previous TEST mark ___________, Overall mark now___________.

Limit of a Function

1. Sometimes you can't work something out directly ... but you can see what it should be as you get closer and closer!

   Why can't you find out the output of the function $f(x)=\frac{x^2-1}{x-1}$ at x=1?

2. Sketch the function $f(x)=\frac{x^2-1}{x-1}$

3. What is the output approaching as $x \to 1$?

4. The notation $\lim_{x\to a} f(x)=L$ is read as:

5. $\lim_{x\to a} f(x)=L$ means that it EXISTS if:

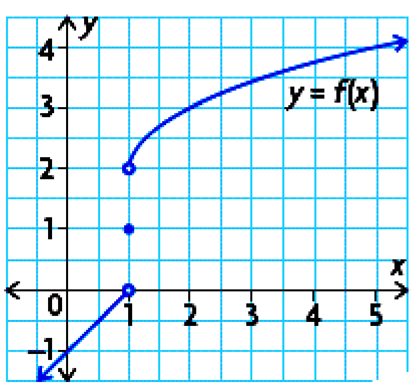
6. Limit will not exist if there is:

jump in the graph

one sided graph

approaching ∞ because of a VA

7. Find the following limits.



a. $\lim_{x\to 5^-} f(x)$

b. $\lim_{x\to 5^+} f(x)$

c. $\lim_{x\to 1^-} f(x)$

d. $\lim_{x\to 1^+} f(x)$

e. $\lim_{x\to 5} f(x)$

f. $\lim_{x\to 1} f(x)$

8. Determine if the limit exists both algebraically and graphically

Algebraic *informal way by trying #s from either side*	**Graphical**
a. $f(x)=\sqrt{4-x}$ limit at $x=4$	

Algebraic *informal way by trying #s on either side*	**Graphical**
b. $f(x)=\dfrac{1}{1-x}$ limit at $x=1$	

Algebraic *formal way by subbing in*	**Graphical**
c. $f(x)=\begin{cases} x^2 & \text{if } x<1 \\ -2 & \text{if } x=1 \\ -x+3 & \text{if } x>1 \end{cases}$ limit at $x=1$	

9. Discuss proper way of recording solutions when subbing in the value limit approaches

Determine if the limit exists both algebraically and graphically

Algebraic *formal way by subbing in*	**Graphical**
d. $f(x)=\dfrac{4-x^2}{x-2}$ limit at $x=2$	

Algebraic *informal way*	**Graphical**
e. $f(x)=\dfrac{1}{x}$ limit at $x\to\infty$	

Algebraic	**Graphical**
f. $f(x)=\dfrac{1}{x+2}$ limit at $x=-3$	

Properties of Limits

For any real number a, suppose that f and g both have limits that exist at $x = a$. **THINGS to try**

1. $\lim_{x \to a} k = k$, for any constant k
2. $\lim_{x \to a} x = a$
3. $\lim_{x \to a} [f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$
4. $\lim_{x \to a} [cf(x)] = c[\lim_{x \to a} f(x)]$, for any constant c
5. $\lim_{x \to a} [f(x)g(x)] = [\lim_{x \to a} f(x)][\lim_{x \to a} g(x)]$
6. $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$, provided that $\lim_{x \to a} g(x) \neq 0$
7. $\lim_{x \to a} [f(x)]^n = [\lim_{x \to a} f(x)]^n$, for any rational number n

Evaluate the limits

1. $\lim_{x \to 5} \sqrt{\frac{x^2}{x-1}}$

(use the properties above to explain why you can just substitute the value in if it is not indeterminate form.)

2. $\lim_{x\to 2}\frac{|2x-4|}{x-2}$

3. $\lim_{x\to 3}\frac{\frac{1}{3}-\frac{1}{x}}{x-3}$

4. $\lim_{x\to 3}\sqrt{9-x^2}$

5. $\lim_{x\to -1}\frac{x}{x^2+x}$

6. $\lim_{x\to 0}\frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1}$ try to use a substitution that will get rid of both roots

7. $\lim_{x\to\infty}\frac{4-3x^3+2x}{8-x^2+7x^3}$

8. $\lim_{x\to 0}\frac{\sqrt{4+x}-2}{x}$

9. $\lim_{x\to -3}\frac{(1+x)^2-4}{x+3}$

10. $\lim_{x\to -\infty}\frac{1-x}{5+x^2}-2$

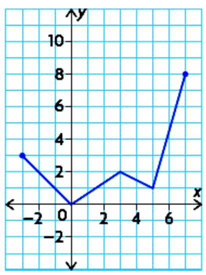
11. $\lim_{x\to 9}\frac{2x}{x^2-9}$

12. $\lim_{x\to 8}\frac{2-\sqrt[3]{x}}{8-x}$

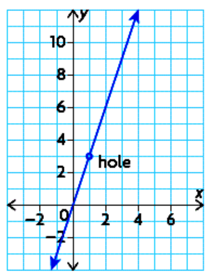
13. $\lim_{x\to 4}\frac{4+x}{16-x^4}$

Continuity

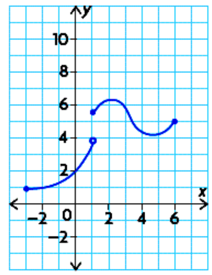
A. Continuous for all values of the domain



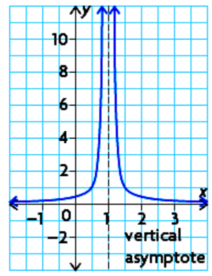
B. Discontinuous at $x = 1$ (point discontinuity)



C. Discontinuous at $x = 1$ (jump discontinuity)



D. Discontinuous at $x = 1$ (infinite discontinuity)



1. State the 3 conditions you must meet for the function to be continuous.

2. Determine if the function is continuous, if not, identify where the discontinuity is. Also show a graphical representation.

a. $f(x) = x^3 - x$

b. $g(x) = \begin{cases} x & \text{if } x \le -4 \\ 5 & \text{if } -4 < x \le 3 \\ x^2 - 4 & \text{if } x > 3 \end{cases}$

Determine if the function is continuous, if not, identify where the discontinuity is. Also show a graphical representation.

c. $h(x)=\dfrac{2x+2}{3x-12}$

d. $i(x)=\dfrac{x^2-9}{x+3}$

e. $f(x)=\dfrac{1}{x+2}$

f. $f(x)=\dfrac{1}{x^2+4}$

3. Find the values of each constant that would make this function continuous.

a.

$$f(x)=\begin{cases} -2x+a & \text{if } x\le -1 \\ x^2+b & \text{if } -1<x\le 2 \\ \dfrac{1}{x}+c & \text{if } 2\le x \end{cases}$$

b.

$$j(x)=\begin{cases} 2x+a & \text{if } x\le -1 \\ 22 & \text{if } -1<x\le 3 \\ -bx^2+31 & \text{if } 3\le x\le 5 \\ \sqrt{cx}+b & \text{if } 5\le x \end{cases}$$

Slope of Secants and Tangents with Limits

1. What is the slope equation for straight lines?

2. How is rate of change of a straight line different from rate of change of a curve?

3. Recall from advanced functions,
   a. the picture and formula of the **average** rate of change of a curve.
   b. the picture and formula of the **instantaneous** rate of change of a curve.

4. Determine the slope of the tangent for rational function $f(x) = \frac{5+x}{x^2}$ at $x = 5$

5. Determine the slope of the tangent for square root function $g(x)=\sqrt{x-4}$ at $x=5$

6. Determine the equation of the line that is perpendicular to the tangent to $y=x^5$ at $x=-2$, and which passes through the tangent point.

1. An outdoor hot tub holds 2700L of water. When a valve at the bottom of the tub is opened, it takes 3h for the water to completely drain. The volume of water in the tub is modelled by the function $V(t)=\frac{1}{12}(180-t)^2$, where *V* is the volume of water in the hot tub, in litres, and *t* is the time, in minutes, that the valve is open.
   a. Determine the average rate of change of volume from 2 min to 10min.
   b. Determine the instantaneous rate of change of the volume of water at 60min. (Use the second version of the definition for rate here)