Intersection - Notes

Tentative TEST date_______________

Big idea/Learning Goals

In this unit, we will introduce perhaps the most important idea associated with vectors – the solution of systems of equations. In grade 10, the solution of systems of equations was introduced in situations dealing with two equations in two unknowns. Geometrically, the solution of two equations in two unknowns is the point of intersection between two lines on the *xy*-plane. In this unit, we are going to extend these ideas and consider systems of equations in R^3 and interpret their meaning. We will be working with systems of up to three equations in three unknowns, and we will demonstrate techniques for solving these systems. At university level you will learn how to use matrices to solve the many equations with less amount of writing than we will do in this unit.

Corrections for the textbook answers:

Review #9a non coplanar vectors, thus meet in a point $(\frac{42}{83}, \frac{-34}{83}, \frac{18}{83})$

Success Criteria

☐ I understand the new topics for this unit if I can do the practice questions in the textbook/handouts

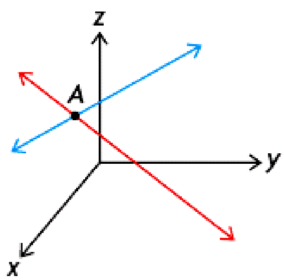
Date	pg	Topics	# of quest. done? You may be asked to show them
	2-4	2 Lines & Line with a Plane 9.1 and 9.2	
	5-9	3 Planes & 2 Planes – 2 days 9.3 and 9.4	
	10-11	Distances 9.5 and 9.6	
		Review	

Reflect – previous TEST mark ___________, Overall mark now___________.

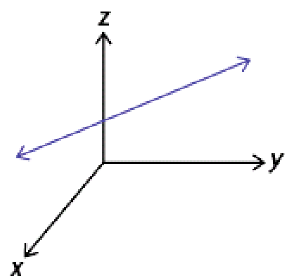
2 Lines

Intersecting Lines

Case 1: Intersecting Lines at a Point

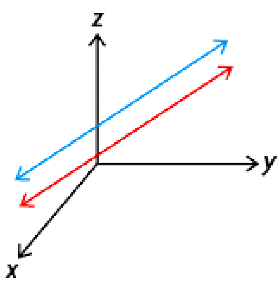


Case 2: Coincident Lines

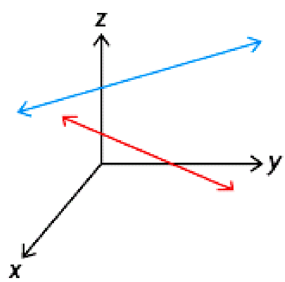


Non-intersecting Lines

Case 3: Parallel Lines



Case 4: Skew Lines



1. Find the intersection of

line 1 $\begin{cases} x = -1 + 3t \\ y = 1 + 4t \\ z = -2t \end{cases}$ and **line 2** $\begin{cases} x = -1 + 2s \\ y = 3s \\ z = -7 + s \end{cases}$

2. Find the intersection of

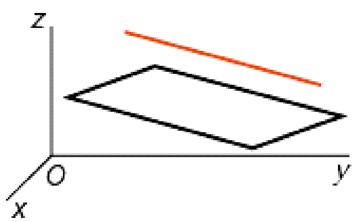
line 1 $\vec{r} = (2, 1, 0) + t(1, -1, 1)$
line 2 $\vec{r} = (3, 0, -1) + s(2, 3, -1)$

3. Find the intersection of each pair of lines. If they do not meet explain why.

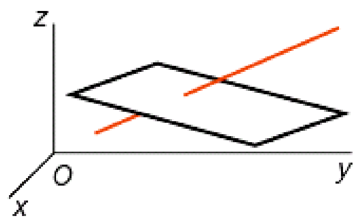
a. $(x, y, z) = (1 + t, 2 + t, -t)$
$(x, y, z) = (3 - 2u, 4 - 2u, -1 + 2u)$

b. $x = 1 + t, y = 1 + 2t, z = 1 - 3t$
$x = 3 - 2u, y = 5 - 4u, z = -5 + 6u$

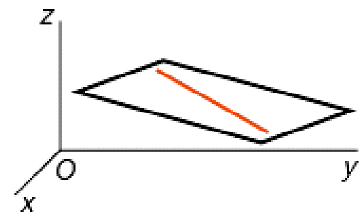
Line with a Plane



line is parallel to the plane



line intersects the plane



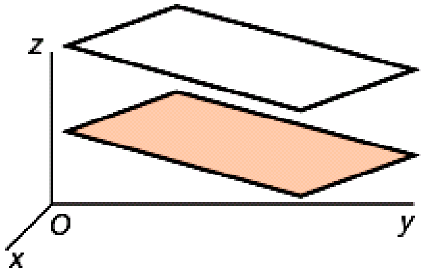
line lies in the plane

1. Find the intersection of the line with parametric equations $x = 1 + 2t$, $y = -6 + 3t$, $z = -5 + 2t$ and the plane whose scalar equation is $4x - 2y + z - 19 = 0$.

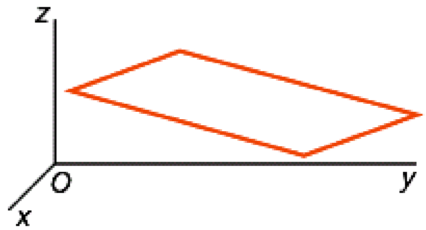
2. Find the intersection of the line $x = 2t$, $y = 1 - t$, $z = -4 + t$ and the plane $x + 4y + 2z - 4 = 0$.

3. Find the intersection of the line $x = -4 + 3t$, $y = 0$, $z = t$ and the plane $x - 2y - 3z + 4 = 0$.

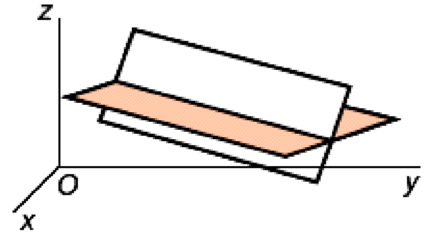
2 Planes



planes are parallel



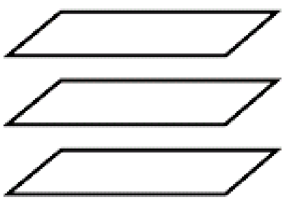
planes are coincident



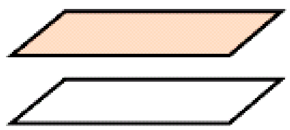
planes intersect in a line

3 Planes

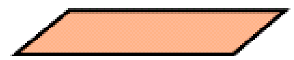
When the normals of all three are parallel, the possibilities are



3 planes are parallel and distinct; no intersection

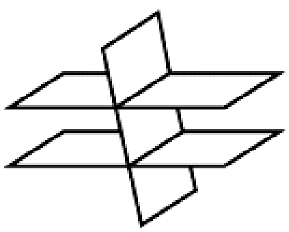


2 planes are coincident, the other parallel; no intersection

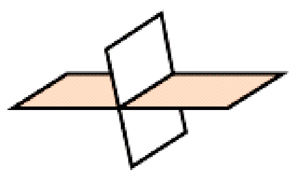


3 planes are coincident;. intersection: a plane

When only two of the normals of the planes are parallel, the possibilities are

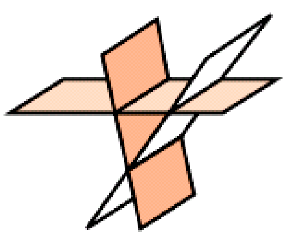


two planes are parallel and distinct, the other crossing; no common intersection

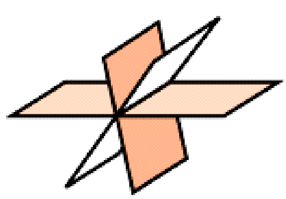


two planes are coincident; the other crossing; intersection: a line

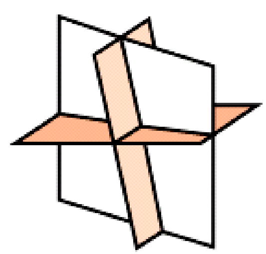
When none of the normals are parallel, the possibilities are



normals coplanar; no intersection



normals coplanar; intersection: a line



normals are not parallel and non-coplanar; intersection: a point

System of Equations with 3 Planes

1. Solve each system and interpret what type of geometrical picture the system represents.

a.

$$\begin{aligned} x + y + 2z &= 2 \\ x - y - 2z &= 5 \\ 3x + 3y + 6z &= 5 \end{aligned}$$

b.

$$\begin{aligned} x + 2y + z &= 12 \\ 2x - y + z &= 5 \\ 3x + y - 2z &= 1 \end{aligned}$$

c.

$$\begin{aligned} -2x + 4y + 6z &= -2 \\ 4x - 8y - 12z &= 4 \\ x - 2y - 3z &= 1 \end{aligned}$$

d.

$$\begin{aligned} x + y - z &= 5 \\ 2x + 2y - 4z &= 6 \\ x + y - 2z &= 3 \end{aligned}$$

e.

$$\begin{aligned} x - 3y - 2z &= 9 \\ x + 11y + 5z &= -5 \\ 2x + 8y + 3z &= 4 \end{aligned}$$

f.

$$\begin{aligned} x - y + z &= 1 \\ 2x + y - z &= 11 \\ 3x + y + 2z &= 12 \end{aligned}$$

g.

$$\begin{aligned} x + y + 2z &= 6 \\ x - y - 4z &= -2 \\ 3x + 5y + 12z &= 27 \end{aligned}$$

System of Equations with 2 Planes

1. Solve each system and interpret what type of geometrical picture the system represents.

a.

$$x + 3y - z - 4 = 0$$
$$2x + 6y - 2z - 8 = 0$$

b.

$$5x - 2y + 2z + 1 = 0$$
$$5x - 2y + 2z - 3 = 0$$

c.

$$x + y - 3z = 4$$
$$x + 2y - z = 1$$

d.

$\vec{r} = (1, 1, 1) + p(0, 0, 1) + q(0, 1, 0)$
and $\vec{r} = (0, 0, 0) + s(0, 0, 1) + t(1, 0, 0)$

 12CV Date:____________ Name: ________________________

Distances

Shortest Distance
from point $Q(x_1,y_1)$ to ***line in*** R^2
$Ax+By+C=0$

Shortest Distance
from point Q to ***line in*** R^3
through P with direction vector $\vec{m}$

Shortest Distance
from point $Q(x_1,y_1,z_1)$
to ***plane in*** R^3 *$Ax+By+Cz+D=0$*

1. a. Determine the distance between the two planes π_1: $2x - y + 2z + 4 = 0$ and π_2: $2x - y + 2z + 16 = 0$.
   b. Determine the equation of the plane that is equidistant from π_1 and π_2.

2. **Calculate the distance between the two parallel lines $5x - 12y + 60 = 0$ and $5x - 12y - 60 = 0$.**

3. Determine the distance between point (–2, 1, 0) and line $\vec{r} = (0,1,0) + t(1,1,2), t \in \mathrm{R}$

4. **Determine the distance between L_1: $\vec{r} = (-2, 1, 0) + s(1, -1, 1)$, $s \in \mathbf{R}$, and L_2: $\vec{r} = (0, 1, 0) + t(1, 1, 2)$, $t \in \mathbf{R}$.**